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ON THE STICK-SLIP VIBRATIONS CONTINUOUS FRICTION MODEL

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Abstract: In this work an alternative (novel) continuous friction model is proposed and presented, which suitable for analysis of stick-slip vibrations caused by dry friction. It takes into account some elements of the known switch model. Advantages of the proposed model are illustrated and discussed using a one degree-of-freedom model with dry friction, which exhibits regular and chaotic dynamics. The system dynamics is monitored via standard characteristics like trajectories of motion in the system's phase space, bifurcation diagrams as well as the Lyapunov exponents. The obtained results exhibit advantages of (the proposed model in comparison to the often used switch model.

1. Introduction

Although dry friction belongs to one of the most known phenomenon exhibited by mechanical systems, but its proper mathematical modeling does not belong to easy tasks. Friction force between sliding surfaces is a complex process and depends on several parameters, e.g. relative velocity of sliding surfaces, normal load, time, temperature. An extensive literature review on the dependence of friction on these parameters and dry friction models can be found in works of [Kragelsky and Shchedrov (1956) [6], Martins, Oden and Simoes (1990) [10], Ibrahim (1994) [5], Awrejcewicz and Lamarque (2003) [2], Awrejcewicz and Olejnik (2005) [3], Andersson, Soderberg and Bjorklund (2007) [1]]. The cited references mainly address dry friction stick-slip oscillations with different models of friction.

The aim of this paper is to propose an alternative continuous friction model suitable for analysis of stick-slip vibrations caused by dry friction. Continuous friction model is introduced and presented in Section 2. As an example of mechanical system a one degree-of-freedom (1-dof) model is considered in Section 3, which exhibits regular and chaotic dynamics. Numerical calculations and results of our study can be found in Section 4, where advantages of the proposed model are illustrated and discussed, and where also phase portraits, bifurcations diagram as well as the Lyapunov exponents are reported. Conclusions of our study are presented in the last Section 5.

2. Continuous Friction Model

Several dry friction formulations have been proposed based on the classical Coulomb model. Owing to the Amonton's assumptions the friction force F_{fr} is defined as a function of the relative velocity v_{rel} of sliding surfaces in the slip phase and as a function of the externally applied force F_{ex} in the stick phase. This model is known as the "signum model with static friction point" and describe dry friction phenomenon in the correct and accurate way. Note that during numerical simulation an exact value of zero is rarely computed. For this reason the "signum model" is equivalent, from a numerical point of view, to the classical Coulomb model.

The dependence of friction force on the relative velocity based on the "signum model" is not continuous function for argument equal to zero and standard numerical procedures devoted for solving differential equations cannot be used. The friction curve is therefore often approximated by a continuous or smooth function. Usually, friction curve approximated by these functions are continuous or even smooth, but for $v_{rel} = 0$ the value of friction force is always equal to zero. In another words, the friction force depends on v_{rel} but not depends on F_{ex} in the stick phase.

In two of the recent papers (Leine et. al. (1998) [8, 9]) devoted to mathematical modeling of dry friction, the so called "switch model" is proposed and used in order to match the obtained numerically simulation results with those given by the experimental investigation of the mechanical bodies exhibiting stick–slip vibrations. Switch model (from a mathematical point of view) is governed by three systems of nonlinear ordinary differential equations: one for the slip phase, a second for the stick phase and a third for the transition from stick to slip. In paper [8] it has been shown (for one system of parameters) that from a computational point of view the smoothing methods are more expensive than the switch model based methods.

Now we introduce an alternative continuous friction model which takes into account some elements of the mentioned switch model. We propose continuous friction model using friction force on the base the switch model. The space F_{ex}-v_{rel} is divided into four regions as follows

$V_1 : |v_{rel}| > \varepsilon$,

$V_2 : [(0 \le v_{rel} \le \varepsilon) \wedge (F_{ex} > F_s)] \vee [(-\varepsilon \le v_{rel} \le 0) \wedge (F_{ex} < -F_s)]$,

$V_3 : [(0 < v_{rel} \le \varepsilon) \wedge (F_{ex} < -F_s)] \vee [(-\varepsilon \le v_{rel} < 0) \wedge (F_{ex} > F_s)]$,

$V_4 : (|v_{rel}| \le \varepsilon) \wedge (|F_{ex}| \le F_s)$.

The continuous friction force is defined by the following way

$$F_{fr} = \begin{cases} F_k \operatorname{sgn}(v_{rel}) & \text{for} \quad V_1 \\ F_s \operatorname{Sgn}_X(v_{rel}, F_{ex}) & \text{for} \quad V_2 \\ (2A-1) F_s \operatorname{sgn}(v_{rel}) & \text{for} \quad V_3 \\ A(-F_{ex} + F_s \operatorname{sgn}(v_{rel})) + F_{ex} & \text{for} \quad V_4 \end{cases}, \tag{1}$$

and

$$A = \frac{v_{rel}^2}{\varepsilon^2}\left(3 - 2\frac{|v_{rel}|}{\varepsilon}\right) \tag{2}$$

is the approximating function. The function

$$\operatorname{Sgn}_X(v_{rel}, F_{ex}) = \begin{cases} \operatorname{sgn}(v_{rel}) & \text{for} \quad v_{rel} \neq 0 \\ \operatorname{sgn}(F_{ex}) & \text{for} \quad v_{rel} = 0 \end{cases} \tag{3}$$

guarantees continuity of friction force in the region of V_2 for a relative velocity equal to zero.

Figure 1 shows the dependences of friction force on the relative velocity v_{rel} in the region of near-zero relative velocity for a few fixed externally applied forces F_{ex}.

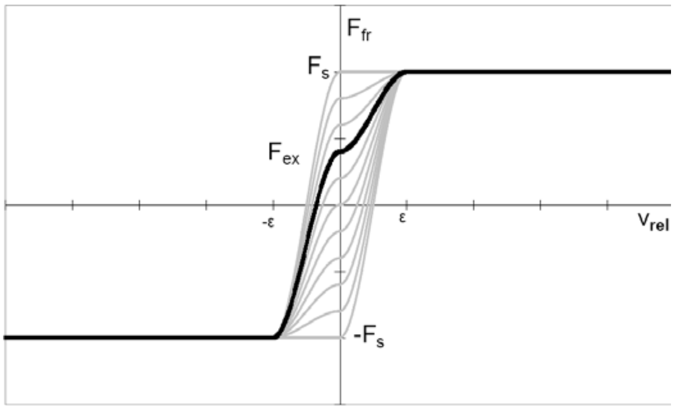

Fig. 1. Friction force of the continuous friction model for fixed externally applied forces as a function of a relative velocity.

In our model friction force is a continuous function on v_{rel} (like in smoothing methods) and for v_{rel} = 0 friction force is equal to externally applied force F_{ex} (like in signum model). In another words, our continuous friction model can be treated as an approximating friction force appeared in switch model using the continuous functions.

3. One Degree-of-Freedom Model

To demonstrate the above presented continuous friction model as an example we consider a single-degree-of-freedom (1-dof) model with dry friction. This model possesses a stick-slip periodic and non-periodic solutions. Our 1-dof system is shown in Figure 2.

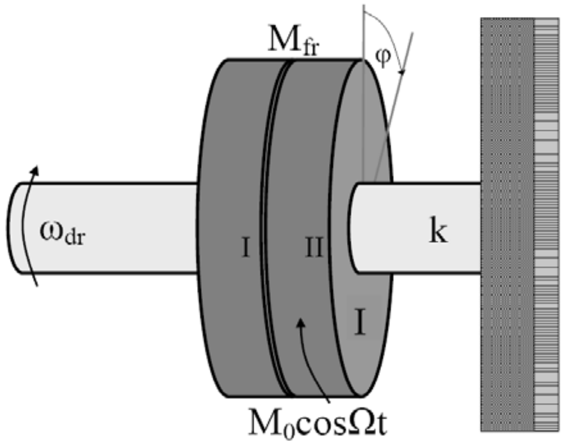

Fig. 2. One degree-of-freedom model with dry friction.

The disc (II) is characterized by linear stiffness k and the mass moment of inertia I. It is riding on a driving disc (I), that is moving at angular velocity ω_{dr}. Between first disc (I) and second one (II) dry friction occurs and generates a moment of friction force M_{fr}. In addition, harmonic excitation with amplitude M_0 and circular frequency Ω is added to our model. The relative angular velocity of the second disc with respect to the first disc is denoted by $\omega_{rel} = \omega_{dr} - \dot{\varphi}$ and $M_{ex} = k\varphi - M_0\cos\Omega t$. The moment of friction force $M_{fr} = F_{fr}\cdot r$, where r is the average radius of friction and F_{fr} is given by equation (1) with friction force in the slip phase

$$F_k = \frac{F_s}{1+\delta\left(\left|\omega_{rel}\right| - \varepsilon\right)} \cdot \tag{4}$$

The following second-order differential equation of this system is

$$\mathrm{I}\ddot{\varphi} + k\varphi = M_{fr} + M_0 \cos\Omega t \cdot \tag{5}$$

This dynamical system can be expressed as a set of first-order ordinary differential equations. The governing equations read

$$\begin{cases} \dot{\varphi} = \omega \\ \dot{\omega} = \left(-k\varphi + M_{fr} + M_0\cos\phi\right)/\mathrm{I} \\ \dot{\phi} = \Omega \end{cases}, \tag{6}$$

where the dot (·) denotes differentiation with respect to time. Below the initial parameters of our model are presented: $I = 2$ kg·m^2, $k = 10$ N·m·rad^{-1}, $F_s = 20$N, $r = 0.1$ m, $\delta = 3$ s·rad^{-1}, $\omega_{dr} = 0.3$ rad·s^{-1}, $\Omega = 2$ rad·s^{-1}, $M_0 = 0.5$ N·m. Numerical parameters are: time step $h = 10^{-3}$ s and steepness parameter $\varepsilon = 10^{-3}$ rad·s^{-1}.

4. Numerical Calculations and Results

In the signum model and switch model friction force is non-continuous function of relative velocity and therefore the methods commonly used to compute the Lyapunov exponents cannot be applied. Continuous friction model as proposed in this paper, does not possess this disadvantage and can be used during analysis of the systems, where the Lyapunov exponents are computed by standard procedures [4, 7]. Note, that while computing Lyapunov exponents, the equations (6) and three additional systems of equations with respect to perturbations are solved.

In order to simulate the stick-slip vibrations using the proposed continuous friction model equations (6) are solved. The state equations for the switch model read

$$\begin{cases} \begin{bmatrix} \dot{\varphi} = \omega \\ \dot{\omega} = \left(-k\varphi + \dfrac{M_s \operatorname{sgn}(\omega_{rel})}{1 + \delta(|\omega_{rel}| - \varepsilon)} + M_0 \cos\phi \right) \Big/ I \\ \dot{\phi} = \Omega \end{bmatrix} & \text{for } |\omega_{rel}| > \varepsilon \\ \begin{bmatrix} \dot{\varphi} = \omega \\ \dot{\omega} = \left(-k\varphi + M_s \operatorname{sgn}(M_{ex}) + M_0 \cos\phi \right)/I \\ \dot{\phi} = \Omega \end{bmatrix} & \text{for } |\omega_{rel}| \le \varepsilon, |M_{ex}| > M_s \\ \begin{bmatrix} \dot{\varphi} = \omega_{dr} \\ \dot{\omega} = \omega_{rel}\sqrt{\dfrac{k}{I}} \\ \dot{\phi} = \Omega \end{bmatrix} & \text{for } |\omega_{rel}| \le \varepsilon, |M_{ex}| \le M_s \end{cases}, \tag{7}$$

In order to solve the derived ordinary differential equations the standard numerical algorithms often used for study dynamics of lumped mechanical systems can be directly applied. In our study differential equations are solved via the fourth order Runge-Kutta method (RK4) with constant time step h and the Gramm-Schmidt ortonormalization method. The dynamics of the system is monitored via standard characteristics like time histories in the system's phase space, bifurcation diagrams as well as the Lyapunov exponents.

Let us consider first dynamics of the system for $M_0 = 0$, i.e. without harmonic excitation. For this case, the phase portraits obtained with both switch model and continuous friction model are shown in Figure 3. The periodic stick-slip oscillations have the sliding velocity almost the same (both for switch model and continuous friction model). It is visible too, that in the sticking phase some differences are observed. The differences occur in result of another approximating friction force application in near-zero relative velocity neighborhood. In comparison with the switch model we obtain better results for this case using our friction model. It allows to obtain the same accuracy as in the switch model, but for larger time step h and steepness parameter ε. Consequently, the switch model is more expensive than continuous friction model for this case, from the computational point of view.

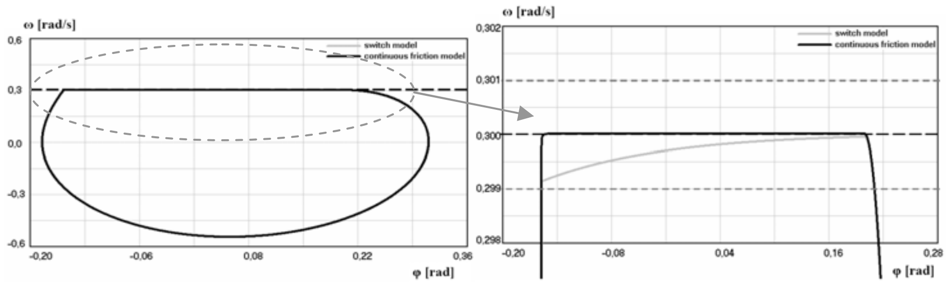

Fig. 3. Phase portraits of the analyzed system without harmonic excitation.

The studied mechanical system (with harmonic excitation) possess both periodic and non-periodic solutions. Figures 4a and 4b presents different behaviors of analyzed mechanical system in time interval $t \in <200, 500>$ s.

a) b)

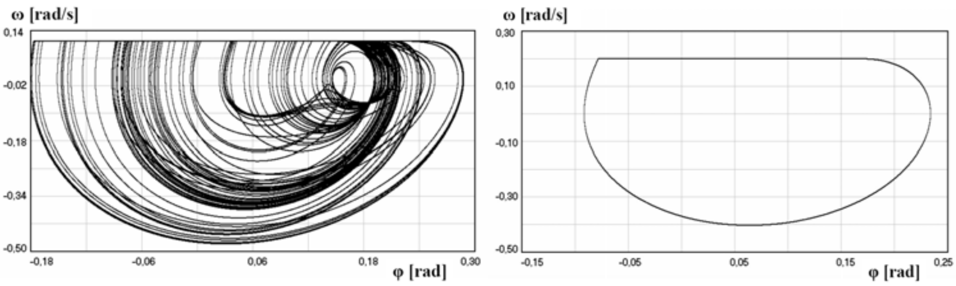

Fig. 4. Phase portraits of the analyzed system with harmonic excitation for various angular velocities ω_{dr}: a) ω_{dr} = 0.11 rad·s^{-1}, b) ω_{dr} = 0.3 rad·s^{-1}.

Below the periodic and non-periodic solutions are detected using bifurcation diagram and the Lyapunov exponents identifications tools. The bifurcation diagram of the system is shown in Figure 5 with the velocity ω_{dr} as control parameter and the angle φ on the vertical axis. In the same plots dependences of the largest Lyapunov exponent λ_1 vs. the control parameters are reported. A study of this bifurcation diagram implies that chaos occurs when the exponent λ_1 is positive. One of the computed Lyapunov exponents is always negative and second is always equal to zero (not shown in Figure 5), since the studied system of equations is autonomous.

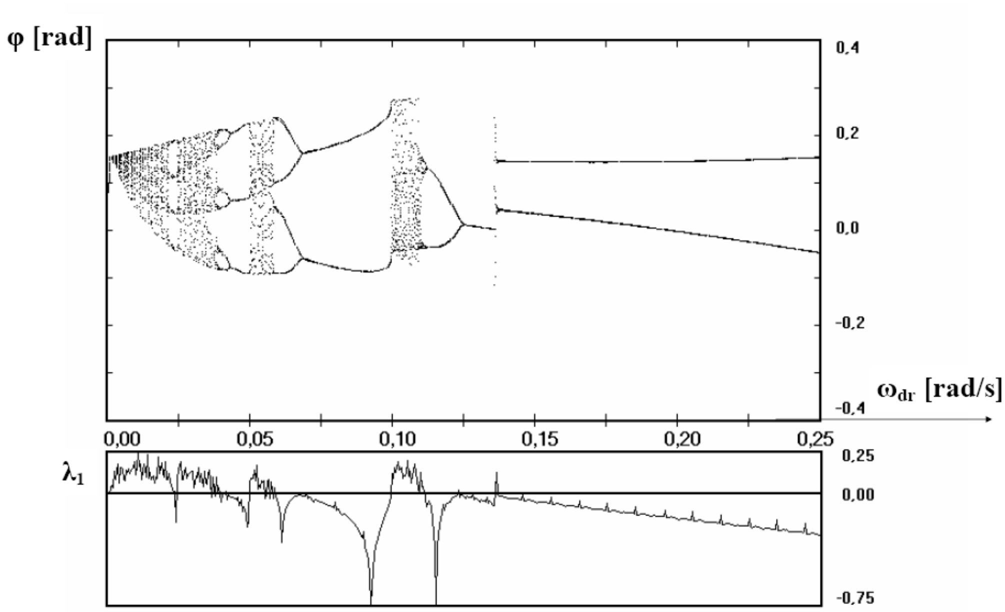

Fig. 5. Bifurcation diagram and the largest Lyapunov exponent of the analyzed 1-dof model with ω_{dr} as control parameter and φ on the vertical axis.

5. Conclusions

The continuous friction model suitable for simulation of the stick-slip vibrations has been proposed and validated using the one degree-of-freedom mechanical system with dry friction. It has been observed that continuous friction model yields engineering accepted results and possesses some advantages in comparison to the switch model. Interesting dynamics of the analyzed system are reported and analyzed, including stick-slip periodic and chaotic behaviors. During analysis the standard techniques, i.e. monitoring of phase portraits, bifurcation diagrams and the Lyapunov exponents have been applied.

One of the important advantages of our novel model is associated with direct application of the standard numerical procedures devoted to either solving differential equations or to computation of the Lyapunov exponents. The obtained results have been compared with those given by switch model application, and they indicate better numerical accuracy of our proposed continuous model. Continuous friction model is validated and it gives correct results, even if the numerical steepness parameter ε is extremely large. It allows obtaining the same accuracy as in the switch model for larger time step h and steepness parameter ε. The proposed continuous friction model may also be suitable for simulation of the stick-slip vibrations and it may be applied to model friction force in any other mechanical systems.

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