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NUMERICAL INVESTIGATION OF NONLOCAL THEORY OF ELASTICITY MODELS FOR 1D LINEAR WAVE PROCESSES

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Abstract: A variety approaches for continuous approximation of a discrete medium are studied using an example of 1D chain of linear oscillators.

1. Introduction

It is well-known that equations governing mechanical behavior of continuous media omit their discrete properties. On the other hand, recent results obtained particularly in the field of high technology materials point out an importance of micro-structural effects exhibited by real material behavior. The discrete-types phenomena in question play a key role during modeling of various crystal, polymer, and composite materials, in fracture and damage mechanics [1-5]. The observed dispersion of waves also in granular materials [3] belongs to the examples of the micro-structural important role influencing material behavior mentioned earlier. The micro-structured material property should be taken into account while analyzing a local material deformation. Finally, the problem studied in this paper also refers to modeling of the nano-effects.

In order to construct the continuous models in physics the statistical modeling is applied, mainly addressing the statistical averaging approach. However, development of this approach has been partially slowed down owing to the unexpected mathematical difficulties that occurred. The same observation can be made in the case of so-called G-homogenization. On the other hand, in many cases the phenomenological approach is used. Namely, some of additional terms are introduced into either energy functional or into the governing relations, and a structure as well as property of the introduced terms is a priori known. In what follows this way of investigation will be further used in our research.

In addition, we study using numerical approaches the applicability of a variety of approximation models, which qualitatively consist of observed in reality discrete type behaviors [6-9] on a basis of the 1D linear wave processes propagation.

2. Discrete model

Let us study wave propagation in the discrete chain of linked masses shown in Figure 1. Assume that at time instant t = 0 a unit force acts on a mass with number 0 in direction of the axis x. Then dynamics governing equations regarding displacements y_k can be obtained in the following way [10] (we take m = 1, c = 1 for simplicity of our considerations):

$$y_{jtt} = y_{j+1} - 2y_j + y_{j-1}, \tag{1}$$

$$y_{ott} = y_1 - 2y_0 + y_{-1} + 1. \tag{2}$$

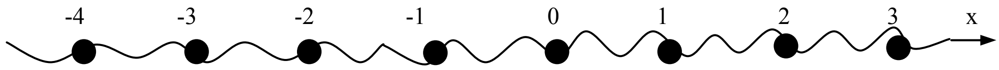

Figure 1. Infinite chain of masses.

In order to solve equations (1),(2) Fourier transformation is applied. Namely, multiplying left and right hand sides of equations (1),(2) by $\exp(iqx)$ and adding them one gets [10]

$$\frac{d^2U}{dt^2} + 4\sin^2\left(\frac{q}{2}\right)U = 1, \quad U = \sum_{j=-\infty}^{\infty} y_j \exp(iqx). \tag{3}$$

Solving equation (3) and applying inversed Fourier transformation one gets relation governing wave velocity propagation of the form

$$\frac{y_x(t)}{dt} = \frac{1}{\pi}\int_0^{\pi/2} \frac{sin(2t\sin z)}{\sin z}\cos(2xz)dz = \frac{1}{\pi}a. \tag{4}$$

3. The classical continuous approximation

Applying the classical continuous approximation instead of system (1), (2), the following wave equation with the Dirac delta $\delta(x)$ right hand-side is obtained in the following form:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \delta(x). \tag{5}$$

Now, applying the Fourier transformation regarding x and the Laplace transformation regarding t, the following velocity governing equation is derived

$$\frac{\partial u}{\partial t} = 0.5H(t - |x|), \tag{6}$$

where $H(...)$ denotes the Heaviside function. As it has been mentioned in reference [10], a wave propagated in a discrete medium differs from that propagated in a continual medium via: (i) occurrence of vibrations being successively damped at x = const; (ii) infinite velocity of perturbations propagation (owing to assumption of a rapid approaching masses interaction during their draw near), occurrence and vanish of the so-called quasi-front domain, where although the stresses increase relatively sharp, but in a continuous way (velocities and deformations exponentially decay while the quasi-front $|x|-t$ increases finally becoming negligibly small).

4. Intermediate continuous model

First of all, we examine the intermediate continuous model [6] governed by the following equation

$$\frac{\partial^2 u}{\partial t^2}-\left(\frac{\partial^2}{\partial x^2}+\frac{1}{12}\frac{\partial^4}{\partial x^4}+\frac{1}{360}\frac{\partial^6}{\partial x^6}\right)u=\frac{\sin(\pi x)}{\pi x}. \tag{7}$$

Applying both Fourier (regarding *x*) and Laplace (regarding t) transformations, the following formula governing wave velocity propagation is obtained

$$\frac{\partial u}{\partial t}=\frac{1}{\pi}\int_0^{\pi/2}\frac{sin(2tz\sqrt{1-(1/12)z^2+(1/380)z^4})}{z\sqrt{1-(1/12)z^2+(1/380)z^4}}\cos(2xz)dz==\frac{1}{\pi}a_1. \tag{8}$$

5. Quasi-continuous approximation

In reference [7] the following improved model, further referred as the quasi-continuous approximation, has been proposed

$$(1+b\frac{\partial^2}{\partial x^2})\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=(1+b\frac{\partial^2}{\partial x^2})\frac{\sin(\pi x)}{\pi x}, \tag{9}$$

where $b=1/12$.

The associated wave velocity propagation follows

$$\frac{\partial u(x,t)}{\partial t}=\frac{1}{\pi}\int_0^{\pi/2}\frac{\sqrt{1+bz^2}\,sin(2t\frac{z}{\sqrt{1+bz^2}}\sin z)}{z}\cos(2xz)dz=\frac{1}{\pi}a_2. \tag{10}$$

6. Improved quasi-continuous approximation

In references [1, 8, 9] the following continuous model is proposed

$$(1+b_1\frac{\partial^2}{\partial x^2})\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=(1+b_1\frac{\partial^2}{\partial x^2})\frac{\sin(\pi x)}{\pi x}, \tag{11}$$

where $b_1=1-4/\pi^2$.

In this case the associated wave velocity propagation reads

$$\frac{\partial u(x,t)}{\partial t} = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sqrt{1+b_1 z^2}\, sin(2t \frac{z}{\sqrt{1+b_1 z^2}} \sin z)}{z} \cos(2xz) dz = \frac{1}{\pi} a_3, \tag{12}$$

where $b_1 = 1 - 4/\pi^2$.

7. Numerical results and conclusions

Results of computations of a, a_1, a_2, a_3 are reported in Figures 2 (a-c) and 3 (a-c).

One may conclude that models (7) and (9) give the same results. On the other hand, model (11) exhibits the best qualitative coincidence.

References

1. A. Askar, *Lattice Dynamical Foundations of Continuum Theories. Elasticity, Piezoelectricity, Viscoelasticity, Plasticity.* World Scientific, Singapore, 1985.
2. A.C. Eringen, *Nonlocal Continuum Field Theories.* Springer, New York, 2002.
3. V.I. Erofeyev, *Wave Processes in Solids with Microstrucutre.* World Scientific, Singapore, 2003.
4. A.M. Kosevich, *The Crystal Lattice: Phonons, Solitons, Dislocations, Superlattices.* Wiley-VCH, Berlin, New York, 2005.
5. I.A. Kunin, *Elastic Media with Microstructure. One-dimensional Models.* Springer, Berlin, New York, 1982.
6. A.M. Filimonov, Continuous approximations of difference operators. *Journal of Difference Equations and Applications,* 2(4), 1996, 411- 422.
7. Ph. Rosenau, Hamiltonian dynamics of dense chains and lattices: or how to correct the continuum. *Physics Letters A,* 311(5), 2003, 39-52.
8. I.V. Andrianov, J. Awrejcewicz, On the average continuous representation of an elastic discrete medium. *Journal of Sound and Vibration,* 264, 2003, 1187-1194.
9. I.V. Andrianov, J. Awrejcewicz, Continuous models for 1D discrete media valid for higher-frequency domain. *Physical Letters A,* 345(1-3), 2005, 55-62.
10. L.I. Slepyan, *Nonstationary Elastic Waves.* Sudostroyenie, Leningrad, 1972.

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