Active Noise and Vibration Control Methods
Krakow - Krasiczyn, Poland
June, 11 - 14, 2007
www.vibrationcontrol.pl

ACTIVE CONTROL OF A TWO DEGREES-OF-FREEDOM BUILDING-GROUND SYSTEM

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Keywords: active control, constructions stabilization, 2-DOF dynamical systems

1. Introduction

Active control of building structures has been widely studied theoretically, using a variety of control strategies [5]. Control of structural dynamics basically means regulation of the corresponding characteristics for the purpose of providing its controlled response to the effect of the external dynamic loads.

During the last decade, a rapid development has been observed in the field of structural control. Although these achievements in civil engineering are quite recent, it needs to ensure the comfort and safety of occupants and protect the integrity of the structure. There are studied many theoretical and experimental problems of control in order to reduce structural vibrations under any unpredictable conditions [1, 2].

The use of active control on a passively base isolated building model is proposed in [4] to counteract vibrations due to a low power excitation. The base isolated building is modeled as a three degrees-of-freedom rigid body. The rotation at the center of this model of building is controlled by means of vertical synchronized actuators. The control methods which are applied to the base isolated model and then compared are as follows: optimal control, eigenvalue assignment using state and output feedbacks.

A method for design of supplemental dampers in multistory structures is presented in [6]. Active optimal control theory is adapted to design linear passive viscous or viscoelastic devices dependent on their deformation and velocity. The theory using a linear quadratic regulator is used to exemplify the procedure. With the use of Ricatti equation the design is aimed at minimizing a performance cost function, which produces a most suitable minimal configuration of devices while minimizing their effect.

A progression visible in investigations of earthquake resistant structures from passively controlled base-isolated structures to actively controlled structures has now led to hybrid structures. As it is shown in [7] a hybrid actuator-damper-bracing control system can be composed of viscoelastic dampers and hydraulic actuators in the form of the passive and active controllers, which are installed on the brace and connected to the building floor. The intelligent control strategy is designed to maximally utilize the passive damper and to minimally utilize the active energy. Thus, the passive controller of the hybrid system is designed for small moderate earthquakes and the active controller works for large earthquakes. The hybrid control system is studied under existing earthquake records and the ground motions together with assumption of tectonic movements of seismic plates.

A generalized minimum variance algorithm for the control of civil engineering structures is described in [3]. The algorithm needs the knowledge of the seismic excitation model to drive the autoregressive moving average exogenous model of the structure. The control is designed such that the variance of the generalized cost function is minimized. To demonstrate the effectiveness of this control technique, some simulation tests using a single degree-of-freedom structure were performed.

2. Problem Statement

The active control of buildings concerns on constructions being analyzed on the base of a general approach. Control of the investigated not subjected to any external loading two degrees-of-freedom continuous dynamical system represents a little particular case of the active control law used in the paper. There is derived a controlling scheme applied to the analyzed 2DOF system, which after an initial placement disturbance x_0 at any initial time t_0 evaluates until the moment of time t_f is reached. The system is not influenced by any external disturbances affecting it from the surrounding environment.

Let the following system of differential equations be given as follows

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \qquad x(t_0) = 0, \tag{1}$$

where: A – $(n \times n)$ matrix of structure parameters,

B – $(n \times n)$ matrix of executing (regulatory) elements,

x – the n-dimensional state vector of the system.

Our task focuses on searching for the control force $u(t)$ that would satisfactorily minimize the cost function J in time $t = t_f$:

$$J = \frac{1}{2}x(t_f)^T \theta(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T \begin{bmatrix} Q & T \\ T^T & R \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt, \tag{2}$$

where: θ, Q, T, R – dependent on time t symmetric weighting coefficients matrices.

3. Riccati equation

Let us assume the existence of such an expression representing the cost function

$$J = \theta(x,t)\Big|_{t_0}^{t_f} + \int_{t_0}^{t_f} \varphi(x,u,t)dt. \tag{3}$$

Hamiltonian of the cost function (3) is defined by the formula

$$H(x,u,\lambda,t) = \varphi - \dot{x}^T \frac{\partial \varphi}{\partial \dot{x}} \equiv \varphi + \dot{x}^T \lambda, \tag{4}$$

where $x(t)$, $\lambda(t)$ are canonical variables.

It results from Eq. (3) that φ takes only scalar values, so the summation in Eq. (4) can be done if and only if the notation $(dx/dt)^T\lambda$ is a scalar value too. A transposition of scalars does not make any difference in Eq. (4), therefore we have

$$H(x,u,\lambda,t) = \varphi + (\dot{x}^T \lambda)^T = \varphi + \lambda^T \dot{x}. \tag{5}$$

Differentiating Eq. (4) with respect to x yields

$$\frac{\partial H}{\partial x} = \frac{\partial \varphi}{\partial x} - \frac{\partial \dot{x}^T}{\partial x}\frac{\partial \varphi}{\partial \dot{x}} - \dot{x}^T \frac{\partial}{\partial x}\frac{\partial \varphi}{\partial \dot{x}}. \tag{6}$$

The composite function theorem allows to simplify Eq. (6) to the following form

$$\frac{\partial H}{\partial x} = -\frac{\partial}{\partial t}\frac{\partial \varphi}{\partial \dot{x}}. \tag{7}$$

One can guess from Eq. (4) that $\lambda = -\partial\varphi/\partial(dx/dt)$. Successively, taking into consideration Eq. (7) we get the following Euler-Lagrange equation

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial x}. \tag{8}$$

The Hamiltonian given in Eq. (4) satisfying Eq. (2) is as follows

$$H(x,u,\lambda,t) = \frac{1}{2}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix}^T \begin{bmatrix} Q & T \\ T^T & R \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \lambda^T \big(Ax(t) + Bu(t)\big). \tag{9}$$

After differentiating the last equation with respect to λ, x and u one gets

$$\frac{\partial H}{\partial \lambda} = \dot{x}(t) = Ax(t) + Bu(t). \tag{10}$$

In the background of Euler-Lagrange equation (8) and with application of the properties $\lambda^T A = A^T\lambda$, $T^T = T$ the following relation is obtained

$$\begin{aligned} \dot{\lambda}(t) = -\frac{\partial H}{\partial x} &= -\frac{\partial}{\partial x}\left[\frac{1}{2}\left(x(t)^T Qx(t) + u(t)^T T^T x(t) + x(t)^T Tu(t) + u(t)^T Ru(t)\right)\right] + A^T\lambda(t) \\ &= -Qx(t) - Tu(t) - A^T\lambda(t). \end{aligned} \tag{11}$$

Derivative of Eq. (9) with regard to u reads

$$\frac{\partial H}{\partial u} = -\frac{\partial}{\partial u}\left[\frac{1}{2}\left(x(t)^T Qx(t) + u(t)^T T^T x(t) + x(t)^T Tu(t) + u(t)^T Ru(t)\right)\right] + B^T \lambda .$$

If $H=const$ then $\partial H/\partial u=0$, and hence

$$\frac{\partial H}{\partial u} = T^T x(t) + Ru(t) + B^T \lambda = 0 . \tag{12}$$

From the above we find

$$u(t) = -R^{-1}\left(T^T x(t) + B^T \lambda\right), \tag{13}$$

and $R(t)$ is assumed to be reversible in the forthcoming analysis. Substitution of Eq. (14) in Eq. (1) yields

$$\dot{x}(t) = x(t)\left(A - BR^{-1}T^T\right) - BR^{-1}B^T \lambda . \tag{14}$$

Equations (11) and (14) constitute a system of $2n$ linear ordinary differential equations of the form

$$\begin{bmatrix} \dot{x}(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} A_n & -BR^{-1}B^T \\ -Q_n & -A_n^T \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix}, \quad A_n = A - BR^{-1}T^T, \quad Q_n = Q - TR^{-1}T^T . \tag{15}$$

Solution to the system (15) can be expressed by means of final (terminal) conditions and the matrix $\varphi(t_f, t)$ decomposed to a four component matrices

$$\begin{bmatrix} x(t_f) \\ \lambda(t_f) \end{bmatrix} = \begin{bmatrix} \varphi_{11}(t_f,t) & \varphi_{12}(t_f,t) \\ \varphi_{21}(t_f,t) & \varphi_{22}(t_f,t) \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix}, \tag{16}$$

where φ_{11}, φ_{12}, φ_{21}, φ_{22} are $n \times n$ component matrices.

Let λ at final time t_f be equal to

$$\lambda(t_f) = \theta(t_f)x(t_f). \tag{17}$$

Taking into account (17) in (16) one obtains

$$x(t_f) = \varphi_{11}(t_f,t)x(t) + \varphi_{12}(t_f,t)\lambda(t), \tag{18}$$

$$\theta(t_f)x(t_f) = \varphi_{21}(t_f,t)x(t_f) + \varphi_{22}(t_f,t)\lambda(t) . \tag{19}$$

Substituting (18) into (19)

$$\theta(t_f)\varphi_{11}(t_f,t)x(t) + \theta(t_f)\varphi_{12}(t_f,t)\lambda(t) = \varphi_{21}(t_f,t)x(t) + \varphi_{22}(t_f,t)\lambda(t), \tag{20}$$

and consequently

$$\lambda(t) = \left(\varphi_{22}(t_f,t) - \theta(t_f)\varphi_{12}(t_f,t)\right)^{-1}\left(\theta(t_f)\varphi_{11}(t_f,t) - \varphi_{21}(t_f,t)\right)x(t). \tag{21}$$

The expressions in braces will be from now denoted by K_x, therefore Eq. (21) can be rewritten in the form

$$\lambda(t) = K_x(t)x(t), \tag{22}$$

where: $K_x - n \times n$ Riccati matrix.

Putting $\lambda(t)$ expressed by Eq. (22) in (13), we get

$$u(t) = -R^{-1}\left(T^T + B^T K_x\right)x(t). \tag{23}$$

Observe that the sought control law $u(t)$ is governed by a linear combination of state vectors. It is known that the best method of determination of the K_x matrix and thereby estimation of $u(t)$ is utilization of a proper convergent numerical procedure. By substitution of Eq. (22) into (15), we have

$$\dot{x}(t) = \left(A_n - BR^{-1}B^T K_x\right)x(t),$$

$$\dot{\lambda}(t) = K_x\dot{x}(t) + \dot{K}_x x(t) = -Q_n x(t) - A_n^T(t)K_x x(t), \tag{24}$$

$$\left(\dot{K}_x + K_x A_n + A_n^T K_x - K_x BR^{-1}B^T K_x + Q_n\right)x(t) = 0. \tag{25}$$

Because validity of Eq. (25) covers all $x(t) \neq 0$, then the expression in braces preceding $x(t)$ must equal zero. Therefore, K_x has to satisfy the following Riccati matrix equation

$$\left(\dot{K}_x - K_x A_n - A_n^T K_x + K_x BR^{-1}B^T K_x - Q_n\right)x(t) = 0, \tag{26}$$

also with inclusion of the final condition $K_x(t_f)=\theta(t_f)$, in which $\theta(t_f)$ is the known value. Of course, Eq. (26) is numerically integrable in (t_0, t_f). Thus, we are able to determine both the Riccati matrix K_x and the gain matrix F_x for all $t \in (t_0, t_f)$:

$$F_x(t) = -R^{-1}\left(T^T + B^T K_x\right). \tag{27}$$

Finally, the following control law can be proposed

$$u(t) = F_x x(t). \tag{28}$$

4. Static control law

Let A, B, Q and R be some time-independent (constant) matrices. Matrix K_x is then a particular solution to such an equation $K_x(t_f)=\theta(t_f)$. If the solution is stable (in accordance to Lyapunov criterion) for the sufficiently large t_f, then the approximate values of K_x converge to a constant value K_c. Evidently, one can assume that the matrix K_x is also time-independent and is in its final state K_c represented by the solution to the Riccati equation

$$-K_c A_n - A_n^T K_c - Q_n + K_c BR^{-1}B^T K_c = 0. \tag{29}$$

For the purpose of illustration of the principle of Riccati matrix stability the simple mechanical system visible in Fig. 1 of two degrees-of-freedom modeling the dynamics of building-ground connection will be analyzed.

In order to control the system under investigation we additionally dispose of a virtual generator of control force $u(t)$ which is placed between the two masses of the building-ground system. Equations of motion with an auxiliary force $u(t)$ controlling the second material point (building) are written in the following form

$$\begin{aligned} m_1\ddot{x}_1(t) &= -k_1x_1(t) - c_1\dot{x}_1(t) - u(t), \\ m_2\ddot{x}_2(t) &= u(t). \end{aligned} \tag{30}$$

We propose a function of balance, which will precise the starting point as an input in the strategy of searching for the control force characteristics. Our fundamental task is to secure the amplitude of vibrations of the second mass minimized as low as possible in the preset interval of time $t\in(t_0, t_f)$.

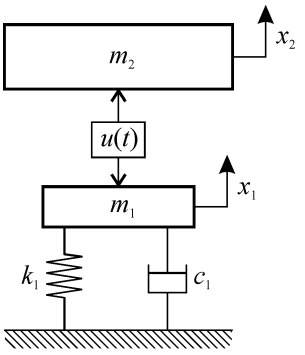

Fig. 1. Active building-ground model.

The cost function (2) will be given in a more consisted form. Therefore, let Q, R and T satisfy

$$Q = \begin{bmatrix} q_1+q_2 & -q_2 & 0 & 0 \\ -q_2 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = r, \quad T = 0, \tag{31}$$

where q_1, q_2, r are weighting coefficients.

Additionally, let θ be equal to zero in (t_0, t_f). Matrices A and B are constant and time-independent:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m_1 & 0 & -c_1/m_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ -1/m_1 \\ 1/m_2 \end{bmatrix}. \tag{32}$$

Taking into consideration Q, R, T and θ in Eq. (2) (given by (31) and (32)), the following form of cost function can be introduced:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \begin{bmatrix} x(t)^T & u(t)^T \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} dt, \tag{33}$$

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left(x^T(t) Q x(t) + u^T(t) R u(t) \right) dt, \tag{34}$$

where both elements of the sum are scalars.

Product $x(t)^T Q x(t)$, where $x(t)^T=[x_1, x_2, dx_1/dt, dx_2/dt]$ (see Fig 4.1) is equal to

$$\begin{aligned} x^T(t) Q x(t) &= \begin{bmatrix} x_1 & x_2 & \dot{x}_1 & \dot{x}_2 \end{bmatrix} \begin{bmatrix} q_1 + q_2 & -q_2 & 0 & 0 \\ -q_2 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \\ &= \left(x_1(q_1 + q_2) - x_2 q_2 \right) x_1 + \left(-x_1 q_2 + x_2 q_2 \right) x_2, \end{aligned} \tag{35}$$

and $u(t)^T R u(t)=ru^2$. Thus, we seek a cost function in the form

$$J(u) = \frac{1}{2}\int_{t_0}^{t_f} \left(q_1 x_1^2 + q_2 (x_2 - x_1)^2 + r u^2 \right) dt. \tag{36}$$

5. Numerical results

The following set of parameters is assumed: m_1=1500kg, m_2=11000kg, k_1=975000N/m, c_1=10800Ns/m. Optimal control will be guaranteed for: q_1=5, q_2=1, r=$10^{-10.3}$. Such estimation of control parameters allows for proving of correctness of the method used as well as for making a good observations of influence of the control force on the investigated 2DOF building-ground system.

Components of the structure matrices A and B in Eq. (32) representing a "control mechanisms" are: $a=k_1/m_1$=650, $b=1/m_1$=6.66e^{-4}, $c=1/m_2$=9,9091e^{-5}, $d=c_1/m_1$=7.2.

Estimation of $u(t)$ will be preceded by a numerical integration of Eq. (26) by means of the standard 4-*th* order Runge-Kutta procedure.

Components of Riccati matrix K_x are:

$$K_x = \left[\alpha_{ij} \right], \qquad \forall\ i, j \in (1...4). \tag{37}$$

Substitution of the symmetric matrix K_x in (26) implies ten subsequent equations:

$$\begin{aligned} \dot{\alpha}_{11} &= -2\alpha_{13} a + q_1 + q_2 + 1/r\,(-\alpha_{13} b + \alpha_{14} c) b \alpha_{13} - 1/r\,(-\alpha_{13} b + \alpha_{14} c) c \alpha_{14}, \\ \dot{\alpha}_{12} &= -\alpha_{23} a - q_2 + 1/r\,(-\alpha_{13} b + \alpha_{14} c) b \alpha_{23} - 1/r\,(-\alpha_{13} b + \alpha_{14} c) c \alpha_{24}, \\ \dot{\alpha}_{13} &= \alpha_{11} - \alpha_{13} d - \alpha_{33} a + 1/r(-\alpha_{13} b + \alpha_{14} c) b \alpha_{33} - 1/r\,(-\alpha_{13} b + \alpha_{14} c) c \alpha_{34}, \\ \dot{\alpha}_{14} &= \alpha_{12} - \alpha_{34} a + 1/r\,(-\alpha_{13} b + \alpha_{14} c) b \alpha_{34} - 1/r\,(-\alpha_{13} b + \alpha_{14} c) c \alpha_{44}, \end{aligned}$$

$$
\begin{aligned}
\dot{\alpha}_{22} &= q_2 + 1/r\,(-\alpha_{23}b + \alpha_{24}c)b\alpha_{23} - 1/r\,(-\alpha_{23}b + \alpha_{24}c)c\alpha_{24}\,,\\
\dot{\alpha}_{23} &= \alpha_{12} - \alpha_{23}d + 1/r\,(-\alpha_{23}b + \alpha_{24}c)b\alpha_{33} - 1/r\,(-\alpha_{23}b + \alpha_{24}c)c\alpha_{34}\,,\\
\dot{\alpha}_{24} &= \alpha_{22} + 1/r(\alpha_{23}b + \alpha_{24}c)b\alpha_{34} - 1/r\,(-\alpha_{23}b + \alpha_{24}c)c\alpha_{44}\,,\\
\dot{\alpha}_{33} &= 2\alpha_{13} - 2\alpha_{33}d + 1/r\,(-\alpha_{33}b + \alpha_{34}c)b\alpha_{33} - 1/r\,(-\alpha_{33}b + \alpha_{34}c)c\alpha_{34}\,,\\
\dot{\alpha}_{34} &= \alpha_{23} + \alpha_{14} - \alpha_{34}d + 1/r(-\alpha_{33}b + \alpha_{34}c)b\alpha_{34} - 1/r\,(-\alpha_{33}b + \alpha_{34}c)c\alpha_{44}\,,\\
\dot{\alpha}_{44} &= 2\alpha_{24} + 1/r(-\alpha_{34}b + \alpha_{44}c)b\alpha_{34} - 1/r(-\alpha_{34}b + \alpha_{44}c)c\alpha_{44}\,).
\end{aligned}
\tag{38}
$$

Equations (38) are then integrated numerically since the constant components Riccati matrix K_c results in the following estimation

$$K_c = \lim_{t\to 0} K_x(t). \tag{39}$$

The knowledge of all components of K_c permits for calculation of the time-dependent matrix of gain, and hence

$$F_x(t) = -R^{-1}B^T K_x = \left[f_1(t), f_2(t), f_3(t), f_4(t)\right]. \tag{40}$$

A final form of the resulting time-dependence of control force (control law) is given in the form

$$u(t) = F_x(t)x(t) = f_1x_1 + f_2x_2 + f_3\dot{x}_1 + f_4\dot{x}_2\,. \tag{41}$$

Note that F_x approaches a constant solution value at F_c for a sufficiently large time t_f.

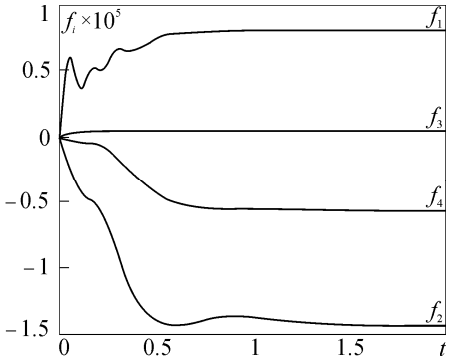

Fig 4.2. Time history of components f_i of the gain matrix $F_x(t)$.

Figure 4.2 presents time histories of components f_i for i=1…4 of gain matrix (40). Numerically computed limits of F_x are found:

$$F_c = -R^{-1}B^T K_c = \left[0.8232\times 10^5,\ -0.14126\times 10^6,\ 0.5926\times 10^4,\ -0.5403\times 10^5\right].$$

Stationary control law is on the basis of the above governed by the general equation

$$u(t) = F_c\, x(t). \tag{42}$$

Our analysis confirms a strong dependence existing between the control force $u(t)$ and the gain matrix F_x. We mentioned that for any building-ground balance coefficients q_1, q_2, r, the system under investigation should be optimally controlled. With regard to Eq. (42) and for the test values $r_1=0.5012\times10^{-10}$, $r_2=10r_1$ and initial conditions $x_1=0.1$, $x_2=0$, $dx_1/dt=0$, $dx_2/dt=0$ two time histories of the control force $u(t)$ are shown in Fig. 4.3.

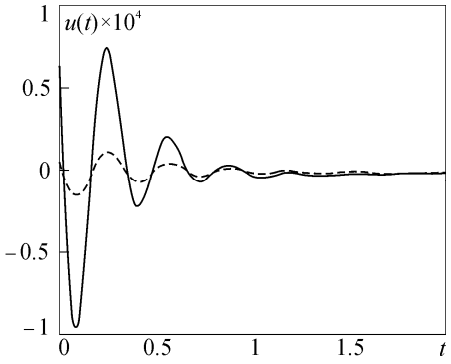

Fig 4.3. Variations of control force for the coefficients r_1 (solid line) and r_2 (dash line).

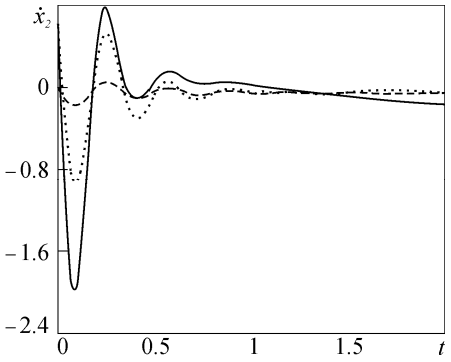

Fig 4.4. Time histories of acceleration d^2x_2/dt^2 for the passive (solid line) and active control of the second mass for $r_1=0.5012\times10^{-10}$ (dot line) and $r_2=10r_1$ (dash line).

Time histories of acceleration of the second mass m_2 are shown in Fig. 4.4. Acceleration of that second controlled mass is definitely better damped for the case of the system being under the active control. The obtained results can be corrected (improved) by

modification of balance coefficients q_i or r. To do this, one of them is assumed to be constant while the second one is searched for the more accurate value.

6. Conclusions

The presented idea of active control of buildings is valid for the general concept of stabilization of some constructions being not under any external excitation. A two degrees-of-freedom mechanical system was used to model the building-ground interactions, and an algorithm of its active control has been implemented and tested as well. Finally, the problem has reduced the investigations conducted in the work to an estimation of the control force which could guarantee the sufficient minimization of the cost function proposed. On the basis of both analytical derivations and numerical computations performed a few time histories of control force convergence, components of the gain matrix and accelerations of one of the system masses have been illustrated and shortly described. One can observe, that system responses stabilize quite quickly (in about 2 sec.), and the shape of control force dependency is characteristic for such similar systems without any external disturbances.

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