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Modeling and Simulation of Real Triple Pendulum System

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Abstract: The dynamics of a plane and periodically forced tree coupled links is investigated both experimentally and numerically. Mathematical modeling includes details taking into account some characteristic features (real characteristics of joints built by the use of roller bearings) as well as some imperfections of the real system. Parameters of the model are obtained by estimation from the experimental data. Then the experimental and numerical analysis of the system is performed.

Keywords: triple pendulum, mathematical modeling, identification, experiment.

1 Introduction

In February, 2005, in the Department of Automatics and Biomechanics, the experimental rig of triple physical pendulum was finished and activated. In order to have more deep insight into dynamics of the real pendulum, the corresponding mathematical model is required. In the work [1] the suitable mathematical modeling and numerical analysis have been performed, where the viscous damping in the pendulum joints (constructed by the use of rolling bearings) has been assumed. In the next step [2], we have also taken into account the dry friction in the joints with many details and variants. The aim of this paper is to present the mathematical model of friction taking into account only essential details.

2 Experimental Rig

The experimental rig (see Fig. 1) of the triple physical pendulum consists of the following subsystems: pendulum, driving subsystem and the measurement subsystem. It is assumed that the pendulum is moving in a plane. The links (1, 2, 3) are suspended on the frame (4) and joined by the use of radial and axial needle bearings. The first link is forced by a special direct-current motor of our own construction with optical commutation consisting of two stators (6) and two rotors (5). The construction ensures avoiding the skewing of the structure and forming the forces and moments in planes different that the plane of the assumed pendulum motion. On the other hand the construction allows the full rotations of all the links of the pendulum.

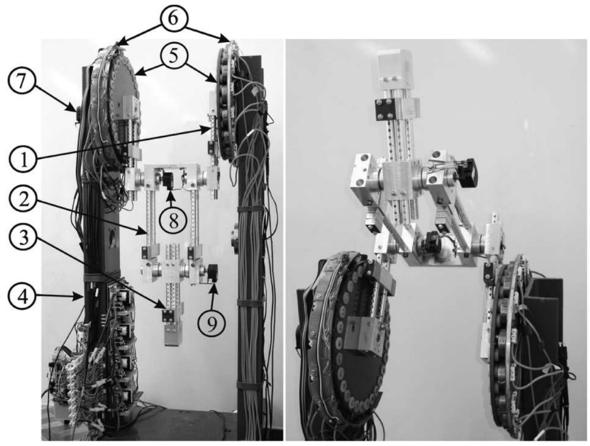

Figure 1. Experimental rig: 1, 2, 3 — links; 4 — stand; 5 — rotors; 6 — stators; 7, 8, 9 — rotational potentiometers

3 Mathematical Model

Details on physical modeling, i.e. idealized physical concept of real pendulum presented in Fig. 1, can be found in works [1, 2]. The system is idealized since it is assumed that it is an ideally plane system of coupled links, moving in the vacuum with the assumed model of friction in joints. The system is governed by the following set of differential equations:

$$\begin{bmatrix} B_1 & N_{12}\cos(\psi_1-\psi_2) & N_{13}\cos(\psi_1-\psi_3) \\ N_{12}\cos(\psi_1-\psi_2) & B_2 & N_{23}\cos(\psi_1-\psi_2) \\ N_{13}\cos(\psi_1-\psi_3) & N_{23}\cos(\psi_1-\psi_2) & B_3 \end{bmatrix} \begin{Bmatrix} \ddot{\psi}_1 \\ \ddot{\psi}_2 \\ \ddot{\psi}_3 \end{Bmatrix} + \\ \begin{bmatrix} 0 & N_{12}\sin(\psi_1-\psi_2) & N_{13}\sin(\psi_1-\psi_3) \\ -N_{12}\sin(\psi_1-\psi_2) & 0 & N_{23}\sin(\psi_1-\psi_2) \\ -N_{13}\sin(\psi_1-\psi_3) & -N_{23}\sin(\psi_1-\psi_2) & 0 \end{bmatrix} \begin{Bmatrix} \dot{\psi}_1^2 \\ \dot{\psi}_2^2 \\ \dot{\psi}_3^2 \end{Bmatrix} + \tag{1}$$

$$
+\left\{\begin{matrix} M_{R1}\left(\dot{\psi}_1\right)-M_{R2}\left(\dot{\psi}_1,\dot{\psi}_2\right) \\ M_{R2}\left(\dot{\psi}_1,\dot{\psi}_2\right)-M_{R3}\left(\dot{\psi}_2,\dot{\psi}_3\right) \\ M_{R3}\left(\dot{\psi}_2,\dot{\psi}_3\right) \end{matrix}\right\}+\left\{\begin{matrix} M_1\sin\psi_1 \\ M_2\sin\psi_2 \\ M_2N_{13}/N_{12}\sin\psi_3 \end{matrix}\right\}=\left\{\begin{matrix} M_e\left(t\right) \\ 0 \\ 0 \end{matrix}\right\}
$$

where the pendulum position is described by the use of three angles ψ_i $(i=1,2,3)$ and where

$$
\begin{aligned}
M_{R1} &= T_1\frac{2}{\pi}\arctan\left(\varepsilon\dot{\psi}_1\right)+2c\dot{\psi}_1 \\
M_{R2} &= T_2\frac{2}{\pi}\arctan\left(\varepsilon\left(\dot{\psi}_2-\dot{\psi}_1\right)\right)+c\left(\dot{\psi}_2-\dot{\psi}_1\right) \\
M_{R3} &= T_3\frac{2}{\pi}\arctan\left(\varepsilon\left(\dot{\psi}_3-\dot{\psi}_2\right)\right)+c\left(\dot{\psi}_3-\dot{\psi}_2\right),
\end{aligned} \tag{2}
$$

are the moments of resistance in the corresponding joints and consisting of two parts: dry friction and viscous damping. The dry friction moment does not depend on the loading of the corresponding bearing and the sign function is approximated by the arctan function. The parameter c is the damping coefficient.

The model parameters are estimated by the global minimum searching of the criterion-function of the model and real system matching.. A minimum is searched applying the simplex method. The model parameters estimated can be found in the works [1, 2]. Here we only address, that we have also treated the parameter as an identified parameter, finally obtaining its optimal value as 6.77 s.

4 Numerical and Experimental Study

Figure 2 shows results of investigation of the forcing frequency region 0.13-0.14 Hz. It is an example that the developed model with their parameters can predict real pendulum dynamics exhibited also for forcing frequencies f outside the region 0.2-1.1 Hz (containing all the periodic solutions taken to the identification process). More details can be found in works [1,2].

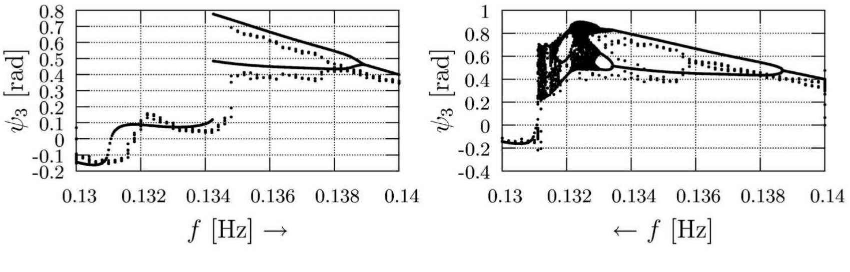

Figure 2. Bifurcation diagrams exhibited by experiment and model with the parameter f growing (→) and decreasing (←)

5 Concluding remarks

Good agreement between both numerical simulation results and experimental measurements have been obtained and presented. It is interesting that we have obtained very low value of the parameter as an optimal one. We are not able to give a physical interpretation of that at this moment. But since it is important to have a model giving results close to experimental observations, we can accept even some artificial improvements of the model having only functional role, no physical sense, particularly if they speed up the simulation process.

Acknowledgments

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References

[1] Awrejcewicz J., Kudra G., Wasilewski G. 2005: Experimental and numerical investigation of chaotic zones exhibited by the triple physical pendulum. *Proc. of the 8th Conference on Dynamical Systems — Theory and Applications*. Łódź, Poland. pp. 183–188.

[2] Awrejcewicz J., Supel B., Kudra G., Wasilewski G., Olejnik P. 2008: Numerical and experimental study of regular and chaotic motion of triple physical pendulum. *International Journal of Bifurcation and Chaos*, **18**(10) – to appear.

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