POLITECHNIKA WARSZAWSKA
Instytut Podstaw Budowy Maszyn
IV Międzynarodowa Konferencja n.t.: **MODELOWANIE I SYMULACJA ZJAWISK TARCIOWYCH W UKŁADACH FIZYCZNYCH STRUKTURACH TECHNICZNYCH**
"TARCIE 2006"

Warszawa, czerwiec 2006

Jan Awrejcewicz, Yuriy Pyryev
Department of Automatics and Biomechanics
Technical University of Łódź

ON THE PERIODIC MOTION EXHIBITED BY A BUSH-SHAFT SYSTEM WITH IMPACT AND FRICTIONAL PROCESSES

1. Introduction

In this work conditions of occurrence of periodic motion in systems with dry friction and impacts are investigated. There are many examples in various mechanisms and machines with gaps between the vibrating objects, where such type of dynamics is generated by dry frictional self-excitation. A proper modeling and control of the mentioned self-excited vibrations play a crucial role during investigation of dry frictional brakes, grinding processes as well as various dynamics of contact pairs between mechanisms elements.

We are aimed on estimation of parameters of the investigated system associated with occurrence of periodic motion. In particular, both restitution coefficient and period of periodic dynamics are defined.

A similar system, however without impacts, has been studied earlier by the authors and it has been described in references [1-3]. In the mentioned works a novel model of vibrations of the bush-shaft system with tribological processes occurring on the contact surface has been proposed. The occurrence of self-excited vibrations in a more simplified system with a gap has also been analyzed in reference [4].

2. Mathematical problem formulation

Attention is focused on modeling of non-linear dynamics of two bodies consisting of a stiff bush with clearance $2\Delta_\varphi$ (see Fig. 1). The bush is coupled with housing by springs with stiffness k_2 and is mounted on the rotating shaft 1. The following assumptions are taken: (i) the shaft rotates with a such enough small angular velocity Ω that centrifugal forces can be omitted; (ii) non-linear kinetic friction occurs between the bush and the shaft.

Let axis Z be a cylinder axis. The equilibrium state of the moments of forces with respect to the shaft axis gives

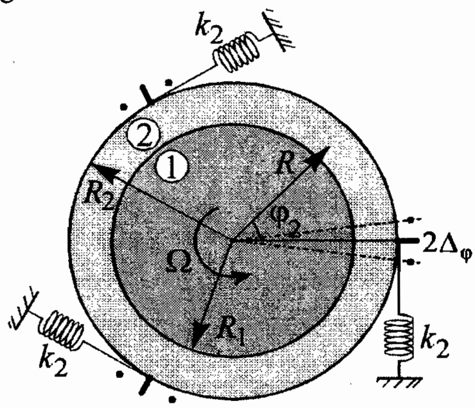

Fig. 1. Analyzed system

$$B_2\ddot{\varphi}_2(t) + k_2R_2^2\varphi_2(t) = f(V_r)2\pi R_1^2 P(t),\ |\varphi_2(t)| < \Delta_\varphi,\ \dot{\varphi}_2(t) \neq \Omega, \tag{1}$$

$$\ddot{\varphi}_2(t) = 0,\ |\varphi_2(t)| < \Delta_\varphi,\ \dot{\varphi}_2(t) = \Omega, \tag{2}$$

$$\dot{\varphi}_2^+ = -k\dot{\varphi}_2^-,\ |\varphi_2| = \Delta_\varphi,\ \dot{\varphi}_2^-\varphi_2 > 0, \tag{3}$$

where: $V_r = R_1\Omega - R_1\dot{\varphi}_2(t)$ relative velocity of the contact bodies, k is the coefficient of restitution, $\dot{\varphi}_2^-$ ($\dot{\varphi}_2^+$) is the bush velocity just before (after) impact, B_2 is the moment of inertia of the bush per length unit, $f(V_r)$ is the kinetic friction coefficient depending on relative velocity, $P(t) = -\sigma_R(R_1,t)$ is the contact pressure. The initial value problem is defined in the following way:

$$\varphi_2(0) = \varphi_2^\circ,\ \dot{\varphi}_2(0) = \omega_2^\circ, \tag{4}$$

Relation approximating curve $f(V_r)$ has the following form

$$f(V_r) = \mathrm{sgn}(V_r)F(|V_r|),\ F(V_r) = \begin{cases} F_0 - \kappa V_r, & 0 < V_r \le V_{\min} \\ F_0 - \kappa V_{\min}, & V_{\min} < V_r \end{cases} \tag{5}$$

where: F_0, κ, $V_{\min}$ are constant coefficients.

Shaft radial stresses $\sigma_R(R,t)$ may be found knowing radial displacement $U(R,t)$ from the following formula

$$\sigma_R(R,t) = \frac{E_1}{1-2\nu_1}\left[\frac{1-\nu_1}{1+\nu_1}\frac{\partial U(R,t)}{\partial R} + \frac{\nu_1}{1+\nu_1}\frac{U(R,t)}{R}\right]. \tag{6}$$

The following notation has been applied: $U(R,t)$ - displacement component along radial direction in the shaft; E_1 - Young modulus; ν_1 - Poisson's ratio.

Upon integration of equation of the theory of elasticity [4]

$$\frac{\partial^2 U(R,t)}{\partial R^2}+\frac{1}{R}\frac{\partial U(R,t)}{\partial R}-\frac{1}{R^2}U(R,t)=0 \tag{7}$$

and taking into account mechanical boundary conditions $U(0,t)=0$, $U(R_1,t)=-U_0 h_U(t)$ and (6), the contact pressure is

$$P(t)=\frac{E_1}{(1-2\nu_1)(1+\nu_1)R_1}U_0 h_U(t)\,. \tag{8}$$

Let us introduce the following dimensionless parameters:

$$\tau=\frac{t}{t_*},\ \varphi(\tau)=\frac{\varphi_2}{\Delta_\varphi},\ p=\frac{P}{P_*},\ \alpha_0=\frac{f_0}{F_0},\ \omega_1=\frac{\Omega t_*}{\Delta_\varphi}=\frac{1}{\gamma\sqrt{\alpha_0}},\ \mu_0=\frac{1-\alpha_0}{\alpha_0},$$

$$\eta_0=\frac{V_0-V_{\min}}{V_0},\ \varepsilon=\mu_0\gamma\sqrt{\alpha_0}=\frac{\mu_0}{\omega_1},\ \gamma=\sqrt{\frac{2\pi R_1^2 P_* F_0\Delta_\varphi}{B_2\Omega^2}},\ \omega_0^2=\frac{k_2 R_2^2 t_*^2}{B_2},\ \Psi=\frac{F}{f_0},$$

$$x=\frac{\overset{\circ}{\varphi}_2}{\Delta_\varphi},\ y=\frac{\overset{\circ}{\omega}_2 t_*}{\Delta_\varphi},\ f(V_0)=f_0,\ h_U(\tau)=h_U(t_*\tau),\ p(\tau)=h_U(\tau)$$

where:

$$t_*=\sqrt{\frac{B_2\Delta_\varphi}{f_0 2\pi R_1^2 P_*}},\ V_*=\frac{R_1\Delta_\varphi}{t_*},\ P_*=\frac{E_1 U_0}{(1-2\nu_1)(1+\nu_1)R_1},\ V_0=\Omega R_1\,.$$

The dimensionless equations governing dynamics of the analyzed system have the form

$$\ddot\varphi(\tau)+\omega_0^2\varphi(\tau)=\operatorname{sgn}(\omega_1-\dot\varphi)\Psi(\dot\varphi)p(\tau),\ |\varphi(\tau)|<1,\ \dot\varphi(\tau)\neq\omega_1 \tag{9}$$

$$\ddot\varphi(\tau)=0,\ |\varphi(\tau)|<1,\ \dot\varphi(\tau)=\omega_1 \tag{10}$$

$$\dot\varphi^+=-k\dot\varphi^-,\ |\varphi|=1,\ \dot\varphi^-\varphi>0, \tag{11}$$

$$\varphi(0)=x,\ \dot\varphi(0)=y \tag{12}$$

where:

$$\Psi(\dot\varphi)=\begin{cases}1+\varepsilon\omega_1\eta_0, & \dot\varphi<\omega_1\eta_0,\ \omega_1(2-\eta_0)<\dot\varphi\\ 1+\varepsilon\dot\varphi, & \omega_1\eta_0<\dot\varphi<\omega_1\\ 1+2\varepsilon\omega_1-\varepsilon\dot\varphi, & \omega_1<\dot\varphi<\omega_1(2-\eta_0)\end{cases}. \tag{13}$$

3. Analysis

First the case of bush vibrations for $p(\tau)=h_U(\tau)=H(\tau)$, $H(\tau)=1$ for $\tau>0$, $H(\tau)=0$ for $\tau<0$. Our system governed by equations (9) may exhibit four different periodic motions. Namely:

(i) periodic orbit $AMNA$ with one impact, where a stick does not appear ($A(x,0)$, $x\in(-1,1)$, $M(1,y_M)$, $N(1,y_N)$, $y_N=-k\,y_M$);

(ii) periodic orbit *ACMNA* with one impact, where a stick-slip occurs ($A(x,0)$, $x \in (-1,1)$, $C(x_C,\omega_1)$, $M(1,\omega_1)$, $N(1,y_N)$, $y_N = -k\omega_1$);
(iii) periodic orbit *BMNDB* with two impacts, where a slip of the contacting bodies occurs ($B(-1,y_B)$, $M(1,y_M)$, $N(1,y_N)$, $y_N = -k\,y_M$, $D(-1,y_D)$, $y_D = -y_B/k$);
(iv) periodic orbit *BCMNDB* with two impacts, where a stick-slip appears ($B(-1,y_B), C(x_C,\omega_1)$, $M(1,\omega_1)$, $N(1,y_N)$, $y_N = -k\omega_1$, $D(-1,y_D)$, $y_D = -y_B/k$).

Below, we assume that $\varepsilon << 1$ and $\omega_0^2 << 1$, and $\eta_0 \le -1$. It means that the system dynamics is exhibited in the interval ($0 < V_0 < V_{\min}$), where a decreasing slope of the kinetic friction coefficient is observed.

Results of our consideration allow us to give formulas for the coefficient of restitution k for a general case of the following case

$$k(x,\omega_1) = \begin{cases} k_{ACMNA}, & 0<\omega_1<2-(4/3)\varepsilon, \quad x_1<x<x_0 \\ k_{AMNA}, & 0<\omega_1<2-(4/3)\varepsilon, \quad x_0<x<1 \\ k_{BCMNDB}, & 2-(4/3)\varepsilon<\omega_1<2, \quad x_1<x<-1 \\ k_{ACMNA}, & 2-(4/3)\varepsilon<\omega_1<2, \quad -1<x<x_0 \\ k_{AMNA}, & 2-(4/3)\varepsilon<\omega_1<2, \quad x_0<x<1 \\ k_{BCMNDB}, & 2<\omega_1<2+(4/3)\varepsilon, \quad x_2<x<-1 \\ k_{ACMNA}, & 2<\omega_1<2+(4/3)\varepsilon, \quad -1<x<x_0 \\ k_{AMNA}, & 2<\omega_1<2+(4/3)\varepsilon, \quad x_0<x<1 \\ k_{BCMNDB}, & 2+(4/3)\varepsilon<\omega_1<\infty, \quad x_2<x<x_0 \\ k_{BMNDB}, & 2+(4/3)\varepsilon<\omega_1<\infty, \quad x_0<x<-1 \\ k_{AMNA}, & 2+(4/3)\varepsilon<\omega_1<\infty, \quad -1<x<1 \end{cases} \tag{14}$$

where the restitution coefficient being sought is given explicitly in the following form

$$k_{AMNA} = 1-(2/3)\tau_1\varepsilon + o(\varepsilon^2), \quad \tau_1 = \sqrt{2(1-x)}, \tag{15}$$

$$k_{BMNDB} = 1 + \frac{\tau_2^3 - \tau_1^3}{3(2+\tau_2^2)}\varepsilon + o(\varepsilon^2), \quad \tau_2 = \sqrt{-2(1+x)}, \tag{16}$$

$$k_{ACMNA} = \tau_1/\omega_1 - (1/3)(\tau_1^2/\omega_1)\varepsilon - (1/8)(\tau_1/\omega_1)(4-\tau_1^2)\delta\varepsilon + o(\varepsilon^2), \tag{17}$$

$$\begin{aligned} k_{BCMNDB} = {} & \frac{\tau_0}{\omega_1} + \frac{-16\tau_0^2 + \tau_2^3\omega_1^2(\tau_0+\omega_1) - \tau_2^2\omega_1^2(4+\tau_0^2)}{6\omega_1(\tau_2^2\omega_1^2 + 2\tau_0^2)}\varepsilon + \\ & \frac{\tau_2^2\tau_0\omega_1(\tau_2^2+4)}{16(\tau_2^2\omega_1^2+2\tau_0^2)}\delta\varepsilon + o(\varepsilon^2), \quad \tau_0 = \sqrt{2+\sqrt{4+\tau_2^2\omega_1^2}} \end{aligned} \tag{18}$$

Note that the periodic motion (i) takes place for $y_M < \omega_1$. The latter observation provides estimation $x_0 < x < 1$, where

$$x_0(\omega_1)=1-(1/2)\omega_1^2+(1/3)\omega_1^3\varepsilon+(1/8)\omega_1^2(\omega_1^2-4)\delta\varepsilon+o(\varepsilon^2). \quad (19)$$

A natural limitation $k_{ACMNA}\le 1$ yields the inequality $x\ge x_1$, where

$$x_1(\omega_1)=1-0.5\omega_1^2-(1/3)\omega_1^3\varepsilon+(1/8)\omega_1^2(\omega_1^2-4)\delta\varepsilon+o(\varepsilon^2). \quad (20)$$

The natural limitation introduced on the restitution coefficient ($k_{BCMNDB}\le 1$) yields $x\ge x_2$, where

$$x_2(\omega_1)=x_1(\omega_1)+(2/3)(\omega_1^2-4)^{3/2}\varepsilon. \quad (21)$$

Parameter zones for which the function $k(x,\omega_1)$ is estimated are graphically presented in Fig. 2.

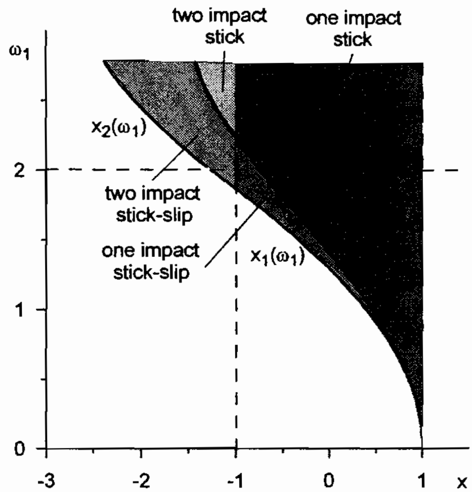

Fig. 2. Zones of different periodic impact motions ($\omega_0^2=0.2$, $\delta=2$, $\varepsilon=0.1$)

Observe that the function $k(x,\omega_1)$ possesses the following values $k(x_1,\omega_1)=1$, $0<\omega_1<2$, $k(x_2,\omega_1)=1$, $2<\omega_1<\infty$, $k(1,\omega_1)=1$ at the boundaries, whereas inside the considered interval it has the following minima

$$\min_{x\in[x_1,1]} k(x,\omega_1)=k(x_0,\omega_1)=1-(2/3)\omega_1\varepsilon,\ 0<\omega_1<2+(4/3)\varepsilon, \quad (22)$$

$$\min_{x\in[x_2,1]} k(x,\omega_1)=k(-1,\omega_1)=1-(4/3)\varepsilon,\ 2+(4/3)\varepsilon<\omega_1<\infty, \quad (23)$$

which can be presented in the form

$$k_{\min}=\begin{cases}1-(2/3)\omega_1\varepsilon, & 0<\omega_1<2+(4/3)\varepsilon\\ 1-(4/3)\varepsilon, & 2+(4/3)<\omega_1<\infty\end{cases} \quad (24)$$

Notice that for an arbitrary $k^*\in(k_{\min},1)$ there are two values of x_1^*, x_2^* ($k(x_1^*,\omega_1)=k(x_2^*,\omega_1)=k^*$). Let us introduce the following intervals

$$x_1<x_1^*<x_0, x_0<x_2^*<1 \text{ for } 0<\omega_1<2, \quad (25)$$

$$x_2<x_1^*<x_0, x_0<x_2^*<1 \text{ for } 2<\omega_1<2+(4/3)\varepsilon, \quad (26)$$

$$x_2 < x_1^* < -1, -1 < x_2^* < 1 \text{ for } 2-(4/3)\varepsilon < \omega_1 < \infty . \tag{27}$$

It is not difficult to check that a periodic orbit associated with x_1^* (decreasing part of the coefficient $k(x)$) is stable, whereas a periodic orbit associated with x_2^* (incresing part of the coefficient $k(x)$) is unstable.

4. Conclusions

We have proposed a novel model of vibrations of the bush-shaft system with inclusion of both impacts. We have estimated analytically the restitution coefficient for which a periodic motion occurs assuming small slope of friction characteristics. We have shown, among the others, various periodic motions exhibited by the analyzed system and we have verified numerically our theoretical considerations and predictions.

For an arbitrary restitution coefficient $k^* \in (k_{\min}, 1)$ two periodic orbits (stable and unstable) appear on the phase plane. Increase of the parameter k^* from $k_{\min}$ to 1 yields increase (decrease) of stable (unstable) periodic orbit. For $k^* = 1$ the unstable periodic orbit is reduced to the point $(1,0)$. Decrease of the parameter k^* causes approaching of both stable and unstable periodic trajectories. For $k^* = k_{\min}$ a bifurcation occurs and a halfly-stable periodic orbit is born substituting two previous stable and unstable orbits. In other words for $k < k_{\min}$ a periodic motion is not exhibited by the studied system.

5. References

1. Awrejcewicz J., Pyryev Yu.: Thermoelastic contact of a rotating shaft with a rigid bush in conditions of bush wear and stick-slip movements, International Journal of Engineering Science, 2002, 40, 1113-1130.
2. Awrejcewicz J., Pyryev Yu.: Contact phenomena in braking and acceleration of bush-shaft system. Journal of Thermal Stresses, 2004, 27, 5, 433–454.
3. Awrejcewicz J., Pyryev Yu.: Tribological periodic processes exhibited by acceleration or braking of a shaft-pad system, Communications in Nonlinear Science and Numerical Simulation, 2004, 9, 6, 603–614.
4. Balandin D.V.: Frictional vibrations in a gap, MTT, 1993, 1, 54-60, in Russian.
5. Nowacki W.: Thermoelasticity. Pergamon Press, Oxford, 1962.

This work has been financially supported by the Polish Ministry of Education and Science for the years 2005-2008 (grant No. 4 TO7C 044 29).