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ON THE PERIODIC BUSH VIBRATIONS IN A SELF-EXCITED SYSTEM WITH IMPACTS

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Abstract: In this work mathematical model of periodic bush vibrations in a bush-shaft self-excited system with impacts is derived and studied. The bush and shaft are coupled via friction, which depends on both normal contact force and relative velocity of the moving bodies. Friction generates heat and wear on the contact surface. The obtained prediction is verified numerically showing surprisingly good agreement. Some application oriented results are also given.

1. Introduction

In this work the model of a contact bush-shaft system with heat and wear generated by friction and/or impacts is proposed and studied. The bush motion is bounded by rigid barriers and it takes place within the introduced clearance. The bush is considered as a rigid body being fixed to a foundation via mass-less springs and dampers, and it is mounted on the rotating with constant angular velocity thermo-elastic shaft. The so far stated problem is reduced to analysis of the equations governing bush dynamics taking into account impacts and nonlinear friction. The latter one is a product of time depended contact pressure and relative velocity of the shaft and the bush. The contact pressure value is estimated by a second order Volterra-type equation. In the case of a small slope of the kinematical friction coefficient, the restitution coefficients required to realize the system periodic impact motion either with one or two impacts are estimated analytically.

Analytically predicted vibro-impact stick-slip and slip-slip dynamics has been also verified numerically.

2. Problem formulation

We are focused on modeling of non-linear dynamics of two bodies consisting of a stiff bush 2 with clearance $2\Delta_\varphi$ (see Figure 1) and solid isotropic circular shaft 1 of radius R_1. The bush external radius R_2, whereas internal bush radius is $R_1 - U_0$ ($U_0/R_1 << 1$), and U_0 corresponds to initial

shaft compression. The bush is linked with housing by springs with stiffness k_2 and the damper with viscous coefficient c, and is mounted on the rotating thermo-elastic shaft 1.

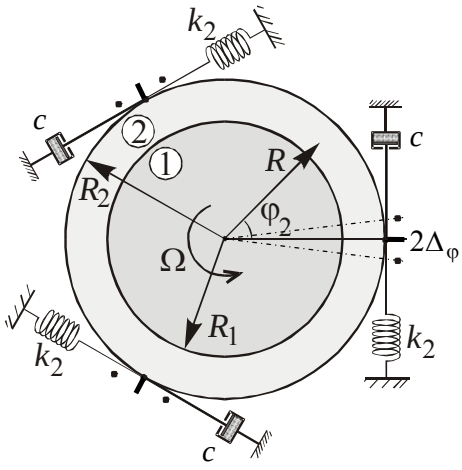

Figure 1. A model of the problem

The following assumptions are taken [1]:

(i) The external excitation of the system allows neglecting of the inertial term in the Lamé equation;

(ii) The vector components related to displacement as well as the shaft temperature $T(R,t)$ depend only on the radial coordinate R and time t;

(iii) The following friction model is introduced between shaft and bush:

$$F_{\text{fr}}(V_r,\varphi_2)=\begin{cases} f(V_r)2\pi R_1 P(t), & V_r\neq 0,\ \text{slip} \\ \min\left(R_1^{-1}R_2^2\left|c\Omega+k_2\varphi_2\right|, F_0 2\pi R_1 P(t)\right)\operatorname{sgn}(c\Omega+k_2\varphi_2), & V_r=0,\ \text{stick} \end{cases}, \tag{1}$$

where: $F_0 2\pi R_1 P(t)$ is static friction force, $F_0 = f(+0)$, $P(t)$ is the contact pressure, $f(V_r)$ is the kinetic friction coefficient depending on relative velocity of the contact bodies $V_r = R_1\Omega - R_1\varphi_2'(t)$, $\varphi_2(t)$ is the angle of bush deviation;

(iv) Approximating curve $f(V_r)$ has the following form

$$f(V_r)=\operatorname{sgn}(V_r)F(|V_r|),\ F(V_r)=\begin{cases} F_0-\kappa V_r, & 0<V_r\le V_{\min} \\ F_0-\kappa V_{\min}, & V_{\min}<V_r \end{cases}, \tag{2}$$

where F_0, κ, $V_{\min}$ are constant coefficients;

(v) Heat flows q_1 and q_2 are generated on the contact surface $R=R_1$ and governed by the equation $q_1+q_2=(1-\eta)f(V_r)P(t)V_r$, where $\eta\in[0,1]$ denotes the heat energy part, which is dissipated by wear [2,3]. Both flows q_1 and q_2 go into shaft and bush, respectively:

$$q_1 = \lambda_1 \frac{\partial T(R_1,t)}{\partial R},\ q_2 = -\alpha_T\big(T_0 - T(R_1,t)\big), \tag{3}$$

where λ_1 is the thermal conductivity, α_T is the heat transfer coefficient between the shaft and the bush, T_0 is the temperature of the bush. Furthermore, we assume that bush transforms heat ideally and $T_0 = 0$.

(vi) Velocity of the bush wear is proportional to a certain power of friction force. We assume Archard's law of wear [4] of the form

$$dU^w(t)/dt = K^w\left|V_r(t)\right|P(t), \tag{4}$$

where the coefficient K^w is usually identified experimentally.

The dimensionless equations governing dynamics of the analyzed system have the form

$$\ddot{\varphi}(\tau) + 2h\dot{\varphi}(\tau) + \omega_0^2\varphi(\tau) = \operatorname{sgn}(\omega_1 - \dot{\varphi})\Psi(\dot{\varphi})\,p(\tau),\ \left|\varphi(\tau)\right| < 1,\ \dot{\varphi}(\tau) \neq \omega_1, \tag{5}$$

$$\ddot{\varphi}(\tau) = 0,\ \left|\varphi(\tau)\right| < 1,\ \dot{\varphi}(\tau) = \omega_1, \tag{6}$$

$$\dot{\varphi}^+ = -k\dot{\varphi}^-,\ \left|\varphi\right| = 1,\ \dot{\varphi}^-\varphi > 0, \tag{7}$$

$$\varphi(0) = x,\ \dot{\varphi}(0) = y, \tag{8}$$

where k is the coefficient of restitution, and $\dot{\varphi}^-$ ($\dot{\varphi}^+$) is the bush velocity just before (after) impact,

$$\Psi(\dot{\varphi}) = \begin{cases} 1 + \varepsilon\omega_1\eta_0, & \dot{\varphi} < \omega_1\eta_0,\ \omega_1(2-\eta_0) < \dot{\varphi} \\ 1 + \varepsilon\dot{\varphi}, & \omega_1\eta_0 < \dot{\varphi} < \omega_1 \\ 1 + 2\varepsilon\omega_1 - \varepsilon\dot{\varphi}, & \omega_1 < \dot{\varphi} < \omega_1(2-\eta_0) \end{cases} \tag{9}$$

In order to solve the motion equations (5) one needs to know contact pressure $p(\tau)$ and wear $u^w(\tau)$ [1]:

$$p(\tau) = H(\tau) - u^w(\tau) + 2\gamma_1\tilde{\omega}^{-1}\int_0^\tau \dot{G}_p(\tau-\xi)\Psi\big(\dot{\varphi}(\xi)\big)\left|\omega_1 - \dot{\varphi}(\xi)\right|p(\xi)d\xi, \tag{10}$$

$$u^w(\tau) = k^w\int_0^\tau \left|\omega_1 - \dot{\varphi}(\tau)\right|p(\tau)d\tau. \tag{11}$$

Then the problem is reduced to consideration of equations (5) and (10), which yield both dimensionless pressure $p(\tau)$ and velocity $\dot{\varphi}(\tau)$. The temperature is defined by the following formula

$$\theta(r,\tau) = \gamma_1\tilde{\omega}^{-1}\int_0^\tau \dot{G}_\theta(r,\tau-\xi)\Psi\big(\dot{\varphi}(\xi)\big)\left|\omega_1 - \dot{\varphi}(\xi)\right|p(\xi)d\xi, \tag{12}$$

where

$$\left\{G_p(\tau), G_\theta(1,\tau)\right\} = \frac{\{0.5, 1\}}{Bi} - \sum_{m=1}^{\infty} \frac{\left\{2Bi, 2\mu_m^2\right\}}{\mu_m^2(Bi^2+\mu_m^2)} e^{-\mu_m^2 \tilde{\omega}\tau}, \tag{13}$$

and μ_m are the roots of the characteristic equation $BiJ_0(\mu) - \mu J_1(\mu) = 0$.

In the above the following dimensionless parameters are introduced:

$$\tau = \frac{t}{t_*},\ r = \frac{R}{R_1},\ \varphi(\tau) = \frac{\varphi_2}{\Delta_\varphi},\ p = \frac{P}{P_*},\ \theta = \frac{T_1}{T_*},\ u^w = \frac{U^w}{U_0},\ \alpha_0 = \frac{f_0}{F_0},\ \omega_1 = \frac{\Omega t_*}{\Delta_\varphi},\ \mu_0 = \frac{1-\alpha_0}{\alpha_0},$$

$$\eta_0 = \frac{V_0 - V_{\min}}{V_0},\ \varepsilon = \frac{\mu_0}{\omega_1},\ \gamma = \sqrt{\frac{2\pi R_1^2 P_* F_0 \Delta_\varphi}{B_2 \Omega^2}},\ \omega_0^2 = \frac{k_2 R_2^2 t_*^2}{B_2},\ 2h = \frac{cR_2^2 t_*}{B_2},\ Bi = \frac{\alpha_T R_1}{\lambda_1},\ \Psi = \frac{F}{f_0},$$

$$f(V_0) = f_0,\ \tilde{\omega} = \frac{t_*}{t_T},\ k^w = \frac{K^w \Delta_\varphi E_1}{(1-2\nu_1)(1+\nu_1)},\ \gamma_1 = \frac{(1-\eta)E_1\alpha_1 R_1^2 f_0 \Delta_\varphi}{\lambda_1(1-2\nu_1)t_T},\ x = \frac{\varphi_2^\circ}{\Delta_\varphi},\ y = \frac{\omega_2^\circ t_*}{\Delta_\varphi}, \tag{14}$$

where:

$$t_* = \sqrt{\frac{B_2 \Delta_\varphi}{f_0 2\pi R_1^2 P_*}},\ V_* = \frac{R_1 \Delta_\varphi}{t_*},\ P_* = \frac{E_1 U_0}{(1-2\nu_1)(1+\nu_1)R_1},\ T_* = \frac{U_0}{\alpha_1(1+\nu_1)R_1},\ t_T = \frac{R_1^2}{a_1},\ V_0 = \Omega R_1.$$

B_2 is the moment of inertia of the bush, $F_s = k_2 \varphi_2 R_2$ is springs force related to the bush, $F_d = cR_2\varphi'(t)$ is the damping force related to the bush, F_{fr} is friction force between the bush and the shaft, and B_2, k_2, c, F_{fr} are the quantities measured per length unit bush.

3. Analysis in the case of lack of tribological processes

First the case of bush vibrations without tribological processes is studied ($\gamma_1 = 0$, $k^w = 0$, $\delta = 4$, $\chi = 0.5$). For this case we have $p(\tau) = H(\tau)$ ($H(\tau) = 1, .\ \tau > 0$, $H(\tau) = 0, .\ \tau \le 0$). Our system governed by equations (20) may exhibit four different periodic motions. Namely: (i) periodic orbit with one impact, where a stick does not appear (Figure 2a), $\omega_1 = 2$, $\varepsilon = 0.01$, $x = -0.9$, $k = 0.993$; (ii) periodic orbit with two impacts, where a slip of the contacting bodies occurs (Figure 2b), $\omega_1 = 2.5$, $\varepsilon = 0.01$, $x = -1.25$, $k = 0.994$; (iii) periodic orbit with one impact, where a stick-slip occurs (Figure 2c), $\omega_1 = 2$, $\varepsilon = 0.1$, $x = -1$, $k = 0.967$; (iv) periodic orbit with two impacts, where a stick-slip appears (Figure 2d), $\omega_1 = 2.5$, $\varepsilon = 0.05$, $x = -2$, $k = 0.994$.

In what follows we assume that $\varepsilon << 1$, $\omega_0^2 << 1$, $2h << 1$ and $\eta_0 \le -1$. It means that the system dynamics is exhibited in the interval ($0 < V_0 < V_{\min}$), where a decreasing slope of the kinetic friction coefficient is observed.

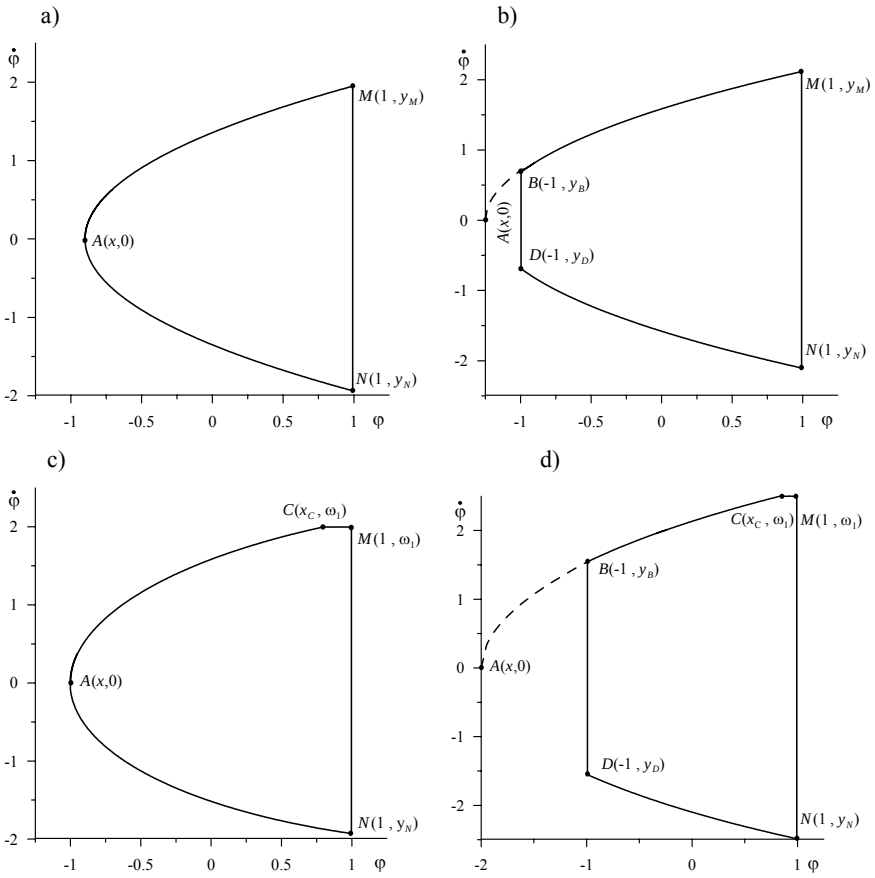

Figure 2. Periodic bush phase orbit

The reported below results are yielded by a standard perturbation approach. It allows us to give formulas for the coefficient of restitution $k(x,\omega_1)$

$$k(x,\omega_1)=\begin{cases} k_{ACMNA}, & 0<\omega_1<2-(4/3)(1-\chi)\varepsilon, \quad x_1<x<x_0 \\ k_{AMNA}, & 0<\omega_1<2-(4/3)(1-\chi)\varepsilon, \quad x_0<x<1 \\ k_{BCMNDB}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad x_1<x<-1 \\ k_{ACMNA}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad -1<x<x_0 \\ k_{AMNA}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad x_0<x<1 \\ k_{BCMNDB}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad x_2<x<-1 \\ k_{ACMNA}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad -1<x<x_0 \\ k_{AMNA}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad x_0<x<1 \\ k_{BCMNDB}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad x_2<x<x_0 \\ k_{BMNDB}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad x_0<x<-1 \\ k_{AMNA}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad -1<x<1 \end{cases} \tag{15}$$

and dimensionless period $\tau(x,\omega_1)$ for various cases of the system periodic dynamics

$$\tau(x,\omega_1)=\begin{cases}\tau_{ACMNA}, & 0<\omega_1<2-(4/3)(1-\chi)\varepsilon, \quad x_1<x<x_0\\ \tau_{AMNA}, & 0<\omega_1<2-(4/3)(1-\chi)\varepsilon, \quad x_0<x<1\\ \tau_{BCMNDB}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad x_1<x<-1\\ \tau_{ACMNA}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad -1<x<x_0\\ \tau_{AMNA}, & 2-(4/3)(1-\chi)\varepsilon<\omega_1<2, \quad x_0<x<1\\ \tau_{BCMNDB}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad x_2<x<-1\\ \tau_{ACMNA}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad -1<x<x_0\\ \tau_{AMNA}, & 2<\omega_1<2+(4/3)(1-\chi)\varepsilon, \quad x_0<x<1\\ \tau_{BCMNDB}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad x_2<x<x_0\\ \tau_{BMNDB}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad x_0<x<-1\\ \tau_{AMNA}, & 2+(4/3)(1-\chi)\varepsilon<\omega_1<\infty, \quad -1<x<1\end{cases} \tag{16}$$

where

$$k_{AMNA}=1-(2/3)\tau_1\varepsilon+o(\varepsilon^2),\ \tau_1=\sqrt{2(1-x)},\ \tau_{AMNA}=2\tau_1-(1/12)\tau_1(5\tau_1^2-12)\delta\varepsilon+o(\varepsilon^2),$$

$$k_{BMNDB}=1+\frac{\tau_2^3-\tau_1^3}{3(2+\tau_2^2)}\varepsilon+o(\varepsilon^2),\ \tau_2=\sqrt{-2(1+x)},\ x_2(\omega_1)=x_1(\omega_1)+(2/3)(\omega_1^2-4)^{3/2}(1-\chi)\varepsilon,$$

$$\tau_{BMNDB}=2(\tau_1-\tau_2)-\frac{4\tau_2\tau_1}{3(2+\tau_2^2)}(1-\chi)\varepsilon+\frac{(5\tau_2^2+12)\tau_2\tau_1-(5\tau_2^4+28\tau_2^2+32)}{12\tau_1}\delta\varepsilon+o(\varepsilon^2),$$

$$k_{ACMNA}=\tau_1/\omega_1-(1/3)(\tau_1^2/\omega_1)\varepsilon-(1/8)(\tau_1/\omega_1)(4-\tau_1^2)\delta\varepsilon+o(\varepsilon^2),\ \tau_0=\sqrt{2+\sqrt{4+\tau_2^2\omega_1^2}},$$

$$\tau_{ACMNA}=(\tau_1+\omega_1)^2/(2\omega_1)+(1/6)(\tau_1^2-\omega_1^2)(1-\chi)\varepsilon+$$
$$(1/24)(\tau_1+\omega_1)(12-5\tau_1^2+\omega_1^2-\omega_1\tau_1)\delta\varepsilon+o(\varepsilon^2),$$

$$k_{BCMNDB}=\frac{\tau_0}{\omega_1}+\frac{-16\tau_0^2+\tau_2^3\omega_1^2(\tau_0+\omega_1)-\tau_2^2\omega_1^2(4+\tau_0^2)}{6\omega_1(\tau_2^2\omega_1^2+2\tau_0^2)}\varepsilon+\frac{\tau_2^2\tau_0\omega_1(\tau_2^2+4)}{16(\tau_2^2\omega_1^2+2\tau_0^2)}\delta\varepsilon+o(\varepsilon^2),$$

$$\tau_{BCMNDB}=(\tau_1^2-4\tau_2\omega_1+2\tau_1\omega_1+\omega_1^2)/(2\omega_1)+$$
$$(1/6)(\tau_2^4-8\tau_2\tau_1-\tau_2^2\omega_1^2+6\tau_2^2-2\omega_1^2+8)(1-\chi)\varepsilon/(2+\tau_2^2)+$$
$$(1/24)(24\tau_2+10\tau_2^3-8\tau_1-12\omega_1+\omega_1^3-5\tau_2^2\tau_1-6\tau_2^2\omega_1)\delta\varepsilon+o(\varepsilon^2),$$

$$x_0(\omega_1)=1-(1/2)\omega_1^2+(1/3)\omega_1^3(1-\chi)\varepsilon+(1/8)\omega_1^2(\omega_1^2-4)\delta\varepsilon+o(\varepsilon^2),$$

$$x_1(\omega_1)=1-0.5\omega_1^2-(1/3)\omega_1^3(1-\chi)\varepsilon+(1/8)\omega_1^2(\omega_1^2-4)\delta\varepsilon+o(\varepsilon^2),$$

A detailed analysis shows that the function $k(x,\omega_1)$ possesses the following values $k(x_1,\omega_1)=1,\ 0<\omega_1<2$, $k(x_2,\omega_1)=1,\ 2<\omega_1<\infty$, $k(1,\omega_1)=1$ at the boundaries, whereas inside the considered interval it has the minima, which can be presented in the form

$$k_{\min} = \begin{cases} 1-(2/3)\omega_1(1-\chi)\varepsilon, & 0 < \omega_1 < 2+(4/3)(1-\chi)\varepsilon \\ 1-(4/3)(1-\chi)\varepsilon, & 2+(4/3)(1-\chi)\varepsilon < \omega_1 < \infty \end{cases}. \quad (17)$$

Note that for an arbitrary $k^* \in (k_{\min},1)$ there are two values of x_1^*, x_2^*. Let us introduce the following intervals

$$x_1 < x_1^* < x_0, x_0 < x_2^* < 1 \text{ for } 0 < \omega_1 < 2, \quad (18)$$

$$x_2 < x_1^* < x_0, x_0 < x_2^* < 1 \text{ for } 2 < \omega_1 < 2+(4/3)(1-\chi)\varepsilon, \quad (19)$$

$$x_2 < x_1^* < -1, -1 < x_2^* < 1 \text{ for } 2-(4/3)(1-\chi)\varepsilon < \omega_1 < \infty. \quad (20)$$

It is not difficult to check that the periodic orbit associated with x_1^* (decreasing part of the coefficient $k(x)$) is stable, whereas the periodic orbit associated with x_2^* (increasing part of the coefficient $k(x)$) is unstable.

4. Numerical analysis

Time histories of contact pressure, temperature on surface contact are reported in Figure 3. Curves 1 correspond to the case when $\gamma_1 = 0$ (lack of heat extension), $k^w = 0$ (lack of bush wear). Curves 2 correspond to the case of heat transfer lack ($\gamma_1 = 0$) and $k^w = 0.01$.

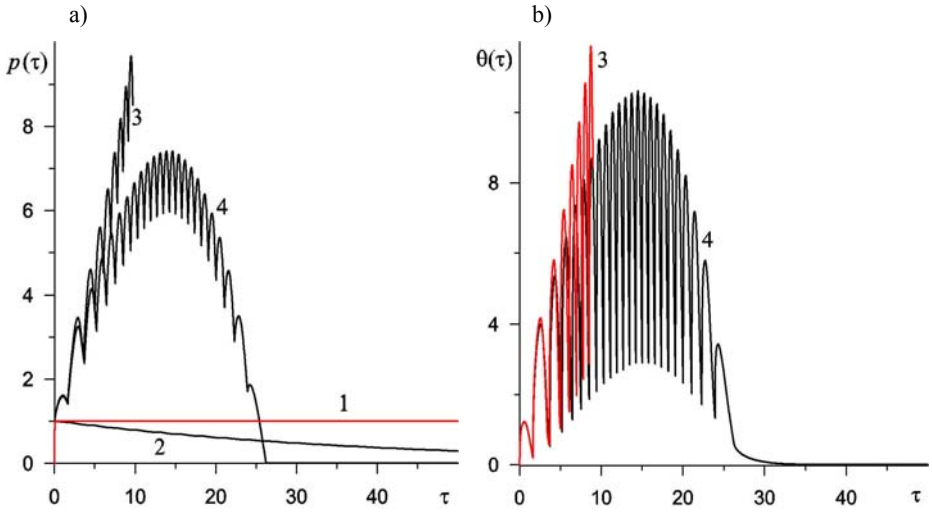

Figure 3. Time histories of dimensionless contact pressure $p(\tau)$ (a) and dimensionless contact temperature $\theta(\tau)$ (b) versus dimensionless time τ for different values of γ_1 and k^w.

Curves 3 correspond to the case, where the shaft heat expansion is taken into account ($\gamma_1 = 0.5$), but the bush wear is neglected ($k^w = 0$). Curves 4 correspond to the case, where both mentioned parameters are taken into account ($\gamma_1 = 0.5$, $k^w = 0.01$).

4. Conclusions

Mathematical model of periodic bush impact-type vibrations of the bush-shaft system taking into account tribological processes is derived. We have also address an analytical approach to the stated problem by introduction a perturbation parameter associated with the friction cinematic slope. The latter technique allowed us to derive the appropriate restitution coefficients responsible for realization of impact periodic vibration of stick-slip and/or slip types and either with one or with two impacts during the motion period. The period is analytically estimated, and the predicted analytically results have been compared with numerical ones showing very good agreement. Observe that this approach extends our earlier studies. Namely, dynamics of the mentioned bush-shaft system without impacts is studied in references [1,2,5], whereas influence of the introduced gap between both bodies on their non-linear dynamics is reported in [6].

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References

1. Awrejcewicz J., Pyryev Yu., Thermoelastic contact of a rotating shaft with a rigid bush in conditions of bush wear and stick-slip movements. *Int. J. Engng. Sci.*, **40**, 2002, 1113-1130.
2. Pyryev Yu., Dynamics of Contacting Systems with an Account of Heat Transfer, Friction and Wear. Łódź, Scientific Bulletin of Łódź Technical University, No 936, 2004, in Polish.
3. Ling F. F., A quasi-iterative method for computing interface temperature distribution. *ZAMP*, **10**, 1959, 461–474.
4. Archard J.F., Contact and rubbing of flat surface. *J. Applied Physics*, **24**, 8, 1953, 981-988.
5. Awrejcewicz J., Pyryev Yu., Contact phenomena in braking and acceleration of bush-shaft system, *Journal of Thermal Stresses*, **27**(5), 2004, 433–454.
6. Balandin D.V., Frictional vibrations in a gap, *MTT* ,1, 1993, 54-60, in Russian.

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