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DYNAMICS ANALYSIS OF A ROTOR MAGNETO-HYDRODYNAMIC BEARING SYSTEM BY MEANS OF THE MULTIPLE SCALES METHOD

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Abstract: The method of multiple scales [1] is used to analyze a dynamics of high-speed rotor symmetrically supported on the magneto-hydrodynamic bearing (MHDB). The governing equations of the 2-dof system are reduced to dimensionless. The right hand sides of the equations have been expanded in the Taylor series in the equilibrium position neighbourhood. The linear and quadratic terms have been kept. The nonresonant and resonant cases are considered. Accordingly to the last case the system is in conditions of the primary and internal resonances. Next, it is shown that hysteretic properties in the system can be taken into consideration by means of Bouc-Wen model.

1. Introduction

The magnetic, magneto-hydrodynamic and also piezoelectric bearings are used in many mechanical engineering applications in order to support a high-speed rotor, provide vibration control, lower rotating friction losses and potentially avoid flutter instability. There are a lot of publications devoted to the dynamics analysis and control of a rotor supported on various bearings systems. The conditions for active close-/open-loop control of a rigid rotor supported on hydrodynamic bearings and subjected to harmonic kinematical excitation are presented in [2, 3]. In [5, 6] a rotor–active magnetic bearings systems with time-varying stiffness are considered. Using the method of multiple scales a governing nonlinear equation of motion for the rotor-AMB system with 1-dof is transformed to the averaged equation and then the bifurcation theory and the method of detection function are used to analyze the bifurcations of multiple limit cycles of the averaged equation. In the present paper the 2-dof motion of the rotor supported on AMHDB system is analysed in the nonresonant case and under conditions of primary and internal resonances.

2. Equations of motion

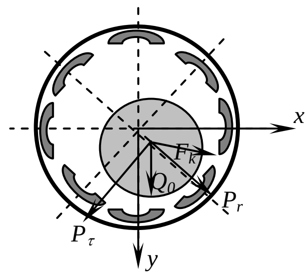

Fig. 1. The cross-section diagram of the rotor symmetrically supported on the magneto-hydrodynamic bearing

A uniform symmetric rigid rotor (Fig. 1) is supported by magneto-hydrodynamic bearing system. The 4-pole legs are symmetrically placed in the stator. F_k is electromagnetic control force produced by the k-th opposed pair of electromagnet coils, Q_0 is the vertical rotor load identified with its weight, (P_r ,P_τ) are the radial and tangential components of the dynamic oil-film action.

Equations of motion for the rotor are represented in the following form [2, 3, 4]

$$m^* \ddot{x}^* = P_r^*\left(\rho,\dot{\rho}^*,\dot{\phi}^*\right)\cos\phi - P_\tau^*\left(\rho,\dot{\phi}^*\right)\sin\phi + \sum_{k=1}^{K} F_k^* \cos\theta_k + Q_x^*(t)$$
$$m^* \ddot{y}^* = P_r^*\left(\rho,\dot{\rho}^*,\dot{\phi}^*\right)\sin\phi + P_\tau^*\left(\rho,\dot{\phi}^*\right)\cos\phi + \sum_{k=1}^{K} F_k^* \sin\theta_k + Q_0^* + Q_y^*(t)$$

$$P_r^*\left(\rho,\dot{\rho}^*,\dot{\phi}^*\right) = -2C^*\left\{\frac{\rho^2\left(\omega^* - 2\dot{\phi}^*\right)}{p(\rho)q(\rho)} + \frac{\rho\dot{\rho}^*}{p(\rho)} + \frac{2\dot{\rho}^*}{\sqrt{p(\rho)}}arctg\sqrt{\frac{1+\rho}{1-\rho}}\right\},\ P_\tau^*\left(\rho,\dot{\phi}^*\right) = \pi C^*\frac{\rho\left(\omega^* - 2\dot{\phi}^*\right)}{q(\rho)\sqrt{p(\rho)}}.$$

Here $Q_x^*(t)$, $Q_y^*(t)$ are external excitation (we are supposing that $Q_x^*(t)=0$, $Q_y^*(t)=Q^*\sin\Omega^* t^*$), $C^* = \frac{6\mu_s R_c L_c}{\delta_s^{\ 2}}$, $p(\rho)=1-\rho^2$, $q(\rho)=2+\rho^2$. The parameters μ_s, δ_s, R_c, L_c denote oil viscosity, relative bearing clearance, journal radius, total bearing length respectively, (ρ, ϕ) are polar coordinates.

To represent the equations of motion in dimensionless form the following changes of variables and parameters are introduced

$$t=\omega^* t^*;\ \dot{\phi}=\frac{\dot{\phi}^*}{\omega^*};\ \dot{\rho}=\frac{\dot{\rho}^*}{\omega^*};\ x=\frac{x^*}{c^*};\ \dot{x}=\frac{\dot{x}^*}{\omega^* c^*};\ \ddot{x}=\frac{\ddot{x}^*}{\omega^{*2}c^*};\ y=\frac{y^*}{c^*};\ \dot{y}=\frac{\dot{y}^*}{\omega^* c^*};\ \ddot{y}=\frac{\ddot{y}^*}{\omega^{*2}c^*};$$

$$C=\frac{C^*}{m^*\omega^* c^*};\ \Omega=\frac{\Omega^*}{\omega^*};\ Q=\frac{Q^*}{m^*\omega^{*2}c^*};\ Q_0=\frac{Q_0^*}{m^*\omega^{*2}c^*};\ F_k=\frac{F_k^*}{m^*\omega^{*2}c^*};\ P_r=\frac{P_r^*}{m^*\omega^{*2}c^*};\ P_\tau=\frac{P_\tau^*}{m^*\omega^{*2}c^*},$$

Where ω^* is rotation speed; c^* is bearing clearance.

Thus the dimensionless equations of motion take the form

$$\begin{aligned}\ddot{x} &= P_r\left(\rho,\dot{\rho},\dot{\phi}\right)\cos\phi - P_\tau\left(\rho,\dot{\phi}\right)\sin\phi + F_{ux}\\ \ddot{y} &= P_r\left(\rho,\dot{\rho},\dot{\phi}\right)\sin\phi + P_\tau\left(\rho,\dot{\phi}\right)\cos\phi + F_{uy} + Q_0 + Q\sin\Omega t\end{aligned} \qquad (1)$$

$$P_r(\rho,\dot{\rho},\dot{\phi}) = -2C\left\{\frac{\rho^2(1-2\dot{\phi})}{p(\rho)q(\rho)} + \frac{\rho\dot{\rho}}{p(\rho)} + \frac{2\dot{\rho}}{\sqrt{p(\rho)}}arctg\sqrt{\frac{1+\rho}{1-\rho}}\right\},\ P_\tau(\rho,\dot{\phi}) = \pi C\frac{\rho(1-2\dot{\phi})}{q(\rho)\sqrt{p(\rho)}}$$

Here $x = \rho\cos\phi$, $y = \rho\sin\phi$, $\dot{\phi} = \frac{\dot{y}x - \dot{x}y}{\rho^2}$, $\dot{\rho} = \frac{x\dot{x} + y\dot{y}}{\rho}$, $\rho = \sqrt{x^2+y^2}$, $\cos\phi = \frac{x}{\sqrt{x^2+y^2}}$, $\sin\phi = \frac{y}{\sqrt{x^2+y^2}}$, the magnetic control forces are expressed as follows $F_{ux} = -\gamma\dot{x} - \lambda(x - x_0)$, $F_{uy} = -\gamma\dot{y} - \lambda(y - y_0)$, $(\gamma,\lambda) = (\gamma_v,\lambda_p)\frac{C_m}{mc}$.

3. The nonresonant case

The right hand sides of the equations (1) have been expanded in the Taylor's series as well as the origin have been shifted to the location of the static equilibrium for the convenience of the investigation. The linear and quadratic terms have been kept. So, the reformed equations of motion are following

$$\begin{aligned}&\ddot{x} + \alpha x - \beta\dot{y} = -2\hat{\mu}_1\dot{x} + \alpha_1 x^2 + \alpha_2 y^2 + \alpha_3 x\dot{x} + \alpha_4 xy\\&+ \alpha_5 x\dot{y} + \alpha_6\dot{x}y + \alpha_7 y\dot{y}\end{aligned} \quad (2)$$

$$\begin{aligned}&\ddot{y} + \alpha y + \beta\dot{x} = -2\hat{\mu}_2\dot{x} + \beta_1 x^2 + \beta_2 y^2 + \beta_3 x\dot{x} + \beta_4 xy\\&+ \beta_5 x\dot{y} + \beta_6\dot{x}y + \beta_7 y\dot{y} + F\cos(\Omega t + \tau)\end{aligned} \quad (2)$$

We seek a first-order solution for small but finite amplitudes in the form

$$\begin{aligned}x &= \varepsilon x_1(T_0,T_1) + \varepsilon^2 x_2(T_0,T_1) + \ldots\\ y &= \varepsilon y_1(T_0,T_1) + \varepsilon^2 y_2(T_0,T_1) + \ldots\end{aligned} \quad (3)$$

where ε is a small, dimensionless parameter related to the amplitudes and $T_n = \varepsilon^n t$ (n=0, 1) are independent variables. It follows that the derivatives with respect to t become expansions in terms of the partial derivatives with respect to T_n according to

$$\frac{d}{dt} = \frac{\partial}{\partial T_0}\frac{\partial T_0}{\partial t} + \frac{\partial}{\partial T_1}\frac{\partial T_1}{\partial t} + \frac{\partial}{\partial T_2}\frac{\partial T_2}{\partial t} + \ldots = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \ldots$$

$$\frac{d^2}{dt^2} = (D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \ldots)^2 = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2(D_1^2 + 2D_0 D_2) + \ldots,\ \text{where}\ D_k = \frac{\partial}{\partial T_k}.$$

To analyze the *nonresonant case* the forcing term is ordered so that it appears at order ε . Thus we recall in (2) $F = \varepsilon f$, $\hat{\mu}_n = \varepsilon\mu_n$. Substituting (3) into (2) and equating coefficients of like powers of ε we obtain

$$\text{Order } \varepsilon \quad \begin{aligned}&D_0^2 x_1 + \alpha x_1 - \beta D_0 y_1 = 0\\&D_0^2 y_1 + \alpha y_1 + \beta D_0 x_1 = f\cos(\Omega T_0 + \tau)\end{aligned} \quad (4)$$

Order ε^2

$$\begin{aligned}&D_0^2x_2+\alpha x_2-\beta D_0y_2=-2D_0\left(D_1x_1+\mu_1x_1\right)+\beta D_1y_1+\alpha_1x_1^2+\alpha_2y_1^2\\&+\alpha_3x_1D_0x_1+\alpha_4x_1y_1+\alpha_5x_1D_0y_1+\alpha_6y_1D_0x_1+\alpha_7y_1D_0y_1\\&D_0^2y_2+\alpha y_2+\beta D_0x_2=-2D_0\left(D_1y_1+\mu_2y_1\right)-\beta D_1x_1+\beta_1x_1^2+\beta_2y_1^2\\&+\beta_3x_1D_0x_1+\beta_4x_1y_1+\beta_5x_1D_0y_1+\beta_6y_1D_0x_1+\beta_7y_1D_0y_1\end{aligned} \quad (5)$$

The solution of (4) is expressed in the form

$$\begin{aligned}x_1&=A_1(T_1)exp(i\omega_1T_0)+A_2(T_1)exp(i\omega_2T_0)+\Phi_1\,exp[i(\Omega T_0+\tau)]+CC\\y_1&=\Lambda_1A_1(T_1)exp(i\omega_1T_0)+\Lambda_2A_2(T_1)exp(i\omega_2T_0)+\Phi_2\,exp[i(\Omega T_0+\tau)]+CC\end{aligned} \quad (6)$$

where CC denotes the complex conjugate of the preceding terms, ω_n^2 are roots of the equation $\omega_n^4-\left(2\alpha+\beta^2\right)\omega_n^2+\alpha^2=0$; $\Lambda_n=\dfrac{\omega_n^2-\alpha}{\omega_n\beta}i$, $\Phi_1=\dfrac{i}{2}\dfrac{\beta\Omega f}{\left(\alpha-\Omega^2\right)^2-\beta^2\Omega^2}$, $\Phi_2=\dfrac{1}{2}\dfrac{f\left(\alpha-\Omega^2\right)}{\left(\alpha-\Omega^2\right)^2-\beta^2\Omega^2}$,

(n=1, 2).

Substituting (6) into (5) yields

$$\begin{aligned}&D_0^2x_2+\alpha x_2-\beta D_0y_2=\left[-2i\omega_1\left(A_1'+\mu_1A_1\right)+\beta\Lambda_1A_1'\right]exp(i\omega_1T_0)+\\&\left[-2i\omega_2\left(A_2'+\mu_1A_2\right)+\beta\Lambda_2A_2'\right]exp(i\omega_2T_0)+\ldots+CC\\&D_0^2y_2+\alpha y_2+\beta D_0x_2=\left[-2i\omega_1\Lambda_1\left(A_1'+\mu_2A_1\right)-\beta A_1'\right]exp(i\omega_1T_0)+\\&\left[-2i\omega_2\Lambda_2\left(A_2'+\mu_2A_2\right)-\beta A_2'\right]exp(i\omega_2T_0)+\ldots+CC\end{aligned}$$

The terms, which doesn't influence on solvability conditions, aren't presented in the last equations and replaced by dots. So, the solvability conditions are

$$\begin{vmatrix}R_{1n} & -i\beta\omega_n\\ R_{2n} & \left(\alpha-\omega_n^2\right)\end{vmatrix}=0\,,$$

where

$$\begin{aligned}&R_{11}=-2i\omega_1\left(A_1'+\mu_1A_1\right)+\beta\Lambda_1A',\quad R_{12}=-2i\omega_2\left(A_2'+\mu_1A_2\right)+\beta\Lambda_2A',\\&R_{11}=-2i\omega_1\Lambda_1\left(A_1'+\mu_2A_1\right)-\beta A_1',\quad R_{12}=-2i\omega_2\Lambda_2\left(A_2'+\mu_2A_2\right)-\beta A_2'\end{aligned}$$

Consequently, the solvability conditions become $R_{1n}=\dfrac{R_{2n}}{\overline{\Lambda}_n}$. And the equations for A_1 and A_2 are following

$$\begin{aligned}&\left(\beta\Lambda_1-2i\omega_1+\frac{2i\omega_1\Lambda_1+\beta}{\overline{\Lambda}_1}\right)A_1'+\left(\frac{2i\omega_1\Lambda_1\mu_2}{\overline{\Lambda}_1}-2i\omega_1\mu_1\right)A_1=0\\&\left(\beta\Lambda_2-2i\omega_2+\frac{2i\omega_2\Lambda_2+\beta}{\overline{\Lambda}_2}\right)A_2'+\left(\frac{2i\omega_2\Lambda_2\mu_2}{\overline{\Lambda}_2}-2i\omega_2\mu_1\right)A_2=0\end{aligned} \quad (7)$$

It follows from (3), (6), (7) that the solution in the complex form is

$$\begin{aligned}x&=\varepsilon\left[exp(-\varepsilon\nu_1t)a_1\,exp(i\omega_1t)+exp(-\varepsilon\nu_2t)a_2\,exp(i\omega_2t)+\Phi_1\,exp[i(\Omega t+\tau)]+CC\right]+O\left(\varepsilon^2\right)\\y&=\varepsilon\left[\Lambda_1\,exp(-\varepsilon\nu_1t)a_1\,exp(i\omega_1t)+\Lambda_2\,exp(-\varepsilon\nu_2t)a_2\,exp(i\omega_2t)+\Phi_2\,exp[i(\Omega t+\tau)]+CC\right]+O\left(\varepsilon^2\right)\end{aligned}$$

Then the real solution is following

$$x=\varepsilon\left[exp(-\varepsilon\nu_1 t)a_1\cos(\omega_1 t+\Theta_1)+exp(-\varepsilon\nu_2 t)a_2\cos(\omega_2 t+\Theta_2)+2\,Im\,\Phi_1\sin(\Omega t+\tau)\right]+O(\varepsilon^2)$$
$$y=\varepsilon\left[\Lambda_1 exp(-\varepsilon\nu_1 t)a_1\sin(\omega_1 t+\Theta_1)+\Lambda_2 exp(-\varepsilon\nu_2 t)a_2\sin(\omega_2 t+\Theta_2)+2\Phi_2\cos(\Omega t+\tau)\right]+O(\varepsilon^2) \quad (8)$$

where $\nu_n=\dfrac{2\omega_n(\mu_1+\mu_2)}{4\omega_n-\beta\left(Im\,\Lambda_n+\dfrac{1}{Im\,\Lambda_n}\right)}$

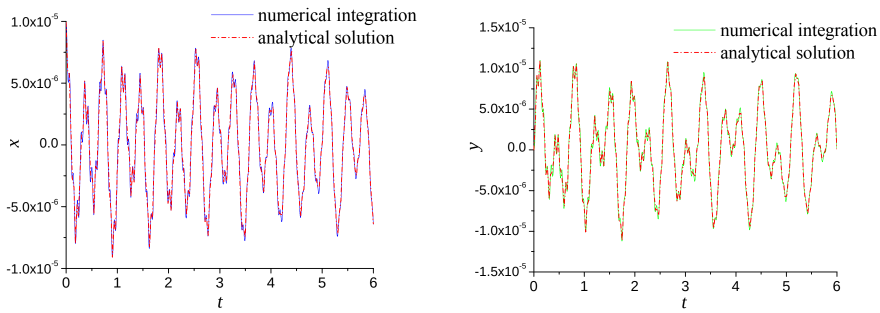

Fig. 2. Comparison of the numerical integration (2) and the perturbation solutions (8)

4. The resonant case $\Omega\approx\omega_2$, $\omega_2\approx 2\omega_1$.

To analyze the resonant case the forcing term is ordered so that it appears at order ε^2. Thus we recall in (2) $F=\varepsilon^2 f$, $\hat{\mu}_n=\varepsilon\mu_n$. It is supposed that $\omega_2>\omega_1$. Also in the resonant case we introduce the detuning parameters σ_1, σ_2. Let put $\Omega=\omega_2+\varepsilon\sigma_1$, $\omega_2=2\omega_1-\varepsilon\sigma_2$ that corresponds to presence of primary and internal resonance in the system.

Substituting (3) into (2) and equating coefficients of like powers of ε we obtain

$$\text{Order } \varepsilon \quad \begin{aligned} &D_0^2x_1+\alpha x_1-\beta D_0 y_1=0\\ &D_0^2y_1+\alpha y_1+\beta D_0 x_1=0\end{aligned} \quad (9)$$

$$\text{Order } \varepsilon^2 \quad \begin{aligned} &D_0^2x_2+\alpha x_2-\beta D_0y_2=-2D_0(D_1x_1+\mu_1x_1)+\beta D_1y_1+\alpha_1x_1^2+\alpha_2y_1^2\\ &+\alpha_3x_1D_0x_1+\alpha_4x_1y_1+\alpha_5x_1D_0y_1+\alpha_6y_1D_0x_1+\alpha_7y_1D_0y_1\\ &D_0^2y_2+\alpha y_2+\beta D_0x_2=-2D_0(D_1y_1+\mu_2y_1)-\beta D_1x_1+\beta_1x_1^2+\beta_2y_1^2\\ &+\beta_3x_1D_0x_1+\beta_4x_1y_1+\beta_5x_1D_0y_1+\beta_6y_1D_0x_1+\beta_7y_1D_0y_1+f\cos(\Omega T_0+\tau)\end{aligned} \quad (10)$$

The solution of (9) is expressed in the form

$$\begin{aligned} x_1&=A_1(T_1)exp(i\omega_1T_0)+A_2(T_1)exp(i\omega_2T_0)+CC\\ y_1&=\Lambda_1A_1(T_1)exp(i\omega_1T_0)+\Lambda_2A_2(T_1)exp(i\omega_2T_0)+CC\end{aligned} \quad (11)$$

where $\Lambda_n = \dfrac{\omega_n^2 - \alpha}{\omega_n \beta} i$.

Substituting (11) into (10) yields

$$
\begin{aligned}
& D_0^2 x_2 + \alpha x_2 - \beta D_0 y_2 = \left[-2i\omega_1\left(A_1' + \mu_1 A_1\right) + \beta\Lambda_1 A_1'\right] exp\left(i\omega_1 T_0\right) + \\
& \left[-2i\omega_2\left(A_2' + \mu_1 A_2\right) + \beta\Lambda_2 A_2'\right] exp\left(i\omega_2 T_0\right) + \\
& A_1^2\left[\alpha_1 + \Lambda_1^2\alpha_2 + i\omega_1\alpha_3 + \Lambda_1\alpha_4 + i\omega_1\Lambda_1\alpha_5 + i\omega_1\Lambda_1\alpha_6 + i\omega_1\Lambda_1^2\alpha_7\right] exp\left(2i\omega_1 T_0\right) \\
& + A_2^2\left[\alpha_1 + \Lambda_2^2\alpha_2 + i\omega_2\alpha_3 + \Lambda_2\alpha_4 + i\omega_2\Lambda_2\alpha_5 + i\omega_2\Lambda_2\alpha_6 + i\omega_2\Lambda_2^2\alpha_7\right] exp\left(2i\omega_2 T_0\right) \\
& + A_1 A_2\left[2\alpha_1 + 2\Lambda_1\Lambda_2\alpha_2 + \left(i\omega_1 + i\omega_2\right)\alpha_3 + \left(\Lambda_1 + \Lambda_2\right)\alpha_4 + \left(i\omega_2\Lambda_2 - i\omega_1\Lambda_1\right)\alpha_5\right. \\
& \left. + \left(i\omega_2\Lambda_1 + i\omega_1\Lambda_2\right)\alpha_6 + \left(i\omega_1 + i\omega_2\right)\Lambda_1\Lambda_2\alpha_7 \quad \right] exp\left(i\left(\omega_1 + \omega_2\right)T_0\right) \\
& + \overline{A}_1 A_2\left[2\alpha_1 + 2\overline{\Lambda}_1\Lambda_2\alpha_2 + \left(i\omega_2 - i\omega_1\right)\alpha_3 + \left(\Lambda_2 + \overline{\Lambda}_1\right)\alpha_4 + \left(i\omega_2\Lambda_2 - i\omega_1\overline{\Lambda}_1\right)\alpha_5\right. \\
& \left. + \left(i\omega_2\overline{\Lambda}_1 - i\omega_1\Lambda_2\right)\alpha_6 + \left(i\omega_2 - i\omega_1\right)\overline{\Lambda}_1\Lambda_2\alpha_7 \quad \right] exp\left(i\left(\omega_2 - \omega_1\right)T_0\right) \\
& + A_1\overline{A}_1\left(\alpha_1 + \Lambda_1\left(\overline{\Lambda}_1\alpha_2 + \alpha_4 + i\omega_1\left(\alpha_5 - \alpha_6\right)\right)\right) + A_2\overline{A}_2\left(\alpha_1 + \Lambda_2\left(\overline{\Lambda}_2\alpha_2 + \alpha_4 + i\omega_2\left(\alpha_5 - \alpha_6\right)\right)\right) + CC
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
& D_0^2 y_2 + \alpha y_2 + \beta D_0 x_2 = \left[-2i\omega_1\Lambda_1\left(A_1' + \mu_2 A_1\right) - \beta A_1'\right] exp\left(i\omega_1 T_0\right) + \\
& \left[-2i\omega_2\Lambda_2\left(A_2' + \mu_2 A_2\right) - \beta A_2'\right] exp\left(i\omega_2 T_0\right) + \\
& A_1^2\left[\beta_1 + \Lambda_1^2\beta_2 + i\omega_1\beta_3 + \Lambda_1\beta_4 + i\omega_1\Lambda_1\beta_5 + i\omega_1\Lambda_1\beta_6 + i\omega_1\Lambda_1^2\beta_7\right] exp\left(2i\omega_1 T_0\right) \\
& + A_2^2\left[\beta_1 + \Lambda_2^2\beta_2 + i\omega_2\beta_3 + \Lambda_2\beta_4 + i\omega_2\Lambda_2\beta_5 + i\omega_2\Lambda_2\beta_6 + i\omega_2\Lambda_2^2\beta_7\right] exp\left(2i\omega_2 T_0\right) \\
& + A_1 A_2\left[2\beta_1 + 2\Lambda_1\Lambda_2\beta_2 + \left(i\omega_1 + i\omega_2\right)\beta_3 + \left(\Lambda_1 + \Lambda_2\right)\beta_4 + \left(i\omega_2\Lambda_2 - i\omega_1\Lambda_1\right)\beta_5\right. \\
& \left. + \left(i\omega_2\Lambda_1 + i\omega_1\Lambda_2\right)\beta_6 + \left(i\omega_1 + i\omega_2\right)\Lambda_1\Lambda_2\beta_7 \quad \right] exp\left(i\left(\omega_1 + \omega_2\right)T_0\right) \\
& + \overline{A}_1 A_2\left[2\beta_1 + 2\overline{\Lambda}_1\Lambda_2\beta_2 + \left(i\omega_2 - i\omega_1\right)\beta_3 + \left(\Lambda_2 + \overline{\Lambda}_1\right)\beta_4 + \left(i\omega_2\Lambda_2 - i\omega_1\overline{\Lambda}_1\right)\beta_5\right. \\
& \left. + \left(i\omega_2\overline{\Lambda}_1 - i\omega_1\Lambda_2\right)\beta_6 + \left(i\omega_2 - i\omega_1\right)\overline{\Lambda}_1\Lambda_2\beta_7 \quad \right] exp\left(i\left(\omega_2 - \omega_1\right)T_0\right) \\
& + A_1\overline{A}_1\left(\beta_1 + \Lambda_1\left(\overline{\Lambda}_1\beta_2 + \beta_4 + i\omega_1\left(\beta_5 - \beta_6\right)\right)\right) + A_2\overline{A}_2\left(\beta_1 + \Lambda_2\left(\overline{\Lambda}_2\beta_2 + \beta_4 + i\omega_2\left(\beta_5 - \beta_6\right)\right)\right) \\
& + \frac{1}{2} f \, exp\left(i\left(\omega_2 T_0 + \sigma_1 T_1 + \tau\right)\right) + CC
\end{aligned} \tag{13}
$$

Consequently, the solvability conditions are

$$
\begin{aligned}
& q_{\omega_1} + \frac{1}{\Lambda_1} p_{\omega_1} + \left(q_{\omega_2 - \omega_1} + \frac{1}{\Lambda_1} p_{\omega_2 - \omega_1}\right)\overline{A}_1 A_2 \, exp\left(-i\sigma_2 T_1\right) = 0 \\
& q_{\omega_2} + \frac{1}{\Lambda_2} p_{\omega_2} + \left(q_{2\omega_1} + \frac{1}{\Lambda_2} p_{2\omega_1}\right) A_1^2 \, exp\left(i\sigma_2 T_1\right) + \frac{1}{2} f \, exp\left(i\left(\sigma_1 T_1\right) + \tau\right) = 0
\end{aligned} \tag{14}
$$

Here the coefficients $q_{\omega 1}$, $q_{\omega 2}$, $q_{\omega 2\text{-}\omega 1}$, $q_{2\omega 1}$ are the expressions in the bracket at the exponents with the corresponding powers (12) and $p_{\omega 1}$, $p_{\omega 2}$, $p_{\omega 2\text{-}\omega 1}$, $p_{2\omega 1}$ are the expressions in the bracket at the exponents with the corresponding powers (13).

Let introduce the polar notation $A_m = \frac{1}{2} a_m \, exp\left(i\Theta_m\right), \quad m = 1, 2$. Substituting this polar expressions into (14) and separating the result into real and imaginary parts, we obtain for the steady-state response $a_n' = \gamma_n' = 0$

$$(\mu_1+\mu_2)a_1-\frac{\phi}{4\omega_1}a_1a_2\sin\gamma_2-\frac{\psi}{4\omega_1}a_1a_2\cos\gamma_2=0$$
$$(\mu_1+\mu_2)a_2+\frac{\zeta}{4\omega_2}a_1^2\sin\gamma_2-\frac{\eta}{4\omega_2}a_1^2\cos\gamma_2-\frac{f}{2\omega_2}\sin\gamma_1=0$$
$$-\frac{l_1}{4\omega_1}(\sigma_1-\sigma_2)a_1+\frac{\phi}{4\omega_1}a_1a_2\cos\gamma_2-\frac{\psi}{4\omega_1}a_1a_2\sin\gamma_2=0$$
$$-\frac{l_2}{2\omega_2}\sigma_1a_2+\frac{\zeta}{4\omega_2}a_1^2\cos\gamma_2-\frac{\eta}{4\omega_2}a_1^2\sin\gamma_2+\frac{f}{2\omega_2}\cos\gamma_1=0$$

Here $\gamma_1=\sigma_1T_1+\tau-\Theta_2$, $\gamma_2=2\Theta_1-\Theta_2+\sigma_2T_1$.

Finally we obtain expressions for a_1 and a_2. Thus unknown functions in (11) have been defined.

$$a_2=\left(\frac{16\omega_1^2(\mu_1+\mu_2)^2+l_1^2(\sigma_1-\sigma_2)^2}{\phi^2+\psi^2}\right)^{\frac{1}{2}};\qquad a_1=\left(\frac{-b\pm\left(b^2-4ac\right)^{\frac{1}{2}}}{2a}\right)^{\frac{1}{2}}$$

were $a=\zeta^2+\eta^2$, $b=8\omega_2(\mu_1+\mu_2)(\zeta\sin\gamma_2-\eta\cos\gamma_2)-4l_2\sigma_1(\zeta\cos\gamma_2+\eta\sin\gamma_2)$;
$c=a_2^2\left(16\omega_2^2(\mu_1+\mu_2)^2-4l_2^2\sigma_1^2\right)-8f^2$.

5. Modeling of the rotor–MHDB system with hysteresis

To take into account hysteretic properties of the rotor–MHDB system the Bouc-Wen model have been successfully applied (Fig. 3). An appropriate choice of parameters and functions of the Bouc-Wen model allow constructing of hysteretic loops of a various form in accordance with an experimental data.

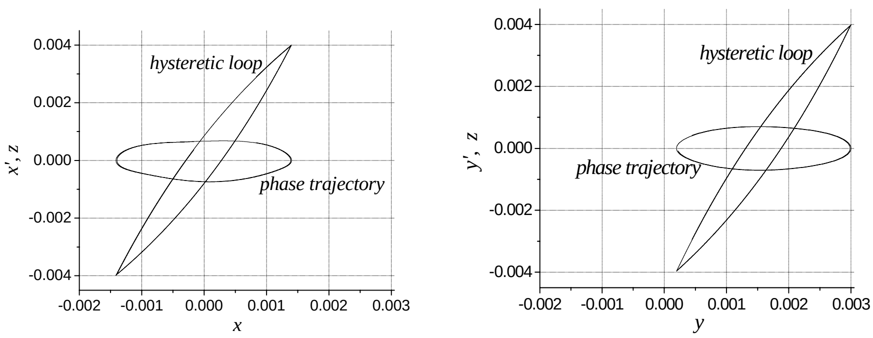

Fig. 3. The phase trajectories and the hysteretic loops of the system (15).

The hysteretic model of the rotor–MHDB system is looked like following

$$
\begin{aligned}
\ddot{x} &= P_r\left(\rho,\dot{\rho},\dot{\phi}\right)cos\,\phi - P_\tau\left(\rho,\dot{\phi}\right)sin\,\phi + F_{ux} \\
\ddot{y} &= P_r\left(\rho,\dot{\rho},\dot{\phi}\right)sin\,\phi + P_\tau\left(\rho,\dot{\phi}\right)cos\,\phi + F_{uy} + Q_0 + Q\,sin\,\Omega t \\
\dot{z}_1 &= \left[k_z - \left(\gamma + \beta\,sgn(\dot{x})\,sgn(z_1)\right)\left|z_1\right|^n\right]\dot{x} \\
\dot{z}_2 &= \left[k_z - \left(\gamma + \beta\,sgn(\dot{y})\,sgn(z_2)\right)\left|z_2\right|^n\right]\dot{y}
\end{aligned} \tag{15}
$$

6. Conclusions

In this paper the dynamics of the rotor–MHDB system with 2-dof is analyzed only for two cases. The first one is the nonresonant case. The second one corresponds to presence of the primary and internal resonance in the system ($\Omega \approx \omega_2$, $\omega_2 \approx 2\omega_1$). The method of multiple scales allows to study the behaviour of the system under conditions of other categories of primary and secondary resonances, to investigate a possibility of a saturation phenomenon.

It is supposed to find conditions for flutter instability (conditions for chaotic responses) of hysteretic model of the rotor–MHDB system by means of the method based on the analysis of wandering trajectories.

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