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CHAOTIC DYNAMICS OF BEAMS DRIVEN PERIODICALLY BY TRANSVERSAL LOADS

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1 INTRODUCTION

Although deterministic temporal-spatial chaos has been already observed in fluid dynamics (turbulence), but it has been recently also exhibited by structural mechanical objects like beams, plates and shells. The problem regarding existence and uniqueness of solutions of the partial differential equations governed dynamics of the Timoshenko type shells has been rigorously studied in references [1, 2]. On the other hand it is observed that although investigations devoted to chaotic dynamics of plates, and conical and spherical shells are being already often investigated, but chaotic vibrations of geometrically nonlinear Euler-Bernoulli beams are rather rarely studied [3]. We are aimed to fill, at least partially, the existing gap of investigation of dynamics of this class of problems of nonlinear continual systems.

2 PROBLEM STATEMENT

Mathematical model of the studied beams is constructed owing to the Euler-Bernoulli hypothesis taking into account both nonlinear coupling between stresses and displacements and nonlinear dissipation properties.

We study the one-layered beam as a 2D object of the space R^2 applying the rectangular co-ordinates introduced in the following way (Figure 1): In the beam body the so called mean line z = 0 is fixed, axis OX is oriented from left to right along the mean line, and the axis OZ (being perpendicular to OX) goes down. Therefore, in the taken system of co-ordinates our studied beam is defined as follows:

$$\Omega = \{x, z/(x) \in [0; a], (z) \in [-h/2; h/2]\}, \ 0 \le t \le \infty$$

The governing beam (having a squared cross-section) dynamics equations are cast in the form [4]

$$\begin{cases} \dfrac{Eh^3}{12}\dfrac{\partial^4 w}{\partial x^4} + K_1 w - K_2 \dfrac{\partial^2 w}{\partial x^2} = q + Eh[l_1(u,w) + l_2(w,w)] - \dfrac{h\gamma}{g}\left[w'' + \varepsilon |w'|^{m-1} w'\right], \\ Eh\left[\dfrac{\partial^2 u}{\partial x^2} + l_3(w,w)\right] = -p_x + \dfrac{h\gamma}{g} u''. \end{cases} \tag{1}$$

where: $d/dt = '$.

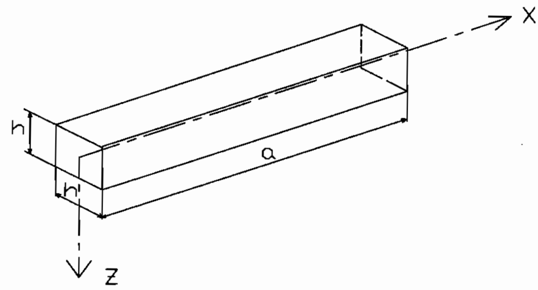

Figure 1. Beam geometry

In system (1) the following non-linear operators are introduced

$$l_1(u,w)=\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2},\quad l_2(w,w)=\frac{3}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2,\quad l_3(w,w)=\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}.$$

Note that equations (1) exhibit two various nonlinearities: (i) nonlinear stress-strain dependence is taken into account [5], and (ii) nonlinear dependence of friction vs. velocity within the Coulomb model ($m = 0$) is applied ($m = 1$ corresponds to linear damping).

Equations (1) require boundary and initial conditions. In this brief report only rigid clamping is further studied and the following boundary conditions are applied

$$w(0,t)=u(0,t)=w'_x(0,t)=w(a,t)=u(a,t)=w'_x(a,t)=0. \tag{2}$$

Initial conditions follow

$$w(x,0)=u(x,0)=u'(x,0)=w'(x,0)=0.$$

Problem (1)-(2) is reduced to the non-dimensional form via the following transformations

$$\bar{t}=t/\tau;\quad \bar{\varepsilon}=\varepsilon\frac{h^{m-1}}{\tau^{m-2}};\quad \tau=\frac{a}{h}\sqrt{\frac{a^2\gamma}{Eg}};\quad \bar{w}=\frac{w}{h};\quad \bar{u}=u\frac{a}{h^2};\quad \bar{x}=\frac{x}{a};$$

$$\bar{q}=\frac{a^4}{Eh^4}q;\quad \bar{p}_x=\frac{a^5}{Eh^5}p_x;\quad b^2=\left(\frac{a}{h}\right)^2;\quad \bar{K}_1=\frac{a^4}{Eh^3}K_1;\quad \bar{K}_2=\frac{a^2}{Eh^3}K_2, \tag{3}$$

where x, t are spatial and time co-ordinates, respectively; ε is the damping coefficient; E, γ are Young modulus and volume specific weight, respectively; q, p_x are transversal and longitudinal loads, respectively; w, u are deflection and displacement mean line functions, respectively; h, a are beam height and length, respectively; K_1, K_2 are first and second foundation coefficients, and m is nonlinear dissipation parameter [5]. For $K_2=0$ equation governing beam dynamics on the Winkler foundation is obtained.

3 ANALYSIS

The studied continuous system is reduced to a lumped system via the finite difference method regarding the spatial co-ordinate with approximation $O(\Delta^2)$. Differential operators regarding x are substituted by finite-difference approximation, and the following system of ordinary differential equations with respect to time (ODEs)is obtained:

$$\begin{cases}\dfrac{1}{12\Delta^4}\lambda_x^2 w_i+K_1w_i-\dfrac{K_2}{\Delta^2}\lambda_x w_i=q+\dfrac{\lambda_x u_i}{\Delta^2}\dfrac{w_{i+1}-w_i}{\Delta}+\dfrac{3}{2}\left(\dfrac{w_{i+1}-w_i}{\Delta}\right)^2\dfrac{\lambda_x w_i}{\Delta^2}+\\ +\dfrac{u_{i+1}-u_i}{\Delta}\dfrac{\lambda_x w_i}{\Delta^2}-\left[w_i''+\varepsilon\left|w_i'\right|^{m-1}w_i'\right]\\ b^2\left[\dfrac{\lambda_x u_i}{\Delta^2}+\dfrac{w_{i+1}-w_i}{\Delta}\dfrac{\lambda_x w_i}{\Delta^2}\right]=-p_x+u_i'';\end{cases} \tag{4}$$

where: $\Delta=1/n$, $0\le i\le n$, n is the number of beam axis partition; λ_x^2, λ_x are second and first order difference operators. Note that for brevity of considerations bars in non-dimensional equations (4) are omitted.

Boundary conditions regarding the finite-difference form read:

$$w_0 = u_0 = w_n = u_n = 0, \quad w_{-1} = w_{n+1} = w_{n-1} = w_1. \quad (5)$$

Initial conditions ($t = 0$) for the transversal load is

$$w_i = u_i = u'_i = w'_i = 0. \quad (6)$$

The applied load has harmonic form $q = q_0 \sin(\omega_p t)$, where ω_p is excitation frequency, and q_0 is the excitation amplitude. Equations (4)-(5) are reduced to a normal form and solved by fourth order Runge-Kutta method.

Our investigations have shown that the qualitative picture of all beam points is the same, therefore further study is carried out only for beam center ($x = 0.5$) and quarter-beam centers ($x = 0.25$). Time histories $w(0.5;t)$, $w'(0.5;t)$, $u(0.25;t)$, $u'(0.25;t)$, phase portraits $w(w')$ and $u(u')$, power frequency spectra (FFT) $S_w(\omega_p)$ and $S_u(\omega_p)$, as well as Poincaré maps and Lyapunov exponents are further studied to monitor the system dynamics.

4 NUMERICAL RESULTS

Numerical results concern steel made beams ($E = 2 \cdot 10^5$ MPa) with the ratio $b = a/h = 50$. The non-dimensional damping coefficient $\bar{\varepsilon} = 1$. Numerical simulation for symmetric boundary conditions (5) allowed for construction of the vibration character charts vs. control parameters $\{q_0, \omega_p\}$ for n=28 and n=40 (Figure 2). The reported charts contain 300×300 points, i.e. the system of ($2n$ – 4) equations has been solved 9×10^4 times (program package has been built to solve this task). Excitation frequency is changed from $\omega_0/2$ to $3\omega_0/2$, where ω_0 is linear frequency of the system (for the rigid clamping $\omega_0 = 6{,}35$). Maximal amplitude of excitation corresponds to the beam deflection equal to $5h$. Identification of beam vibration type required for a chart $\{q_0, \omega_p\}$ construction for each time series $w(t)$ has been carried out using the frequency power spectrum $S_w(\omega_p)$ and the Lyapunov exponents. Analysis of the chart $\{q_0, \omega_p\}$ indicates that for low values of excitation amplitude, the studied beam exhibits regular periodic vibrations. Increase of q_0 (for low values of the excitation frequency ω_p) yields bifurcation zones occurrence with negligible chaotic zones. Increasing of ω_p causes increase of bifurcation zones, and in the regular zone also small parts of both quasi-periodic and chaotic vibrations appear. It is clear that depending on the partition number n the mutual relation between the mentioned zones undergoes changes: Chaotic zones are shrunk, whereas zones of bifurcations are enlarged and start to merge. Since the qualitative behavior of the system is similar for n = 28 and n = 40, partition number n = 28 is further used during our investigations.

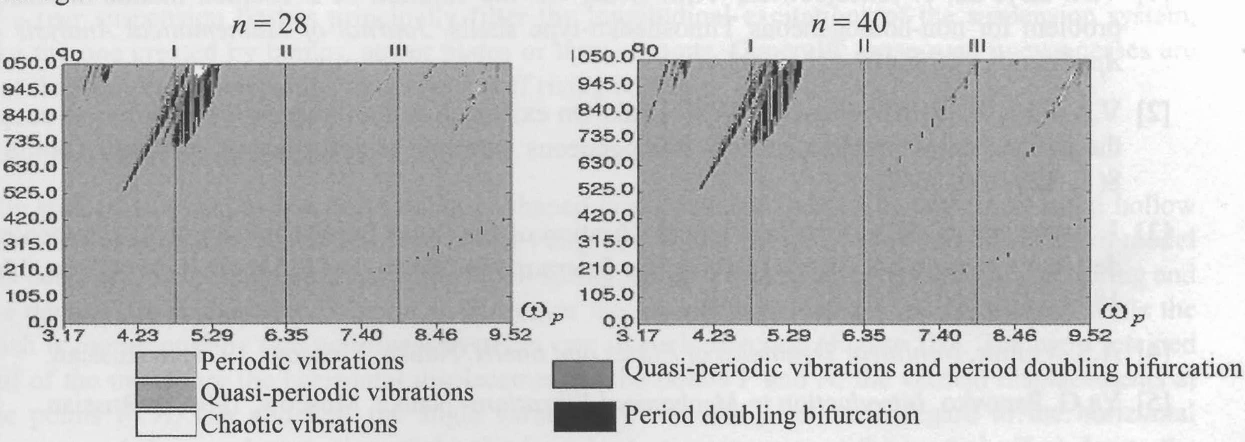

Figure 2. Charts of vibrations vs. control parameters

For $n = 28$ dependencies $W_{\max}(q_0)$ have been constructed with the attached vibration character scales regarding frequencies denoted by lines I-I, II-II and III-III (Figure 3). The applied scale meaning is given in Figure 2.

Note that any arbitrary change of vibration character, frequency power spectra or phase space yields a qualitative change of the global system dynamics.

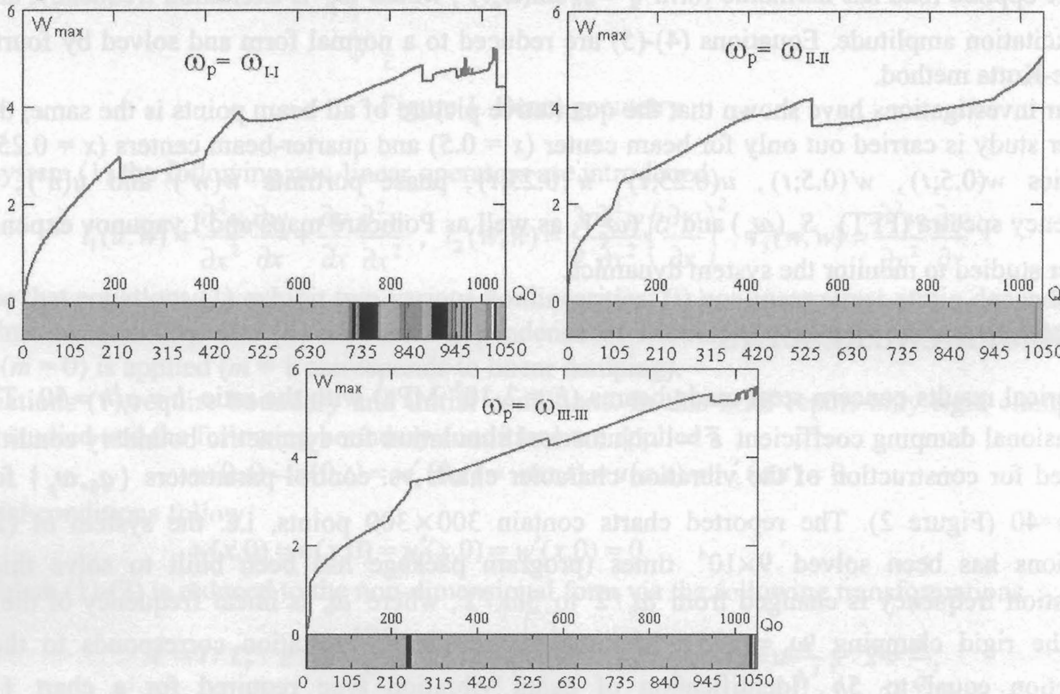

Figure 3. Dependence $W_{\max}(q_0)$ (up) and vibration character scale (down)

5 CONCLUDING REMARKS

The method of investigation of chaotic vibrations of geometrically nonlinear beams is proposed and illustrated on frame of qualitative theory of differential equations and nonlinear dynamics. Chaotic, regular and bifurcation dynamics of the beam vs. control parameters (amplitude and excitation of transversal harmonic load) is reported and discussed.

REFERENCES

[1] V.A. Krys'ko, J. Awrejcewicz, V.M. Bruk, On the solution of a coupled thermo-mechanical problem for non-homogeneous Timoshenko-type shells. *Journal of Mathematical Analysis and Applications*, 273, 409-416, 2003.

[2] V.A. Krys'ko, J. Awrejcewicz, V.M. Bruk, On existence and uniqueness of solutions to coupled thermomechanics problem of non- homogeneous isotropic plates. *Journal of Applied Analysis*, 8(1), 129-139, 2002.

[3] J. Awrejcewicz, V.A. Krys'ko, On the vibration of the Euler-Bernoulli beam with clamped ends deflection constraints, *Special Issue of the International Journal of Bifurcation and Chaos*, 15(6), 1867-1878, 2005.

[4] A.S. Volmir, *Nonlinear Dynamics of Plates and Shells*. Nauka, Moscow, 1972, in Russian.

[5] Ya.G. Panovko, *Introduction to Mechanical Vibrations*. Nauka, Moscow, 1980, in Russian.