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CHAOTIC VIBRATIONS AND STIFF STABILITY LOSS OF SHELLS WITH CONSTANT AND VARIABLE THICKNESSES

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Abstract: In this work chaotic vibrations of deterministic geometrically nonlinear elastic spherical and conical axially symmetric shells with constant and non-constant thicknesses and subjected to transversal sign changeable load using the variation principle are studied. Shell material is isotropic and the Hook law holds. Inertial forces in direction of tangents to a middle surface and the rotational inertia of normal shell cross sections are neglected. Transition from partial to ordinary differential equations (Cauchy problem) is carried out with a help of the Ritz procedure. Then the obtained Cauchy problem is solved using the fourth order Runge-Kutta method. Numerical analysis is supported by theory of nonlinear dynamical systems and the qualitative theory of differential equations.

1. Introduction

Rapid development of nonlinear theory of shells is motivated by practical needs. Wide application of new materials, application of shells in unusual circumstances which are externally driven with a high intensity requires very subtle methods of computations. Although there is a lot of papers and books devoted to dynamics of shells, but mainly linear investigations were carried out so far, and the problems of dynamical stability loss of a shell subjected to transversal finite or infinite length time impulses are less investigated. In addition, chaotic vibrations of plates and shells belong to less investigated too (see for instance [1-7]).

In the works [4-6] transitions from regular to chaotic dynamics of shells vs. various parameters (shell rise, amplitude and frequency of excitation, number of modes, thickness variation, and geometrical properties) are studied. Among others, approximation of continual systems by their lumped models is addressed. This work extends the mentioned study and control of both chaos and stiff stability loss of a shell by variation of its thickness is proposed.

2. Problem formulation and its solution

Applying the principle of virtual displacements of the form:

$$-\delta(U_u + U_c) + \iint R\delta w ds = 0 ,$$

where: U_u, U_c – potential bending energy and potential energy of middle surface, respectively; R – load and inertial force. Using relations of geometrically nonlinear theory of shallow rotational shells [7, 8] the following variation equation is obtained

$$\begin{gathered}\delta\iint_S \left\{ \frac{D}{2}\left[\left(\Delta w\right)^2 - \left(1-\nu\right) L_2\left(w,w\right)\right] - \left[\Delta_k \varphi + L_2\left(\frac{1}{2}w + w_0 , \varphi\right)\right] w - \right. \\ \left. -\frac{1}{2Eh}\left[\left(\Delta\varphi\right)^2 - \left(1+\nu\right) L_2\left(\varphi,\varphi\right)\right]\right\} ds - \iint_S \left[q - \frac{h\gamma}{g}(\ddot{w} + \varepsilon\dot{w})\right]\delta w ds = 0,\end{gathered} \tag{1}$$

where for the axially symmetric problem the operators follow

$$\Delta = \frac{\partial^2}{\partial\rho} + \frac{1}{\rho}\frac{\partial}{\partial\rho} , \; L(w,\varphi) = \frac{\partial^2 w}{\partial\rho^2}\cdot\frac{1}{\rho}\cdot\frac{\partial\varphi}{\partial\rho} + \frac{1}{\rho}\cdot\frac{\partial w}{\partial\rho}\cdot\frac{\partial^2\varphi}{\partial\rho^2} , \; \Delta_k w = \Delta_k \varphi = 0 .$$

In the above w, φ denote deflection and stress, respectively, w_0 is the initial shell deflection, h is the shell thickness, ε denotes the damping, D is the cylindrical stiffness, and E is Young modulus.

We develop the deflection w and the stress function φ in two complete and linearly independent systems, satisfying main boundary conditions, and a finite sums approximation is taken. In the case of axially symmetric problem one gets

$$w = \sum_{i=1}^{n} x_i(t) w_i(\rho) , \qquad \varphi = \sum_{i=1}^{n} y_i(t)\varphi_i(\rho) . \tag{2}$$

The coefficients $x_i(t)$ and $y_i(t)$ are the being sought functions of time. Substituting relations (2) into (1), carrying out variation, and next comparing to zero the coefficients standing by δx_i and δy_i, the following ordinary differential equations are obtained in relation to the functions $x_i(t)$ and $y_i(t)$:

$$\begin{gathered}\mathbf{A}(\ddot{\mathbf{X}} + \varepsilon\dot{\mathbf{X}}) + \mathbf{BX} + \mathbf{CY} + \mathbf{DXY} = \mathbf{Q}q_0 , \\ \mathbf{CX} + \mathbf{EY} + \frac{1}{2}\mathbf{DXX} = 0,\end{gathered} \tag{3}$$

where: $\mathbf{A} = \left\|A_{ij}\right\|$, $\mathbf{B} = \left\|B_{ij}\right\|$, $\mathbf{C} = \left\|C_{ij}\right\|$, $\mathbf{D} = \left\|D_{ijp}\right\|$, $\mathbf{E} = \left\|E_{ij}\right\|$, $\mathbf{X} = \left\|x_i\right\|$, $\mathbf{Y} = \left\|y_j\right\|$, $\mathbf{Q} = \left\|Q_i\right\|$.

Solving second equation of (3) for $\mathbf{Y}$, multiplying first equation of (3) by $\mathbf{A}^{-1}$ and denoting $\dot{\mathbf{X}} = \mathbf{R}$, the Cauchy problem is defined for nonlinear system of first order differential equations. The mentioned transformation is allowed, since the matrices $\mathbf{A}^{-1}$ and $\mathbf{E}^{-1}$ exist for linearly independent coordinate functions

$$\dot{\mathbf{R}} = -\bar{\varepsilon}\mathbf{R} + \left[\mathbf{A}^{-1}\mathbf{C} \;\; + \left(\mathbf{A}^{-1}\mathbf{DX}\right)\right] \cdot \mathbf{Y} - \mathbf{A}^{-1}\mathbf{BX} + q_0(\bar{t})\mathbf{A}^{-1}\mathbf{Q}, \qquad (4)$$
$$\dot{\mathbf{X}} = \mathbf{R},$$

$$q_0(t) = \frac{q(t)a^4}{Eh_0^4}, \; \bar{w} = \frac{w}{h_0}, \; \bar{x}_i = \frac{x_i}{h_0}, \; \bar{\varphi} = \frac{\varphi}{Eh_0^3}, \quad \bar{y}_i = \frac{y_i}{Eh_0^3},$$

$$\bar{h} = \frac{h(\rho)}{h_0}, \; t = \bar{t}\,\tau, \; \varepsilon = \bar{\varepsilon}\big/\tau, \; \tau = \frac{a}{h_0}\sqrt{\frac{a^2\gamma}{Eg}}.$$

In what follows bars standing over non-dimensional quantities are omitted. The obtained equations are solved using the fourth order differential equations with initial conditions $x_i = 0, \; \dot{x}_i = 0$ for $t = 0$. Reliability of the obtained results is studied in detail in the reference [2]

3. Analysis of the obtained results

We study vibrations of shallow conical and spherical shells treating them as plates with initial deflection $w_0 = -k(1-\bar{\rho})$ and $w_0 = -k(1-\bar{\rho}^2)$, where $k = H\big/h_o$ is the arrow of shell height. As boundary conditions we take moving clamping for $\rho = 1$: $w(1) = w'_\rho(1) = \varphi(1) = \varphi'_\rho(1) = 0$. The approximating functions are taken in the following form: $w_i(\rho) = (1-\rho^2)^{i+1}$, $\varphi_i(\rho) = (1-\rho^2)^{i+1}$, and $n = 6$ in series (2) [2,3].

We analyze behaviour of shells with thickness

$$h(\rho) = h_0(1 + c\rho), \qquad (5)$$

and subjected to uniformly distributed load distributed on the shell surface of the form

$$q = q_0 \sin \omega_p t. \qquad (6)$$

In reference [2] is shown that during a transition of shell vibrations from one regime to another one a stability loss occurs, which can be treated as dynamical stability loss of shells subjected to an action of periodic sign changeable loads. In order to investigate shells driven by the mentioned load, a program package has been developed allowing for construction of charts exhibiting vibration character vs. control parameters $\{q_0, \omega_p\}$ [1-3]. In Figure 1 charts of stiff stability loss putting on vibration character charts for a conical shell with constant thickness ($c = 0$ in (5)) and with height arrow $k = 5$ are reported. White dots are associated with stiff stability loss. Study of the figure indicates that the stiff stability loss occurs for a change of vibration regime. Most dangerous cases appear during transition of vibrations with excitation frequency to vibration of half excitation frequency (see the right corner of Figure 1) as well as transitions from two-frequencies to one-frequency regimes. During the mentioned transitions, vibrations amplitude increases on amount of 15%.

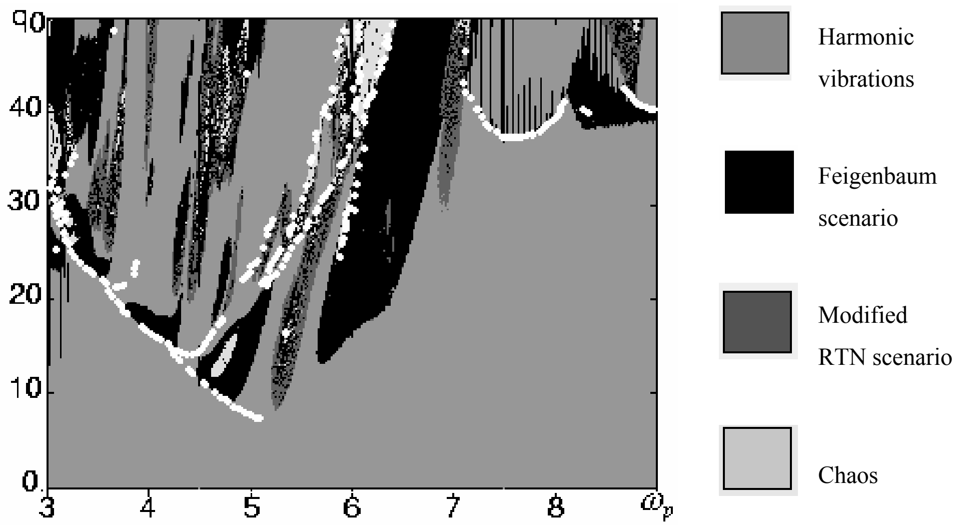
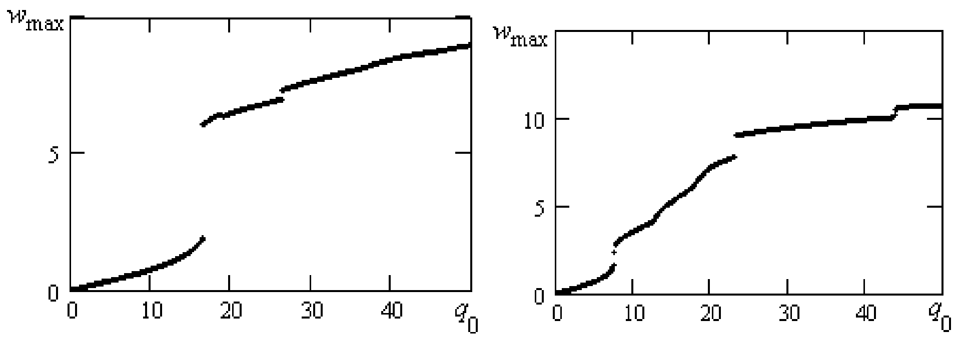


Figure 1. Vibration chart of conical shell with constant thickness

In Figure 1 there are points in zones of harmonic vibrations, where stiff stability loss appears. More detailed analysis shows that we deal not with a first order discontinuity (Fig. 2a), but with inflection point (Fig. 2b) reported on the dependency $w_{\max}(q)$ and the occurred jump is less that 5%.



a) $\omega_p = 4$ b) $\omega_p = 5$

Figure 2. Dependencies $w_{max}(q)$ for (a) $\omega_p = 4$ and (b) $\omega_p = 5$

In Figure 3 the chart of vibration character for the conical shell with variable thickness is shown ($c = 0.1$ in (5), $k = 5$). Increase of thickness on the shell ends decreased chaotic zones and bifurcation zones. Vibration zone associated with half of the frequency is enlarged, and hence stiff stability loss occurred for larger values of exciting amplitude than in the previous case.

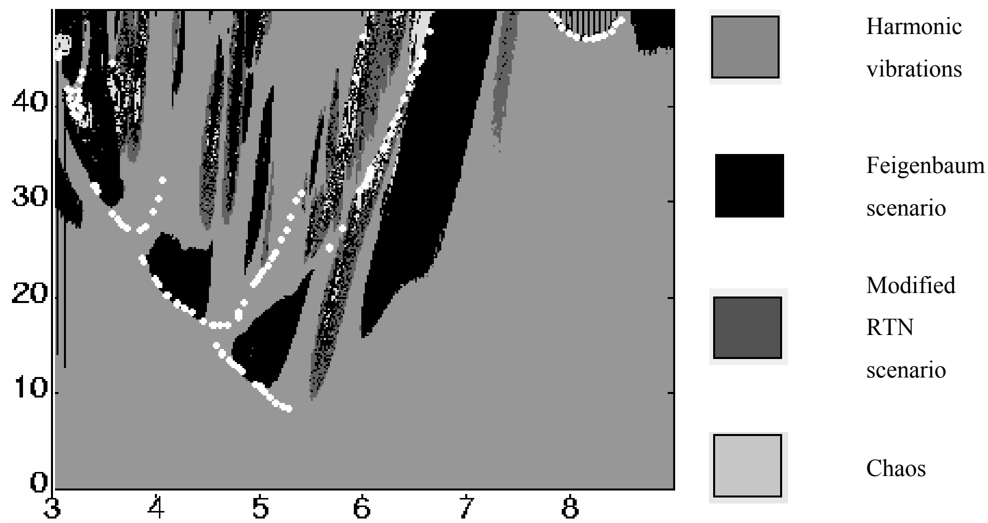


Figure 3. Vibration chart of conical shell with non-constant thickness

One may observe that the number of points, where stiff stability loss occurs, is smaller in comparison to the case of the conical shell with constant stiffness.

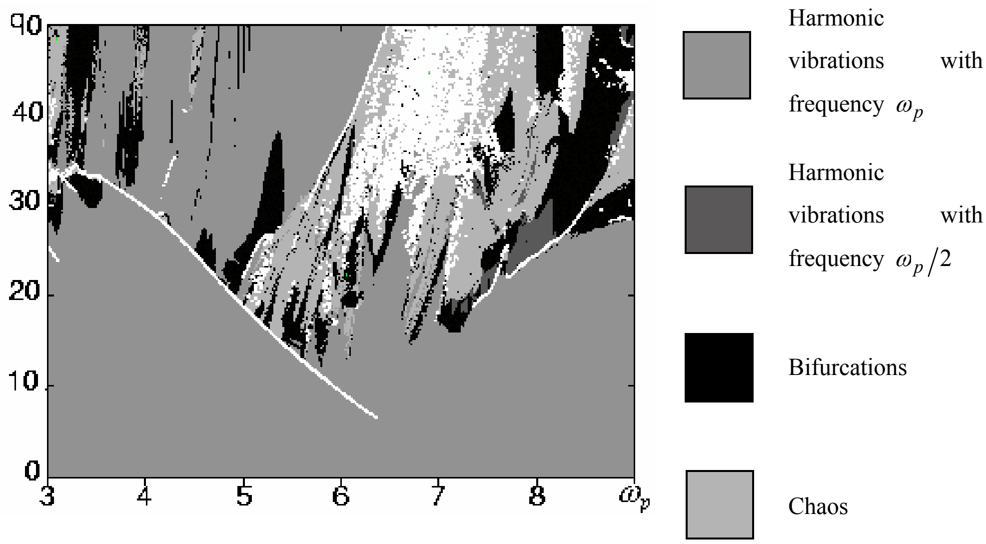


Figure 4. Vibration chart of spherical shell with constant thickness

Let us consider a spherical shell with constant thickness and with arrow height $k = 5$. In Figure 4 the vibration character chart depended on two control parameters is shown. White color denotes stiff stability loss. Here the situation is similar to that reported in Figure 2. Namely, stiff stability loss appears when vibration regime is changed, and largest amplitude jumps occur in the same points as for the conical shell vibrations. Let us change the shell thickness taking $c = 0.1$ in (5), and let us construct a chart of vibration character depending on two control parameters $\{q_0, \omega_p\}$ with an emphasis on stiff stability loss. As it was in the case of the conical shell, increase of its end thicknesses yields essential decrease of zones of bifurcation, chaos and the points of stiff stability loss in comparison to the shall with constant thickness.

Comparison of results obtained for the conical and spherical shells show that shell with boundary condition of the moving clamping and with increased thickness of their ends exhibit smaller both chaotic and stiff stability zones in comparison to shells with constant thickness. Having this in mind one may change the shell thickness and then choose parameters q_0 and ω_p appropriately in order to remove sudden increase of shell vibration amplitude.

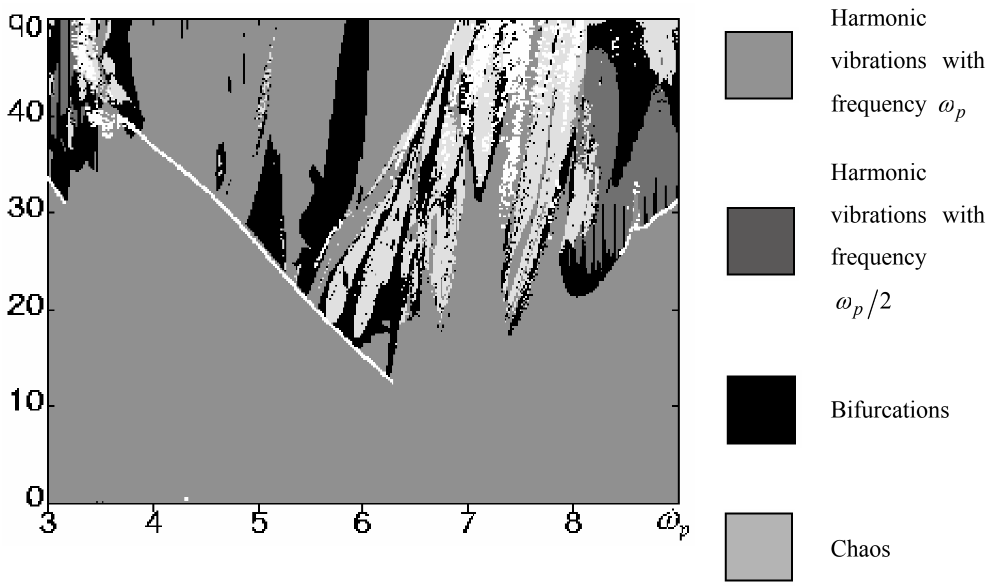


Figure 5. Vibration chart of spherical shell with non-constant thickness

4. Conclusions

In this work the influence of a shell thickness on its vibration character and stiff stability loss has been investigated indicating a possibility to control both chaos and stiff stability loss by applying two control parameters and changing the shell thickness. Vibration charts for both conical and spherical shells with constant and non-constant thicknesses are reported for two control parameters $\{q_0, \omega_p\}$.

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