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STATIC AND DYNAMICS OF CLOSED CYLINDRICAL SHELLS SUBJECTED TO LOCAL LOADS

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Abstract: A closed cylindrical shell of circled cross section and of finite length with constant stiffness and density subjected to non-homogeneous sign-changeable load is analysed. The governing equations are formulated, and then the Bubnov-Galerkin method is applied. Reliability of the obtained results is illustrated and discussed. Static and dynamics problems are studied. In the first case (static) convergence of the used method is achieved in pre- and post-critical cases, whereas in the second case (dynamics) convergence is achieved only for the pre-critical state. The so called shell vibration charts are constructed allowing to predict a real shell behavior.

1. A mathematical model

In the frame of nonlinear classical theory of shallow shells we study a closed cylindrical shell of circled cross section of finite length with constant stiffness and density subjected to non-homogeneous sign-changeable load.

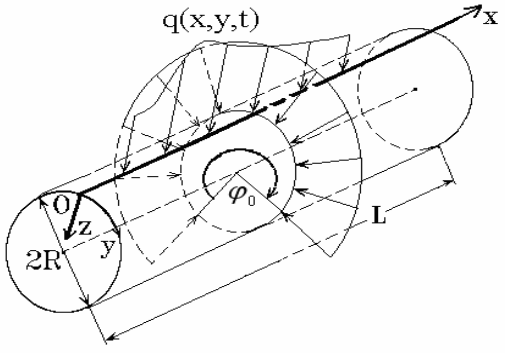


Figure 1. Computational scheme

We introduce the following system of coordinates: axis x overlaps a longitudinal coordinate, axis y overlaps a circled coordinate, and axis z overlaps a normal to middle shell surface (Figure 1). A cylindrical shell as a 3D object Ω in given coordinates is defined in the following way: $\Omega = \{x, y, z \mid (x, y) \in [0; l] \times [0; 2\pi], -h \le z \le h\}$. Governing equations are taken from the theory of shallow shells in the non-dimensional form [1]:

$$\begin{aligned}
&\frac{2h^3 E}{3(1-\mu^2)}\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\cdot)}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2(\cdot)}{\partial y^2}+2(1-\mu)\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2(\cdot)}{\partial x\partial y}+\mu\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\cdot)}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2(\cdot)}{\partial x^2}\right)\right]- \\
&\qquad -\nabla_k^2 F - L(w,F) + k_y{}^2 q(t) - \frac{2h\rho}{g}\left(\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{\partial w}{\partial t}\right)=0, \\
&\frac{1}{2hE}\left[\left(\frac{\partial^2 F}{\partial y^2}-\mu\frac{\partial^2 F}{\partial x^2}\right)\frac{\partial^2(\cdot)}{\partial y^2}+\left(\frac{\partial^2 F}{\partial x^2}-\mu\frac{\partial^2 F}{\partial y^2}\right)\frac{\partial^2(\cdot)}{\partial x^2}+2(1+\mu)\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2(\cdot)}{\partial x\partial y}\right]+ \\
&\qquad +\nabla_k^2 w+\frac{1}{2}L(w,w)=0,
\end{aligned} \tag{1}$$

where $L(w,F)$ is the known nonlinear operator, E is the Young modulus and μ is Poisson's coefficient.

System (1) is transformed into a non-dimensional form using the known non-dimensional parameters: $w = h\overline{w}$, $F = Eh^2\overline{F}$, $t = t_0\bar{t}$, $\varepsilon = \bar{\varepsilon}/\tau$, $x = L\overline{x}$, $y = R\overline{y}$, $k_y = \bar{k}_y\dfrac{h}{R^2}$ ($k_x = 0$), $q = \bar{q}\dfrac{Eh^4}{L^2R^2}$, $\tau = \dfrac{LR}{h}\sqrt{\dfrac{\rho}{Eg}}$, $\lambda = \dfrac{L}{R}$, where L and $R = R_y$ denote shell length and radius, respectively, t is the time, ε denotes damping coefficients, $\mu = 0.3$, and $q(x,y,t)$ denotes transversal load. Bars over non-dimensional quantities are further omitted:

$$\begin{aligned}
&\left[\frac{1}{\lambda^2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\cdot)}{\partial x^2}+\lambda^2\frac{\partial^2 w}{\partial y^2}\frac{\partial^2(\cdot)}{\partial y^2}+2(1-\mu)\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2(\cdot)}{\partial x\partial y}+\mu\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2(\cdot)}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2(\cdot)}{\partial x^2}\right)\right]- \\
&\qquad -\nabla_k^2 F - L(w,F) + k_y{}^2 q(t) - \left(\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{\partial w}{\partial t}\right)=0, \\
&\left[\left(\lambda^2\frac{\partial^2 F}{\partial y^2}-\mu\frac{\partial^2 F}{\partial x^2}\right)\frac{\partial^2(\cdot)}{\partial y^2}+\left(\frac{1}{\lambda^2}\frac{\partial^2 F}{\partial x^2}-\mu\frac{\partial^2 F}{\partial y^2}\right)\frac{\partial^2(\cdot)}{\partial x^2}+2(1+\mu)\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2(\cdot)}{\partial x\partial y}\right]+ \\
&\qquad +\nabla_k^2 w+\frac{1}{2}L(w,w)=0.
\end{aligned} \tag{2}$$

Boundary conditions have the form

$$w = \frac{\partial^2 w}{\partial x^2} = 0; \quad F = \frac{\partial^2 F}{\partial x^2} = 0 \text{ for } x = 0;1,$$

$$w = \frac{\partial^2 w}{\partial y^2} = 0; \quad F = \frac{\partial^2 F}{\partial y^2} = 0 \text{ for } y = 0;2\pi, \tag{3}$$

whereas initial conditions read

$$w(x,y)|_{t=0} = 0, \quad \dot{w}|_{t=0} = 0 . \tag{4}$$

2. Algorithm of the Bubnov-Galerkin method

In order to solve equations (2), functions *w* and *F* are approximated by products of functions depending on time and coordinates:

$$w = \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} A_{ij}(t)\varphi_{ij}(x,y), \quad F = \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} B_{ij}(t)\psi_{ij}(x,y) . \tag{5}$$

In order to find an approximate of an element of *w* and *F*, we take functions $\{\varphi_{ij}(x,y), \psi_{ij}(x,y)\}$ ($i, j = 0,1,2...$) in (5), satisfying the following requirements:

1. $\varphi_{ij}(x,y) \in H_A$, $\psi_{ij}(x,y) \in H_A$, where H_A is Hilbert space, which is further referred to as energy space;
2. $\forall i, j \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ functions $\varphi_{ij}(x,y)$ and $\psi_{ij}(x,y)$ linearly independent, continuous together with its partial derivatives up to the fourth order in space Ω;
3. $\varphi_{ij}(x,y)$ and $\psi_{ij}(x,y)$ satisfy main boundary conditions (and initial conditions) exactly;
4. $\varphi_{ij}(x,y)$ and $\psi_{ij}(x,y)$ are complete in H_A.
5. $\varphi_{ij}(x,y)$ and $\psi_{ij}(x,y)$ should represent *N* first elements of complete functions system.

Coefficients $A_{ij}(t)$ and $B_{ij}(t)$ are the functions of time being sought. We denote, for convenience, left hand sides of equation (2) by Φ_1 and Φ_2, respectively:

$$\Phi_1(w, F, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 F}{\partial x^2}, ...) + k_y{}^2 q(x,y,t) = 0, \quad \Phi_2(w, F, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 F}{\partial x^2}, ...) = 0. \tag{6}$$

Applying the Bubnov-Galerkin procedure to (6) one gets

$$\int_0^1\int_0^\xi \Phi_1\varphi_{rs}(x,y)dxdy+\int_{x_1}^{x_2}\int_{y_1}^{y_2} k_y{}^2 q(x,y,t)\varphi_{rs}(x,y)dxdy=0,$$
$$\int_0^1\int_0^\xi \Phi_2\psi_{rs}(x,y)dxdy=0, \qquad r=0,1,...,N_1;\quad s=0,1,...N_2. \tag{7}$$

Taking into account (5), equations (6) take the form

$$\sum_{rs}[\sum_{ij}A_{ij}\sum_{kl}H_{ijklrs}+\sum_{ij}B_{ij}C_{1,ijrs}+k_y{}^2qQ_{rs}+\sum_{ij}A_{ij}\sum_{kl}B_{kl}D_{1,ijklrs}+$$
$$+\sum_{ij}\left[\frac{d^2A_{ij}}{dt^2}+\varepsilon\frac{dA_{ij}}{dt}\right]G_{ijrs}]=0, \tag{8}$$
$$\sum_{rs}[\sum_{ij}A_{ij}C_{2,ijrs}+\sum_{ij}B_{ij}\sum_{kl}P_{ijklrs}+\sum_{ij}A_{ij}\sum_{kl}A_{rs}D_{2,ijklrs}]=0.$$

Symbol $\sum_{rs}[*]$ standing by each equation of system (8) means that each such equation represents a system of rs equations, and integrals of the Bubnov-Galerkin procedure have the form:

$$H_{ijklrs}=\int_0^1\int_0^\xi \frac{1}{12(1-\mu^2)}\left[\frac{1}{\lambda^2}\frac{\partial^2\varphi_{ij}}{\partial x^2}\frac{\partial^2\varphi_{kl}}{\partial x^2}+\lambda^2\frac{\partial^2\varphi_{ij}}{\partial y^2}\frac{\partial^2\varphi_{kl}}{\partial y^2}+2\frac{\partial^2\varphi_{ij}}{\partial x\partial y}\frac{\partial^2\varphi_{kl}}{\partial x\partial x}\right]\varphi_{rs}dxdy,$$
$$C_{1,ijrs}=\int_0^1\int_0^\xi\left[-k_y\frac{\partial^2\psi_{ij}}{\partial x^2}-k_x\frac{\partial^2\psi_{ij}}{\partial y^2}\right]\varphi_{rs}dxdy,\ C_{2,ijrs}=\int_0^1\int_0^\xi\left[k_y\frac{\partial^2\varphi_{ij}}{\partial x^2}+k_x\frac{\partial^2\varphi_{ij}}{\partial y^2}\right]\psi_{rs}dxdy,$$
$$D_{1,ijklrs}=\int_0^1\int_0^\xi\left[-L(\varphi_{ij},\psi_{kl})\right]\varphi_{rs}dxdy,\ D_{2,ijklrs}=\int_0^1\int_0^\xi\frac{1}{2}L(\varphi_{ij},\varphi_{kl})\psi_{rs}dxdy, \tag{9}$$
$$P_{ijklrs}=\int_0^1\int_0^\xi\left[\frac{1}{\lambda^2}\frac{\partial^2\psi_{ij}}{\partial x^2}\frac{\partial^2\psi_{kl}}{\partial x^2}+\lambda^2\frac{\partial^2\psi_{ij}}{\partial y^2}\frac{\partial^2\psi_{kl}}{\partial y^2}+2\frac{\partial^2\psi_{ij}}{\partial x\partial y}\frac{\partial^2\psi_{kl}}{\partial x\partial x}\right]\psi_{rs}dxdy,$$
$$G_{ijrs}=\int_0^1\int_0^\xi\left[-\varphi_{ij}\varphi_{rs}\right]dxdy,\ Q_{rs}=\int_{x_1}^{x_2}\int_{y_1}^{y_2}k_y{}^2q(x,y,t)\varphi_{rs}dxdy.$$

Integrals (9), perhaps except of Q_{rs}, if transversal load is applied to part of the shell surface, are computed on the whole shell surface. After application of the Bubnov-Galerkin procedure a system of ODEs is obtained regarding $A_{ij}(t)$ and $B_{ij}(t)$, and has the following matrix form:

$$\begin{aligned}&\mathbf{G}(\ddot{\mathbf{A}}+\varepsilon\dot{\mathbf{A}})+\mathbf{HA}+\mathbf{C}_1\mathbf{B}+\mathbf{D}_1\mathbf{AB}=\mathbf{Q}q(t),\\&\mathbf{C}_2\mathbf{A}+\mathbf{PB}+\mathbf{D}_2\mathbf{AA}=0,\end{aligned}\tag{10}$$

where $\mathbf{H}=\left\|H_{ijrs}\right\|$, $\mathbf{G}=\left\|G_{ijrs}\right\|$, $\mathbf{C}_1=\left\|C_{1ijrs}\right\|$, $\mathbf{C}_2=\left\|C_{2ijrs}\right\|$, $\mathbf{D}_1=\left\|D_{1ijklrs}\right\|$, $\mathbf{D}_2=\left\|D_{2ijklrs}\right\|$, $\mathbf{P}=\left\|P_{ijrs}\right\|$ – square matrices of dimension $2\cdot N_1\cdot N_2\times 2\cdot N_1\cdot N_2$, $\mathbf{A}=\left\|A_{ij}\right\|$, $\mathbf{B}=\left\|B_{ij}\right\|$, $\mathbf{Q}=\left\|Q_{ij}\right\|$ – matrices of dimension $2\cdot N_1\cdot N_2\times 1$.

Further, the second equation of system (10) is solved regarding $\mathbf{B}$ in the following way

$$\mathbf{B}=\left[-\mathbf{P}^{-1}\mathbf{D}_2\mathbf{A}-\mathbf{P}^{-1}\mathbf{C}_2\right]\mathbf{A}\tag{11}$$

Multiplying by $\mathbf{G}^{-1}$ the first equation of system (10), the following first order set of ODEs is obtained:

$$\begin{cases}\dot{\mathbf{A}}=\mathbf{R}\ ,\\ \dot{\mathbf{R}}=-\bar{\varepsilon}\mathbf{R}-\left[\mathbf{G}^{-1}\mathbf{C}_1+\mathbf{G}^{-1}\mathbf{D}_1\mathbf{A}\right]\cdot\mathbf{B}-\mathbf{G}^{-1}\mathbf{HA}+\mathbf{G}^{-1}\mathbf{Q}\bar{q}(t).\end{cases}\tag{12}$$

Note that inversed matrices $\mathbf{G}^{-1}$ and $\mathbf{P}^{-1}$ exist if the coordinate functions are linearly independent.

Equations (12) are supplemented by one of the boundary conditions (3) and initial condition (4), and the obtained Cauchy problem is solved via the fourth order Runge-Kutta method. Time step is chosen by the Runge principle.

Below, we study a hinged cylindrical shell with homogeneous boundary conditions (3) and zero value initial conditions (4), and give details of the procedure. For this purpose functions φ_{ij},ψ_{ij} from (5) are represented by a product of two functions, each of them depending only on one argument, and satisfying boundary conditions (3) regarding *x*:

$$w=\sum_{i=1}^{N_1}\sum_{j=0}^{N_2}A_{ij}(t)\sin(i\pi x)\cos(jy),\qquad F=\sum_{i=1}^{N_1}\sum_{j=0}^{N_2}B_{ij}(t)\sin(i\pi x)\cos(jy)\tag{13}$$

Integrals of the Bubnov-Galerkin procedure are computed using the formulas:

$$I_{1,r} = \int_{x_1}^{x_2} \sin(r\pi x)dx = \frac{\cos(r\pi x_1) - \cos(r\pi x_2)}{r\pi},$$

$$I_{2,s} = \int_{y_1}^{y_2} \cos(s\pi y)dy = \frac{\sin(s\pi y_2) - \sin(s\pi y_1)}{s\pi},$$

$$I_{3,ir} = \int_0^1 \sin(i\pi x)\sin(r\pi x)dx = \begin{cases} \frac{1}{2}, & i = r, \\ 0, & i \neq r, \end{cases}$$

$$I_{4,js} = \int_0^{2\pi} \cos(j\pi y)\cos(s\pi y)dy = \begin{cases} 2\pi, & j = s = 0, \\ \pi, & j = s \neq 0, \\ 0, & , j \neq s, \end{cases}$$

$$I_{5,ikr} = \int_0^1 \sin(i\pi x)\sin(r\pi x)\sin(k\pi x)dx =$$

$$= \begin{cases} \frac{1}{4\pi}[-\frac{\cos(\alpha_1\pi)}{\alpha_1} - \frac{\cos(\alpha_2\pi)}{\alpha_2} - \frac{\cos(\alpha_3\pi)}{\alpha_3} - \frac{\cos(\alpha_4\pi)}{\alpha_4} + \\ \qquad + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} - \frac{1}{\alpha_4}], & \alpha_v \neq 0; \\ [\frac{\cos(\alpha_v\pi)}{\alpha_v} \approx 0, \quad \frac{1}{\alpha_v} \approx 0], \quad v = 1,2,3, & \alpha_v = 0; \end{cases}$$

$$I_{7,ikr} = \int_0^1 \cos(i\pi x)\cos(k\pi x)\sin(r\pi x)dx =$$

$$= \begin{cases} \frac{1}{4\pi}[\frac{\cos(\alpha_1\pi)}{\alpha_1} - \frac{\cos(\alpha_2\pi)}{\alpha_2} - \frac{\cos(\alpha_3\pi)}{\alpha_3} - \frac{\cos(\alpha_4\pi)}{\alpha_4} - \\ \qquad - \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} + \frac{1}{\alpha_4}], & \alpha_v \neq 0; \\ [\frac{\cos(\alpha_v\pi)}{\alpha_v} \approx 0, \quad \frac{1}{\alpha_v} \approx 0], \; v = 1,2,3, & \alpha_v = 0; \end{cases}$$

$$I_{8,jls} = \int_0^{2\pi} \sin(jy)\sin(ly)\cos(sy)dy =$$

$$= \begin{cases} \frac{1}{4}[-\frac{\sin(\beta_1\pi)}{\beta_1} + \frac{\sin(\beta_2\pi)}{\beta_2} + \frac{\sin(\beta_3\pi)}{\beta_3} - \frac{\sin(\beta_4\pi)}{\beta_4} + \\ \qquad + \frac{1}{\beta_1} - \frac{1}{\beta_2} - \frac{1}{\beta_3} + \frac{1}{\beta_4}], & \beta_v \neq 0; \\ [\frac{\sin(\beta_v\pi)}{\beta_v} \approx 0, \quad \frac{1}{\beta_v} \approx 0], \ v = 1,2,3,4, & \beta_v = 0; \end{cases}$$

$$I_{6,jls} = \int_0^{2\pi} \cos(jy)\cos(ly)\sin(sy)dy = 0\,,$$

where:

$$\alpha_1 = j + k - r, \quad \alpha_2 = k + r - i, \quad \alpha_3 = r + i - k, \quad \alpha_4 = i + k + r,$$

$$\beta_1 = j + l - s, \quad \beta_2 = l + s - j, \quad \beta_3 = s + j - l, \quad \beta_4 = j + l + s.$$

After introduction of the following notation

$$I_Q^{rs} = k_y^{\,2} \cdot I_{1r} I_{2s}, \quad I_{AB}^{rs} = \left(s^2 k_x + r^2 k_y\right)\pi^2 I_{3,ir} I_{4,js},$$

$$I_{ijklrs} = \pi^2\left[\left(i^2 l^2 + j^2 k^2\right) I_{5ikr} I_{6jls} - 2ijlk I_{7ikr} I_{8jls}\right], \qquad I_{rs}^t = I_{3,ir} I_{4,js},$$

$$J_{1,ijkl}^{rs} = \frac{\pi^2}{12(1-\mu^2)}\left[\frac{r^4}{\lambda^2} + 2r^2 s^2 + \lambda^2 s^4\right] \cdot I_{3,ir} I_{4,js},$$

$$J_{2,ijkl}^{rs} = \left[\frac{r^4}{\lambda^2} + 2r^2 s^2 + \lambda^2 s^4\right] \cdot \pi^2 I_{3,ir} I_{4,js},$$

and taking into account integrals in the above system (12) takes the following form

$$\sum_{rs}\left\{\sum_{ij}\sum_{kl}\left[J_{1,ijkl}^{rs} A_{ij} + I_{AB}^{rs} B_{rs} + I_Q^{rs} q(t) + A_{ij} B_{kl} I_{ijklrs} + \left(\frac{d^2 A_{ij}}{dt^2} + \varepsilon\frac{dA_{ij}}{dt}\right) I_{rs}^t\right]\right\} = 0,$$

$$\sum_{rs}\left\{\sum_{ij}\sum_{kl}\left[J_{2,ijkl}^{rs} B_{ij} + I_{AB}^{rs} A_{rs} + \frac{1}{2} A_{ij} A_{kl} I_{ijklrs}\right]\right\} = 0\,. \qquad (14)$$

3. Reliability of the obtained results

The presented Bubnov-Galerkin method yields solutions for a wide class of static and dynamic problems. Static problems can be solved via the set up method first time applied by Feodosev [2] for shells. In order to solve static problems of plates and shells various approximate methods are applied, which enable reduction of partial differential equations (PDEs) to algebraic nonlinear equations (AE) that are further transformed to a linear system.

Let us briefly point out advantages of the set up method. From the mathematical point of view set up can be understood as an iteration method for solution of nonlinear algebraic equations, where in each time step we achieve an exact solution. The method has high accuracy and is not sensitive to the choice of initial approximation. In addition, it is not difficult to find multiple solutions of static problems. Namely, different initial conditions may result in getting different results. Besides, finding solutions of homogeneous systems by traditional methods one has to introduce artificial imperfection like a small transversal load, or small curvature, or small initial deflection, etc., which in many cases influences the obtained results. In the set up method the initial imperfections are represented by non-homogeneous initial conditions and small changes of initial conditions do not influence a static solution which is sought.

Solving the Cauchy problem for $\varepsilon = \varepsilon_{cr}$ and for various values of external load amplitude we get $\{q_i\} \to \{w_i\}$, which allows us to construct relation $q(w)$ and to investigate a stress-strain state of the construction.

We apply this approach for closed cylindrical shells and study a linear dissipative system. Namely, we take transversal pressure $q(t) = q_0$, distributed within a zone with central angle φ_0, $0 \le x \le 0.5$. Next, we study the cylindrical shell behavior depending on pressure wideness characterized by φ_0. We compare our numerical results with those obtained analytically and presented in [3]. For this purpose we take in equations (14) nonlinear terms equal to zero, i.e. $I_{ijklrs} = 0$. Applying such a simplified mathematical model one may check that for various loading angles functions $w_{\max}(q_0)$ are linear, i.e. $y = kx$, where $y = \overline{w}$, $x = \overline{q}_0$. Therefore, we get $y = \overline{w} \big/ q_0 \; x$, and analytical solution $\dfrac{w}{qR \big/ E}(\varphi)$ can be transformed to numerical one $\overline{w} \big/ q \, (\varphi)$ using non-dimensional parameters in the following way:

$$\overline{w} = \frac{w}{2h}; \qquad \overline{q} = \frac{L^2 R^2}{E(2h)^4 \overline{k}_y^2} q \,, \quad \text{hence} \quad \frac{\overline{w}}{\overline{q}} = \frac{w}{qR/E}\left(\frac{2h}{R}\right)\frac{1}{\lambda^2}.$$

Figure 2 shows relations $\frac{w}{qR/E}(\varphi)$ (an analytical solution) and $\overline{w}/\overline{q}(\varphi_0)$ (a numerical solution), where $\overline{w}$ and $\overline{q}$ are the non-dimensional deflection and load in the cylindrical shell centre (0.5; 0.0). Table 1 gives the numerical solution yielded by the Bubnov-Galerkin method for high approximations $(N_1 = N_2 = N = 5)$, and analytical solution for some loading angles, as well as relative errors of the obtained results.

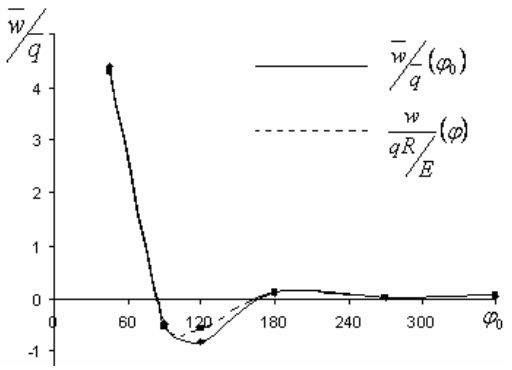


Figure 2. Relations $\overline{w}/\overline{q}(\varphi_0)$ and $\frac{w}{qR/E}(\varphi)$

Taking into account data given in Figure 2 one may conclude that the analytical results overlap the numerical ones, which proves validity of the numerical approach. Relative error is composed of the errors introduced by numerical methods (the method of matrix inverse, the Runge-Kutta method, the set up method) and by computations. In conclusion, full reliability of the results obtained via the Bubnov-Galerkin method with higher approximations $(N_1 = N_2 = N = 5)$ has been illustrated while solving ODEs governing static linear problems.

In order to estimate the critical load values one may apply either the Volmir criterion [4] (either sudden deflection increase vs. slight load variation or occurrence of the inflexion point of $q_0(w)$) or the criterion proposed by Shian et al. [5]. The last one is associated with load estimation where an inversed time process enables finding of a maximum load on the "load-time" dependence.

Table 1. Analytical vs. numerical results

φ_0	Analytical solution $\frac{w}{qR/E}$	Numerical solution $\frac{\bar{w}}{q}\cdot\left(R/2h\right)\cdot\lambda^2$	Ratio $\frac{\bar{w}}{q}$	Error
2π	50	52.8417	0.058713	5%
$\frac{3\pi}{2}$	6.97	7.4808	0.008312	7%
π	101.07	108.9054	0.121006	7%
$\frac{2\pi}{3}$	-524.2	-744.2469	-0.826941	28%
$\frac{\pi}{2}$	-479.47	-436.212	-0.48468	8%
$\frac{\pi}{4}$	3886.27	3948.354	4.38706	1.5%

4. **Static problems of shells theory**

We study the closed cylindrical shell with $\lambda = \frac{L}{R} = 3$, and we apply the transversal load $q(t) = q_0$, distributed within a zone with central angle $\varphi_0 = \frac{3}{4}\pi$, $0.1 \le x \le 0.5$ (see Figure 3). In series (13) we take $N_1 = N_2 = N$. We analyze a maximum shell deflection vs. amplitude of the applied load $w_{\max}(q_0)$ depending on the Bubnov-Galerkin approximation order (Figure 4). It is clear that the method convergence increases with an increase of approximation. Namely, an increase of terms number in (13) yields convergence of successive critical loads, i.e. for $N = 7$ we have pre-critical load $q_1 = 0.36$, for $N = 9$ we have $q_1 = 0.34$, and for $N = 11$ we have $q_1 = 0.33$ (the difference of results reaches 0.01).

For a more detailed analysis we study the shell vibrations modes in space (Table 2) in pre- and post-critical states. Analysis of the obtained results shows that convergence of the modes is achieved already for $N = 5$ in both analyzed cases. Owing to this, one may conclude that an

increase of series terms in (13) does not yield qualitative changes of vibrations and $N = 5$ is sufficient to investigate static local load acting on the shell.

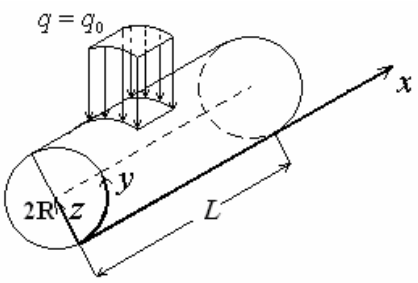


Figure 3. Computational scheme

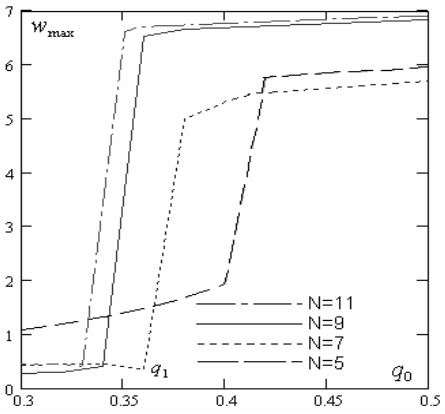
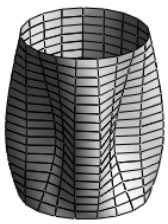
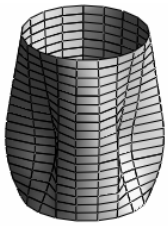
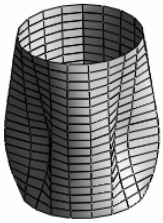
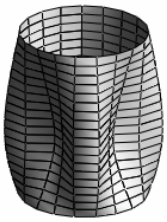
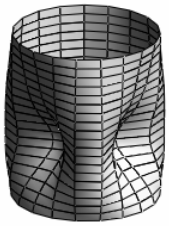
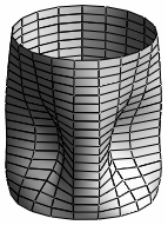
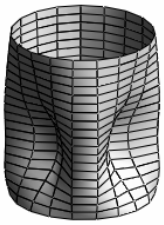
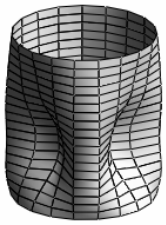


Figure 4. Function $w_{max}(q_0)$

Application of the Bubnov-Galerkin method in higher approximations in the case of our problem requires high computational time and hence is not economical, and therefore computations for large values of $N_1 = N_2 = N$ are not carried out.

Table 2. Pre- and post-critical shell configurations

	N=5	N=7	N=9	N=11
Pre-critical load				
Post-critical load				

5. Conclusions

In the case of local load acting on the closed cylindrical shell and in the case of static load the qualitative convergence of the Bubnov-Galerkin method is achieved regarding vibration modes in pre- and post-critical states. However, in the case of local dynamic load action and in the case of infinite length impulse action (these results will be shown during the paper presentation) the qualitative method convergence is achieved only for the pre-critical state (in the post-critical state with an increase of the series terms of the fundamental functions development the longitudinal waves number is increased and the general shell vibration chart is more complicated).

Therefore, one needs to increase the number of series terms, but it requires extremely long computational time.

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