A NOVEL DRY FRICTION MODELING AND ITS IMPACT ON DIFFERENTIAL EQUATIONS COMPUTATION AND LYAPUNOV EXPONENTS ESTIMATION

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1. Introduction

Computing Lyapunov exponents is one of the most known and most important identification tools in nonlinear dynamics allowing for identification and investigation of chaotic dynamics in nonlinear dynamical systems. Although the Lyapunov's theory for estimation chaotic dynamics was used by Oseledec as early as forty years ego and there are numerous works in the scientific literature dedicated to chaotic vibrations, in general there are not many effective methods available for the Lyapunov exponents estimation. The classical Lyapunov exponents definition can be used with success only in systems governed by differential equations with smooth right-hand sides. On the other hand, methods commonly used for the Lyapunov exponents computation require smooth vector fields as a necessary condition to be satisfied. There are many discontinuous systems arising due to physical discontinuities such as dry friction, impact, and backlash in mechanical systems or diode elements in electrical circuits. For these cases another numerical/analytical approaches may be also applied, i.e. phase portraits, bifurcation diagrams, the Melnikov-type methods, Fourier spectra, as well as various methods experimentally and numerically oriented.

2. A novel dry friction model

In this work we are focused on the systems subject to action of dry friction. The mentioned dry friction belongs to one of the most known phenomena in mechanical systems and its proper mathematical modeling is not an easy task. An extensive literature review on applied friction models can be found in the works [1, 4, 5, 6, 7] and others.

In our work an alternative (novel) numerical algorithm (dry friction modeling) devoted to solving equations of motion and Lyapunow exponents estimation in systems with dry friction is proposed and presented. It takes into account some elements of the known "switch model". The friction force F_{fr} is defined as a function of the relative velocity ω_r of sliding surfaces and externally applied force F_{ex} (all forces acting in the system excluding friction force with maximum static friction force F_s). The proposed continuous friction model has the following form

$$F_{fr}(\omega_r, F_{ex}) = \begin{cases} F(|\omega_r|)\operatorname{sgn}\omega_r, & V_1 \\ F_s \operatorname{sgn} F_{ex}, & V_2 \\ (2A_3 - 1)F_s \operatorname{sgn}\omega_r, & V_3 \\ A_3(-F_{ex} + F_s \operatorname{sgn}\omega_r) + F_{ex}, & V_4 \end{cases}, \qquad A_3 = \frac{\omega_r^2}{\varepsilon^2}\left(3 - 2\frac{|\omega_r|}{\varepsilon}\right), \qquad (1)$$

M. Zakrzhevsky (ed.), The International Symposium on Rare Attractors and Rare Phenomena in Nonlinear Dynamics, 1-3.

where the space $F_{ex} - \omega_r$ is divided into four regions as follows

$V_1 : |\omega_r| > \varepsilon$,

$V_2 : [(0 \le \omega_r \le \varepsilon) \cap (F_{ex} > F_s)] \cup [(-\varepsilon \le \omega_r \le 0) \cap (F_{ex} < -F_s)]$,

$V_3 : [(0 < \omega_r \le \varepsilon) \cap (F_{ex} < -F_s)] \cup [(-\varepsilon \le \omega_r < 0) \cap (F_{ex} > F_s)]$,

$V_4 : (|\omega_r| \le \varepsilon) \cap (|F_{ex}| \le F_s)$.

This model of friction has been already used by the authors in studies [2, 3, 8].
In the classical friction models based on the Coulomb model a friction force is non-continuous function of relative velocity and therefore the methods commonly used to compute the Lyapunov exponents cannot be applied. Continuous friction model (as proposed in this paper) does not possess this disadvantage and can be used during analysis of the systems, where the Lyapunov exponents are computed by standard procedures [6].

3. Computational Methods and Results

Advantages of the proposed algorithm are illustrated and discussed using several one or more degree-of-freedom models with dry friction and external excitation, which exhibit both regular and chaotic dynamics. Although there are numerous works in the scientific literature dedicated to stick-slip vibrations, a rigid body lying on a belt which moves at non-constant velocity and Lyapunov exponents estimation in these systems are less investigated.
The so far mentioned models are described by dimensionless differential equations. These equations are rewritten as the system of first order n ordinary differential equations. While computing Lyapunov exponents, besides the following n equations also n systems of equations with respect to perturbations are solved. Finally, $n(n+1)$ equations re solved via the fourth order Runge-Kutta-Fehlberg (RKF 45) method with varied time step and the Gramm-Schmidt ortonormalization technique.
The so far mentioned models described by dimensionless differential equations are solved. Nonlinear dynamics is monitored via standard time histories in the system's phase space, bifurcation diagrams with the different control parameters applied as well as the Lyapunov exponents.

4. Conclusions

The obtained results exhibit advantages of the proposed algorithm in comparison to the algorithm using smoothing approaches. We have obtained almost exact solutions (high precision numerical computations) even for extremely large steepness parameter. In addition, our algorithm is less expensive and indicate better numerical accuracy and efficiency from the computational point of view than the algorithm associated with various smoothing approaches. One of the most important advantages of our novel model relies on direct application of the standard numerical procedures devoted for solving nonlinear differential equations including computation and monitoring of the Lyapunov exponents.

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