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OPTIMIZATION PROCEDURE APPLIED TO RING-STIFFENED SHELLS

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Abstract: An influence of ring stiffness distribution along shell defined through $k = k(x)$ on the shell stiffness on example on axially symmetric problem for cylindrical shell is investigated.

1. Introduction

In order to compute stiffened shells usually two approaches are applied. The first one is based on discretization of a studied construction using either FEM or FDM. An associated inversed problem is reduced to mathematical programming. Difficulties in getting a reliable solution increase with increase of rings number *N*. Note that for non-uniformly stiffened shell *N* is equal to the number of design parameters.

The second approach is based on homogenization of the differential equations and it attracts recently attention of both mathematicians and mechanical engineers (see, for instance, [1-4]). For the inversed problems this approach is reduced to an optimal design of a construction with distributed parameters [5, 6].

2. Main results

The differential equation governing deflection between rings of considered shell has the following form

$$w^{IV} + bw = q\,, \tag{1}$$

where: $b = 12\left(1-\nu^2\right)/R^2h^2; q = P(x)/D; D = Eh^3/12(1-\nu^2)$; R is shell radius; h is shell thickness; E, ν are Young modulus and Poisson's coefficient of the shell and rings materials.

The coupling condition of the *i*-th ring can be formulated in the following manner

$$w^- = w^+; (w')^- = (w')^+; (w'')^- = (w'')^+; (w''')^+ - (w''')^- = k(x)w_{x=is}\,, \tag{2}$$

where $(\ )^+$, $(\ )^-$ are intervals located to the right and to the left of the point $x = is$, where s is the distance between rings; $k(x) = EF(x) \big/ (R^2 D)$, $F(x)$ is the area of transversal rings cross section.

The boundary conditions on edges $x = 0, L$, for sake of simplicity, are taken in the form

$$w = w'' = 0 \quad (3)$$

If the rings number is large $\left(s / L = \varepsilon << 1\right)$, then in order to solve the problem (1)-(3), one can apply the asymptotic method of homogenization [1, 2].

Let us introduce the variable

$$\xi = x / \varepsilon \quad (4)$$

which is independent on x, and therefore, the associated differential operator reads

$$w' = w'_x + \varepsilon^{-1} w'_\xi \quad (5)$$

The deflection w is sought in the form

$$w = w_0\left(x\right) + \varepsilon^4 w_1\left(x, \xi\right) + \varepsilon^5 w\ \left(x, \xi\right) + \ldots.., \quad (6)$$

where $w_i\left(i = 1,2...\right)$ are periodic functions with the period L and with respect to ξ.

Substituting (5), (6) into (1)-(4), and carrying out the asymptotic splitting with respect to ε powers, the following relations are obtained (periodicity conditions for w_i with respect to ξ are also applied):

$$w_{1,\xi}^{IV} + w_{0,x}^{IV} + \beta w_0 = q; \quad (7)$$

$$\left(w; w'_{1,\xi}; w''_{1,\xi}\right)_{\xi=0} = \left(w_1; w'_{1,\xi}; w''_{1,\xi}\right)_{\xi=L}; \quad (8)$$

$$w'''_{1,\xi / \xi=L} - w'''_{1,\xi / \xi=0} = K^*\left(x\right) w_0; \quad (9)$$

$$w_{0 / x=0,L} = w''_{0,L / x=0,L} = 0. \quad (10)$$

Note that during derivation of relation (9), it has been assumed that $K^*(x) = LK / S \sim 1$.

Integrating (7) with respect to ξ, one gets

$$w_1 = \left(q - w_{0,x}^{IV} - \beta w_0\right)\xi^4 / 24 + C_1(x)\xi^3 + C_2(x)\xi^2 + C_3(x)\xi + C_4(x).$$

Determining $C_1 - C_4$ from conditions (8), one gets

$$w_1 = K * \left(x\right)\left(q - w_{0,x}^{IV} - \beta w_0\right)\xi^2 \left(\xi - L\right)^2 / 24. \quad (11)$$

Substituting (11) into (9), the following homogenized equation for w_0 is obtained

$$w_{0,x}^{IV} + \left(K^*\left(x\right) + \beta\right) w_0 = q. \quad (12)$$

Equation (12) governs the axially symmetric deformation of a structurally orthotropic shell with continuously distributed rings stiffness along the whole shell length. The corrector (11) accounts discreteness of rings distribution.

Consider first the case when variation of the rings stiffness is small, i.e.

$$K^*(x)+\beta = a+\varepsilon_1\varphi(x), \tag{13}$$

where: $a = const$; $\varepsilon_1 << 1$.

The following series is assumed

$$w_0 = w_{00} + \varepsilon_1 w_{01} + \varepsilon_1^2 w_{02} + ... \ . \tag{14}$$

Substituting relations (13), (14) into equation (12) and comparing the coefficients standing by the same power of ε_1 to zero, one gets

$$w_{00,x}^{IV} + aw_{00} = q \tag{15}$$

$$w_{0i,x}^{IV} + aw_{0i} = -\varphi(x)w_{0i-1}, \quad i = 1,2.... \ . \tag{16}$$

Developing the functions $q(x), \varphi(x), w_{0i}(x)$, $w_{0i}(x)$ into the Fourier series in the interval [0, L] one obtains

$$q = \sum_{n-1}^{\infty} q_n \sin\alpha nx \qquad \varphi = \sum_{n-1}^{\infty} \varphi_n \cos\alpha nx \qquad W_{oi} = \sum_{n-1}^{\infty} A_{in} \sin\alpha nx\ , \tag{17}$$

where: q_n, φ_n, A_{in} - const, $\alpha = 2\pi / L$.

Substituting (17) into (15), (16) one obtains

$$A_{0n} = q_n / \left(\alpha^4 n^4 + a\right), \quad A_{in} = B_{i-1n} / \left(\alpha^4 n^4 + a\right), \tag{18}$$
$$B_{in} = 0.5\varphi_k \left(A_{ik+1} - A_{ik-n}\right),$$

and in result the following approximation holds

$$w_0 = \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} \varepsilon_1^i A_{in} \sin\alpha nx\ . \tag{19}$$

The corrector w_i is found from the relation (11).

The solution (19) can be also extended into the case of non-small rings stiffness variations ($\varepsilon_1 \sim 1$) after an application of the Padé approximations [7].

In what follows the Padé approximation [1/1] for the series coefficients (19) gives

$$w_0 = \left[\left[A_{0n}A_{1n} + \varepsilon_1(A_{1n}^2 - A_{0n}A_{2n})\right] \Big/ (A_{1n} - \varepsilon_1 A_{2n})\right]\sin\alpha nx\ . \tag{20}$$

Consider the case $\alpha + \varepsilon_1\varphi(x) = c(1-\varepsilon_1 \cos 2\alpha x),\ \ c, q - const$. Integral rings stiffness is constant for any ε_1 in this case. The coefficients of the series (18) take the form (for $\alpha = 1$):

$$A_{02n-1} = 4q/[(2n-1)\pi[(2n-1)^2 + c]],$$

$$A_{i,2n-1} = 0.5c(A_{i-1,2n-3} + A_{i-1,2n+1})/[(2n-1)^4 + c]; \quad A_{i,2n} = 0.$$

Now one can investigate how a change of ring stiffness influences a change of stiffness of the whole shell. For this purpose one must compare w_0 with w_0^* (note that shell deflection possesses rings with the same stiffness ($\varepsilon = 0$)):

$$w_0 - w_0^* = D_{2n-1} \sin(2n-1)x,$$

where: $D_{2n-1} = \varepsilon_1 A_{1,2n-1} / (A_{1,2n-1} - \varepsilon_1 A_{2,2n-1})$.

The dependence D_1 / A_{01} characterizes shell stiffness variation (for $q = const$ the fundamental contribution into deflection is introduced by the first harmonic of the series (20)), is reported in Figure 1.

The curves 1-5 correspond to $\varepsilon_1 = 0.1; 0.3; 0.5; 0.8; 1$, respectively.

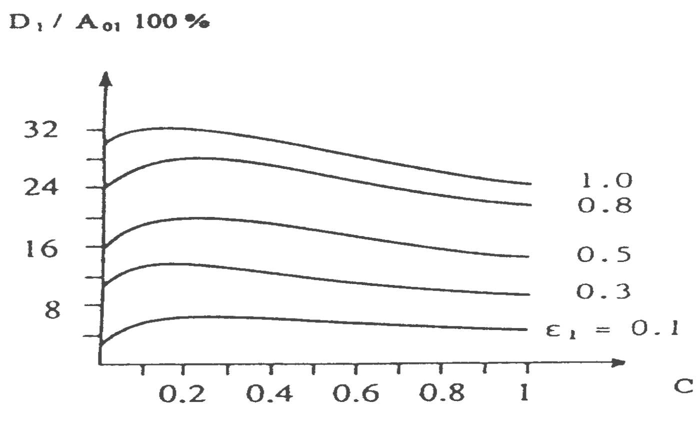

Figure 1. Dependence D_1 / A_{01} vs *c*.

Analysis of the results shown in Figure 1 yields a conclusion that for given load $q = const$, the given rings stiffness distribution is particularly suitable in the interval $0.05 < c < 0.15$, and allows to decrease the largest shell deflection on amount of 30%.

Consider now the problem of optimisation, where the shell flexibility is taken as the being minimized functional of the form

$$I = \int_0^L qwdx \to \min_{\kappa} , \tag{21}$$

with the constraints

$$\int_0^L k \cdot \sum_{n=0}^{N} \delta(x - nl)dx = c . \tag{22}$$

If zero order approximation is used $(w = w_0)$, then one has to add (10), (12) to the constraints. Therefore, following [5], the following new control function $\varphi(x)$ is applied

$$k = \alpha + \gamma \sin\varphi , \quad \alpha = 0.5(k_{\min} + k_{\max}), \quad \gamma = 0.5(k_{\min} - k_{\max}). \tag{23}$$

The inversed problem reads

$$I = \int_0^L qwdx \to \min_{\phi} ; \quad I_1 = \int_0^L \sin\varphi dx = \frac{c - \alpha}{\gamma} ; \tag{24}$$

$$w_{0,x}^{IV} + (\alpha + \gamma \sin\varphi) \quad w_0 = q ; \tag{25}$$

$$w_{0/x=0,L} = w''_{0/x=0,L} = 0 . \tag{26}$$

Following the approaches applied in the theory of optimal control with one variable, one gets the optimality condition of the problem (24)-(26). For this purpose one can write the expressions governing first integrals (24) variations and equation in variations corresponding to (26), of the forms

$$\delta I = \int_0^L q\delta wdx ; \quad \delta I_1 = \int_0^L \cos\varphi\delta\varphi dx ; \tag{27}$$

$$\delta w_{0,x}^{IV} + (\alpha + \gamma \sin\varphi) \quad \delta w_0 + \gamma \cos\varphi \;\; w_0 \quad \delta\varphi = 0 . \tag{28}$$

Notice that equation (28) is obtained first after substitution $w_0 + \delta w_{0,}$ $\varphi + \delta\varphi$ instead of w_0 and φ_0 in (25), and after extraction of the terms linear with respect to δw_0 and $\delta\varphi$.

In what follows we are going to express the first variation of the minimized functional through the variation $\delta\varphi$.

For this purpose the conjugated variable $v(x)$ is introduced, which is defined through the condition that the expression for variation of the minimized functional does not include δw_0 . Multiplying the left hand side of equation (28) by $v(x)$, and integrating it from 0 to *L*, one gets

$$\int_0^L v\left[\delta w_{0,x}^{IV} + (\alpha + \gamma \sin\varphi)\delta w_0 + \gamma \cos\varphi w_0 \delta\varphi\right]dx = 0 .$$

Next carrying out the integration by parts with inclusion of (25), (26), the above integral is transformed to the following form

$$\int_0^L \left[\left(v^{IV} + (\alpha + \gamma \sin\varphi) v\right)\delta w_0 + \gamma\, v w_0 \cos\varphi\, \delta\varphi\right] dx \,, \tag{29}$$

and the following boundary conditions are applied

$$v_{/\,x=0,L} = v''_{/\,x=0,L} = 0 \,. \tag{30}$$

3. Conclusions

To conclude, the considered problem of optimization has been reduced to that of solution of the boundary value problems (12), (3) and (35), (30).

The obtained non-linear boundary value problem can be solved numerically using either one of the successive optimization method [5] or the perturbation technique already used while solving the direct problem (13)-(16).

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