REGULAR AND CHAOTIC MOTION EXHIBITED BY A SHAFT-BUSH SYSTEM WITH HEAT, WEAR AND FRICTION PROCESSES

Jan Awrejcewicz
Department of Automatics and Biomechanics
Technical University of Łódź
Poland
awrejcew@p.lodz.pl

Yuriy Pyryev
Department of Automatics and Biomechanics
Technical University of Łódź
Poland
jupyrjew@p.lodz.pl

Abstract

In this work, methods of analysis and model of dynamics of contact bush-shaft systems, including heat generation and wear exhibited by such systems, are presented. The considered problem is reduced to analysis of ordinary differential equations governing the change of velocities of the contacting bodies, and to study of the integral Volterra type equation governing contact pressure behavior. Thresholds of chaos have been found due to analysis of Lyapunov exponents, phase portraits, Poincaré maps and power spectra. The following theoretical approaches are applied: perturbation methods, Melnikov techniques, Laplace transformations, theory of integral equations and various variants of numerical analysis.

Key words

Frictions, heat, wear bush-shaft system.

1 Introduction

Friction, wear, heat generation accompanying friction, and heat expansion (or contraction) are all very complex phenomena that interact and form one complex multidimensional dynamic system analyzed together with friction. For the non-stationary friction process, all of its time depending parameters are interrelated.

Classical problem concerning vibration of a friction pair consisted of pad and rotating shaft fixed to a frame by mass-less springs (simple model of typical braking pad or the so called Pronny's brake) has been investigated in references [Andronov et al.,(1966), Neimark (1978)]. In works of [Pyryev et al., (1995), Pyryev, Grylitskiy, (1995)] the so called thermo-elastic contact between a rotating cylinder and a fixed non-inertial pad has been studied. In what follows more complicated axially symmetric problem of both regular and chaotic self-excited vibrations (caused by friction) and wear of the rotating cylinder and the pad (fixed to a frame by springs and viscous damping elements) is investigated.

It should be emphasized that usually either tribological processes occurring on the contact surfaces are not accounted, or inertial effects are neglected. In other words, both mentioned processes are treated separately. In this work both elements of complex contact behavior are simultaneously included into consideration, which allows for a proper modeling of the real contact system dynamics. Analytical and numerical analyses are carried out in a wide aspect through investigation of various types of nonlinearities, damping and excitations applied to the analyzed system. A Duffing type elastic nonlinearity, a nonlinear density of the frictional energy stream, a nonlinear friction dependence versus velocity and a nonlinear contact temperature characteristic, as well as nonlinear character of a wear are accounted, among others.

We consider thermo-elastic contact of a solid isotropic circular shaft (cylinder) with a cylindrical tube-like rigid bush, where the bush is linked with the housing by springs and a damper.

This paper extends analysis carried out in reference [Awrejcewicz, Pyryev, 2003]. Contrary to the previous results, a novel mechanism of contact between bush and shaft is proposed, a viscous damping is added, and an influence of tribologic factors as well as chaotic dynamics is analyzed. It has been shown that (owing to wear) chaos vanishes, since there is a lack of contact between both bodies. Owing to heat generation through friction, either chaos vanishes or thermal instability appears.

2 The system under analysis

Consider thermo-elastic contact of a solid isotropic circular shaft of radius R_1 with a cylindrical tube-like rigid bush of external radius R_2, which is fitted to the cylinder according to the expression $U_* h_U(t)$ ($h_U(t) \to 1$, $t \to \infty$). The internal bush radius is: $R_1 - U_*$ ($U_*/R_1 << 1$) (Figure 1). The bush is linked with the housing by springs and the damper with viscous coefficient c.

We assume, that the bush is a perfect rigid body, and that radial springs have the stiffness coefficient k_1, whereas tangent springs are characterized by non-linear stiffness k_2 and k_3 of Duffing type. In addition, the bush is subjected to a damping force action in tangent direction. The cylinder rotates with a such angular velocity $\Omega(t) = t_*^{-1}\omega_1(t)$, that the centrifugal forces may be neglected. We assume that the angular speed of the shaft rotation changes in accordance with $\omega_1 = \omega_k + \zeta_k \sin\omega' t$. We assume that between bush and shaft dry friction appears defined by the function $F_t(V_r)$, where V_r is a relative velocity between the two given bodies $V_r = \Omega R_1 - \dot{\varphi}_2 R_1$. B_2 denotes the mass moment of inertia. We assume also that in accordance with the Amontos assumption the friction force reads: $F_t = f(V_r)N(t)$ ($f(V_r)$ is the kinetic friction coefficient).

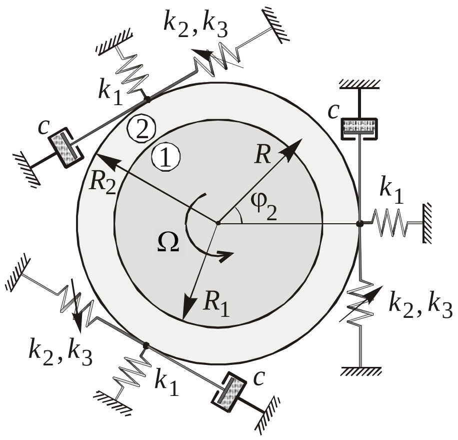

Figure 1 The analyzed system

The friction force F_t yields heat generated by friction on the contact surface $R = R_1$, and wear U^w of the bush occurs. Observe that the frictional work is transformed to heat energy. Let the shaft temperature, denoted by $T_1(r,t)$, be initially equal to T_0. It is further assumed that the bush transfers heat ideally, and that between both shaft and bush the Newton's heat exchange occurs and that the bush has constant temperature T_0. As a first approximation to the studied problem we assume that $T_0 = const$.

3 Mathematical formulation of the problem

Vibrations of the bush being in thermoelastic contact with the rotating shaft are governed by the following dimensionless equation (inclusions) [Awrejcewicz, Pyryev, 2002]:

$$\ddot{\varphi}(\tau) + 2h\dot{\varphi}(\tau) - \varphi(\tau) + b\varphi^3(\tau) \in \varepsilon F(\omega_1 - \dot{\varphi})\,p(\tau) \qquad (1)$$

with the initial condition

$$\varphi(0) = \varphi^\circ,\ \dot{\varphi}(0) = \omega_k\,. \qquad (2)$$

The thermoelastic problem under consideration takes the following dimensionless form [Nowacki, 1986]

$$\frac{\partial^2\theta(r,\tau)}{\partial r^2} + \frac{1}{r}\frac{\partial\theta(r,\tau)}{\partial r} = \frac{1}{\tilde{\omega}}\frac{\partial\theta(r,\tau)}{\partial\tau},\ 0 < r < 1 \qquad (3)$$

$$\frac{\partial\theta(1,\tau)}{\partial r} + Bi\,\theta(1,\tau) = \gamma F\big(\omega_1 - \dot{\varphi}(\tau)\big)\big(\omega_1 - \dot{\varphi}(\tau)\big)p(\tau) \qquad (4)$$

$$r\frac{\partial\theta(r,\tau)}{\partial r}\bigg|_{r\to 0} = 0,\ 0 < \tau < \tau_c \qquad (5)$$

$$\theta(r,0) = 0,\ 0 < r < 1 \qquad (6)$$

$$p(\tau) = h_U(\tau) - u^w(\tau) + 2\int_0^1 \theta(\xi,\tau)\,\xi\,d\xi \qquad (7)$$

$$u^w(\tau) = k^w\int_0^\tau \left|\omega_1 - \dot{\varphi}(\tau)\right| p(\tau)d\tau \qquad (8)$$

Applying the Laplace transformation [Carslaw, Jaeger, 1959] to equations (3)-(7), the following integral equation is found

$$p(\tau) = h_U(\tau) - u^w(\tau) + 2\gamma\tilde{\omega}\int_0^\tau \dot{G}_p(\tau - \xi)F(\omega_1 - \dot{\varphi})\,p(\xi)(\omega_1 - \dot{\varphi})d\xi \qquad (9)$$

Temperature occurred on the contacting surface reads

$$\theta(r,\tau) = \gamma\tilde{\omega}\int_0^\tau \dot{G}_\theta(r,\tau - \xi)F(\omega_1 - \dot{\varphi})\,p(\xi)(\omega_1 - \dot{\varphi})d\xi \qquad (10)$$

where

$$\left\{G_p(\tau), G_\theta(1,\tau)\right\} = \frac{\{0.5,1\}}{Bi\,\tilde{\omega}} - \sum_{m=1}^{\infty}\frac{\{2Bi, 2\mu_m^2\}}{\mu_m^2\tilde{\omega}(Bi^2 + \mu_m^2)}e^{-\mu_m^2\tilde{\omega}\tau} \qquad (11)$$

μ_m $(m = 1,2,3,...)$ are the roots of characteristic equation $BiJ_0(\mu) - \mu J_1(\mu) = 0$.

In equations (1)-(11) the following dimensionless quantities are introduced

$$\tau = \frac{t}{t_*},\ r = \frac{R}{R_1},\ p = \frac{P}{P_*},\ \theta = \frac{T_1 - T_0}{T_*},\ u^w = \frac{U^w}{U_*},$$

$$\varepsilon = \frac{P_* t_*^2 2\pi R_1^2}{B_2},\ h = \frac{cR_2^2}{2B_2 t_*},\ k^w = \frac{P_* K^w R_1}{U_*},\ \tau_c = \frac{t_c}{t_*},$$

$$Bi = \frac{\alpha_T R_1}{\lambda_1},\ \gamma = \frac{(1-\eta)E_1\alpha_1 R_1^2}{\lambda_1(1 - 2\nu_1)t_*},\ \tilde{\omega} = \frac{t_* a_1}{R_1^2},$$

$$\omega_0 = \omega' t_*,\ \varphi(\tau) = \varphi_2(t_*\tau),\ h_U(\tau) = h_U(t_*\tau),$$

$$F(\omega_1 - \dot{\varphi}) = f(V_*(\omega_1 - \dot{\varphi})),$$

$$b = (k_3 R_2^4 - (2/3)k_2 R_2^2 + (l_1 + R_2)R_2((l_0/l_1)(1 + 3(R_2/l_1) + 3(R_2/l_1)^2) - 1)k_1/6)(t_*^2/B_2)$$

where

$$V_* = \frac{R_1}{t_*},\ t_* = \sqrt{\frac{B_2}{k_* R_2^2}},\ T_* = \frac{U_*}{\alpha_1(1 + \nu_1)R_1},$$

$$k_* = k_1\left(\frac{l_0}{l_1} - 1\right)\left(1 + \frac{l_1}{R_2}\right) - k_2,\ P_* = \frac{\alpha_1 E_1 T_*}{(1 - 2\nu_1)},$$

and l_0 is the un-stretched spring length, l_1 is the length of the compressed spring for $\varphi_2 = 0$, ($k_* > 0$), E_1 is the elasticity modulus, ν_1 is the Poisson coefficient, α_1 is the coefficient of thermal expansion of the shaft, α_T is heat transfer coefficient, a_1 is thermal diffusivity, λ_1 is thermal conductivity, $\varphi_2(t)$ is the angle of bush rotation, K^w is the wear coefficient, η is denotes the part of heat energy associated with wear $\eta \in [0,1]$, t_c is time of contact ($0 < t < t_c$, $P(t) > 0$).

Notice that the stated problem is modeled by the both nonlinear differential equation (inclusions) (1) and integral equation (9) governing rotational velocity $\dot{\varphi}(\tau)$ and contact pressure $p(\tau)$. Temperature and wear is defined through equations (10) and (8).

4 Analysis of the investigated process

Let us assume that the relative velocity dependence is approximated by the function [Awrejcewicz, 1996]

$$F(y) = F_0 \operatorname{Sgn}(y) - \alpha y + \beta y^3 \tag{12}$$

where

$$\operatorname{Sgn}(y) = \begin{cases} y/|y| & \text{if } y \neq 0 \\ [-1,1] & \text{if } y = 0 \end{cases} \tag{13}$$

In the numerical analysis, function $\operatorname{Sgn}(y)$ is approximated by [Martins et al., 1990]

$$\operatorname{sgn}_{\varepsilon_0}(y) = \begin{cases} y/|y| & \text{if } |y| > \varepsilon_0 \\ \left(2 - \dfrac{|y|}{\varepsilon_0}\right)\dfrac{y}{\varepsilon_0} & \text{if } |y| < \varepsilon_0 \end{cases} \tag{14}$$

4.1 Melnikov's method

A particular case of our problem is further studied ($\gamma = 0$, $k^w = 0$, $p(\tau) \to 1$). In order to estimate analytically the critical parameters responsible for chaos occurrence often the Melnikov's technique [Melnikov, 1963] is used. In this case the Melnikov function reads (see [Awrejcewicz, Pyryev, 2003])

$$M(\tau_0) = -\int_{-\infty}^{+\infty} y_0(t)\left[F(\omega_r) - h_1 y_0(t)\right]dt = I(\tau_0) + J(\tau_0) \tag{15}$$

where:

$$\omega_r(t) = \omega_k + \zeta_k \sin\left(\omega_0(t + \tau_0)\right) - y_0(t),$$

$$y_0(\tau) = -\sqrt{2/b}\,\sinh(\tau)/\cosh^2(\tau),$$

$$J(\tau_0) = 2C + 2\zeta_k\sqrt{A^2 + B^2}\sin(\omega_0\tau_0 + \varphi_0) + 6\beta\zeta_k^2\left(I_{220}\cos^2\omega_0\tau_0 + I_{202}\sin^2\omega_0\tau_0 - 2\omega_* I_{111}\sin\omega_0\tau_0\cos\omega_0\tau_0\right) + 2\beta\zeta_k^3\left(-I_{130}\cos^3\omega_0\tau_0 - 3I_{112}\sin^2\omega_0\tau_0\cos\omega_0\tau_0\right)$$

$$A = (\alpha - 3\beta\omega_k^2)I_{110} - 3\beta I_{310}, \quad B = 6\beta\omega_k I_{201},$$

$$C = \beta I_{400} - (\alpha - h_1 - 3\beta\omega_k^2)I_{200}, \quad \varphi_0 = \arctan(A/B).$$

$$I_{200} = \frac{2}{3b}, \quad I_{400} = \frac{8}{35b^2}, \quad I_{201} = \frac{\pi\omega_0(2 - \omega_0{}^2)}{6b\sinh(\pi\omega_0/2)},$$

$$I_{110} = -\frac{\pi\omega_0}{\sqrt{2b}\cosh(\pi\omega_0/2)},$$

$$I_{112} = \frac{\pi\omega_0\cosh(\pi\omega_0/2)}{\sqrt{2b}\left(1 - 2\cosh(\pi\omega_0)\right)},$$

$$I_{111} = -\frac{\pi\omega_0}{\sqrt{2b}\cosh(\pi\omega_0)},$$

$$I_{220} = \frac{\pi\omega_0(2\omega_0^2 - 1) + \sinh(\pi\omega_0)}{3b\sinh(\pi\omega_0)},$$

$$I_{202} = \frac{\pi\omega_0(1 - 2\omega_0^2) + \sinh(\pi\omega_0)}{3b\sinh(\pi\omega_0)},$$

$$I_{310} = \frac{\omega_0(11 + 10\omega_0^2 - \omega_0^4)}{120b\sqrt{2b}}\left\{\psi\left(\frac{1 - i\omega_0}{4}\right) - \psi\left(\frac{3 - i\omega_0}{4}\right) + \psi\left(\frac{1 + i\omega_0}{4}\right) - \psi\left(\frac{3 + i\omega_0}{4}\right)\right\},$$

$$I_{130} = -\frac{3\pi\omega_0}{8\sqrt{2b}}\left\{\cot\left(\frac{\pi(1 - i\omega_0)}{4}\right) + \cot\left(\frac{3\pi(1 - i\omega_0)}{4}\right) - \cot\left(\frac{\pi(3 - i\omega_0)}{4}\right) - \cot\left(\frac{\pi(1 - 3i\omega_0)}{4}\right)\right\}, \quad \psi(z) = \frac{\Gamma'(z)}{\Gamma(z)},$$

In equation (15) the term $I(\tau_0)$ is defined by the formula

$$I(\tau_0) = -F_0\int_{-\infty}^{+\infty} y_0(t)\operatorname{sgn}(\omega_r)\,dt = 2F_0\sqrt{\frac{2}{b}}\sum_m \frac{\operatorname{sgn}(\omega_r'(t_m))}{\cosh t_m} \tag{16}$$

where t_m are the roots of the equation

$$\omega_r(t_m) = \omega_k + \zeta_k\sin\left(\omega_0(t_m + \tau_0)\right) - y_0(t_m) = 0, \tag{17}$$

and

$$\omega_r'(t) = \zeta_k\omega_0\cos\left(\omega_0(t + \tau_0)\right) - x_0(t) + bx_0^3(t). \tag{18}$$

If the Melnikov function (15) changes sign, then chaos may occur.

4.2 Calculation of Lyapunov exponents

A particular case of our problem is further studied ($\gamma = 0$, $k^w = 0$, $p(\tau) \to 1$).

Note that while computing Lyapunov exponents, besides the following equations

$$\dot{x} = y\,,$$
$$\dot{y} = x - bx^3 + \varepsilon[F_0 \,\mathrm{sgn}_{\varepsilon_0}(\mathrm{v}_r) - \alpha \mathrm{v}_r + \beta \mathrm{v}_r^3] - \varepsilon h_1 y\,,$$
$$\dot{z} = \omega_0\,, \tag{19}$$

also three additional system of equations ($n = 1,2,3$) with respect to perturbations are solved

$$\dot{\tilde{x}}^{(n)} = \tilde{y}^{(n)}\,,$$
$$\dot{\tilde{y}}^{(n)} = \tilde{x}^{(n)} - 3bx^2\tilde{x}^{(n)} + \varepsilon[F_0\delta_{\varepsilon_0}(\mathrm{v}_r) - \alpha + 3\beta \mathrm{v}_r^2]\tilde{\mathrm{v}}_r^{(n)} - \varepsilon h_1 \tilde{y}^{(n)} \tag{20}$$
$$\dot{\tilde{z}}^{(n)} = 0$$

where:

$$x = \varphi(\tau)\,,\; y = \dot{\varphi}(\tau)\,,\; z = \omega_0 \tau\,,$$
$$\mathrm{v}_r = \omega_k + \zeta_k \sin z - y\,,$$
$$\tilde{\mathrm{v}}_r^{(n)} = \zeta_k \tilde{z}^{(n)} \cos z - \tilde{y}^{(n)}\,,\; h_1 = 2h/\varepsilon\,,$$
$$\delta_{\varepsilon_0}(y) = \begin{cases} 0, & |y| > \varepsilon_0, \\ (2/\varepsilon_0)(1 - |y|/\varepsilon_0), & |y| < \varepsilon_0. \end{cases}$$

Twelve equations of system (19),(20) are solved using the fourth order Runge-Kutta method and Gram-Schmidt reorthonormalization procedure.

Let $\tilde{\mathbf{x}}_0^0$, $\tilde{\mathbf{y}}_0^0$, $\tilde{\mathbf{z}}_0^0$ be initial values of perturbation vectors which are orthonormal. After time T an orbit $\mathbf{x}(\tau)$ reaches the point $\mathbf{x}_1$ with the associated perturbbations $\tilde{\mathbf{x}}_1$, $\tilde{\mathbf{y}}_1$, $\tilde{\mathbf{z}}_1$. Then the so called Gram-Schmidt reorthonormalization procedure is carried out and the following new initial set of conditions is formulated

$$\tilde{\mathbf{x}}_1^0 = \frac{\tilde{\mathbf{x}}_1}{\|\tilde{\mathbf{x}}_1\|}$$
$$\tilde{\mathbf{y}}_1^0 = \frac{\tilde{\mathbf{y}}_1'}{\|\tilde{\mathbf{y}}_1'\|}\,,\; \tilde{\mathbf{y}}_1' = \tilde{\mathbf{y}}_1 - (\tilde{\mathbf{y}}_1, \tilde{\mathbf{x}}_1^0)\tilde{\mathbf{x}}_1^0\,, \tag{21}$$
$$\tilde{\mathbf{z}}_1^0 = \frac{\tilde{\mathbf{z}}_1'}{\|\tilde{\mathbf{z}}_1'\|}\,,\; \tilde{\mathbf{z}}_1' = \tilde{\mathbf{z}}_1 - (\tilde{\mathbf{z}}_1, \tilde{\mathbf{x}}_1^0)\tilde{\mathbf{x}}_1^0 - (\tilde{\mathbf{z}}_1, \tilde{\mathbf{y}}_1^0)\tilde{\mathbf{y}}_1^0\,.$$

In what follows, after time interval T , a new set of perturbation vectors $\tilde{\mathbf{x}}_2$, $\tilde{\mathbf{y}}_2$, $\tilde{\mathbf{z}}_2$ is defined, which is also reorthonormalized due to the Gram-Schmidt procedure (21). This algorithm is repeated M times. Notice that $(\tilde{\mathbf{x}}_1^0, \tilde{\mathbf{y}}_1^0) = 0$, $(\tilde{\mathbf{x}}_1^0, \tilde{\mathbf{z}}_1^0) = 0$, $(\tilde{\mathbf{y}}_1^0, \tilde{\mathbf{z}}_1^0) = 0$ and if $\mathbf{x} = (x, y, z)$, $\mathbf{y} = (x_1, y_1, z_1)$ then $\|\mathbf{x}\| = \sqrt{x^2 + y^2 + z^2}$, and the scalar product $(\mathbf{x}, \mathbf{y}) = xx_1 + yy_1 + zz_1$. Finally, a spectrum of three Lyapunov exponents is computed via formulas

$$\lambda_1 = \frac{1}{MT}\sum_{i=1}^{M} \ln\|\tilde{\mathbf{x}}_i\|\,, \tag{22}$$

$$\lambda_2 = \frac{1}{MT}\sum_{i=1}^{M} \ln\|\tilde{\mathbf{y}}_i'\|\,,\quad \lambda_3 = \frac{1}{MT}\sum_{i=1}^{M} \ln\|\tilde{\mathbf{z}}_i'\|\,, \tag{23}$$

where the occurred vectors are taken before the normalization procedure.

3.3 Numerical Analysis

Our numerical computation are carried out for the particular case ($\gamma = 0$, $k^w = 0$). The following dimensionless parameters are taken: $F_0 = \alpha = \beta = 0.3$, $\omega_0 = 2$, $\omega_k = 0.4$, $b = 1$, $\varepsilon = 0.1$.

In Figure 2 the Melnikov's function $M(\tau_0)$ for different values of parameter ζ_k before and after sign change of $M(\tau_0)$ is reported. One may convince himself that both analytical and numerical predictions of chaos coincide.

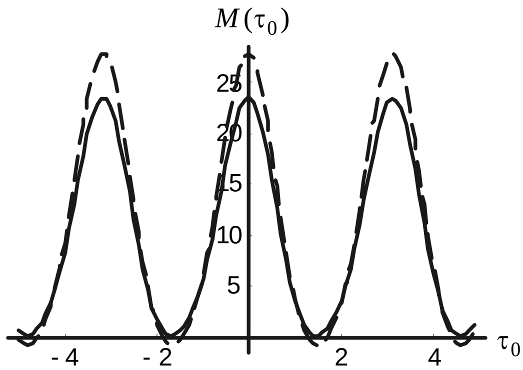

Figure 2 The Melnikov's function $M(\tau_0)$ versus parameter τ_0 for $\zeta_k = 3.9$ (continues curves) and for $\zeta_k = 4.2$ (dashed curves), $h_1 = 1$.

Numerical analysis is carried out for the bifurcation diagram with respect to x vs. ζ_k for $\zeta_k \in (0, 12)$. The obtained results are shown in Figures 3 for the case $h_1 = 1$.

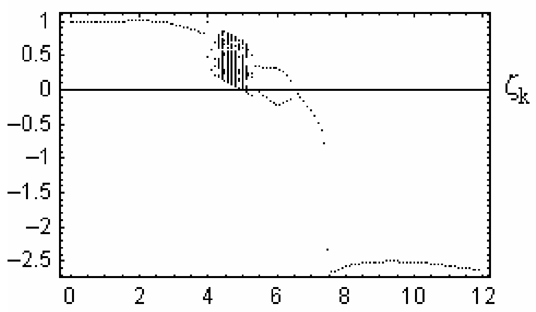

Figure 3 Bifurcation diagrams using ζ_k as control parameter, $\gamma = 0$, $k^w = 0$, $b = 1$, $h_1 = 1$.

The Lyapunov exponents in time interval $\tau \in (1200, 1514)$ ($\tilde{\mathbf{x}}_0^0 = (1, 0, 0)$, $\tilde{\mathbf{y}}_0^0 = (0, 1, 0)$, $\tilde{\mathbf{z}}_0^0 = (0, 0, 1)$, $T = 0.005$, $M = 80000$) are computed due to formulas (13) for the same values of parameters. In Figures 4 dependencies of Lyapunov exponents on the control parameter ζ_k are reported. A study of both Lyapunov exponents and bifurcation diagrams implies that chaos begins for $\zeta_k = 4.25$, for $h_1 = 1$ (notice that the largest Lyapunov exponent λ_1 is positive). An increase of the parameter h_1 responsible for damping yields an increase of the amplitude of the bush, where chaos is birthed.

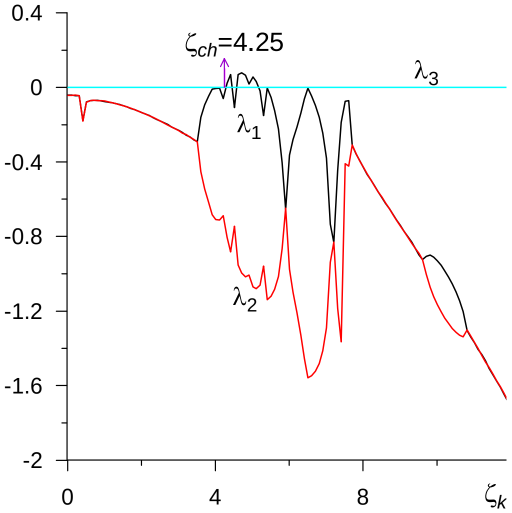

Figure 4 Lyapunov exponents using ζ_k as control parameter, $\gamma = 0$, $k^w = 0$, $b = 1$, $h_1 = 1$.

In a general case, numerical analysis is carried out for a steel made shaft ($\alpha_1 = 14 \cdot 10^{-6}\,^\circ\mathrm{C}^{-1}$, $\lambda_1 = 21 W/(\mathrm{m} \cdot ^\circ\mathrm{C})$, $\nu_1 = 0.3$, $a_1 = 5.9 \cdot 10^{-6} \mathrm{m}^2/s$, $E_1 = 19 \cdot 10^{10}\mathrm{Pa}$). Observe that no accounting of tribological processes ($h_1 = 0.5$, $\zeta_k = 3.9$, $\gamma = 0$, $k^w = 0$) yields chaotic dynamics (Figure 5, curve 2).

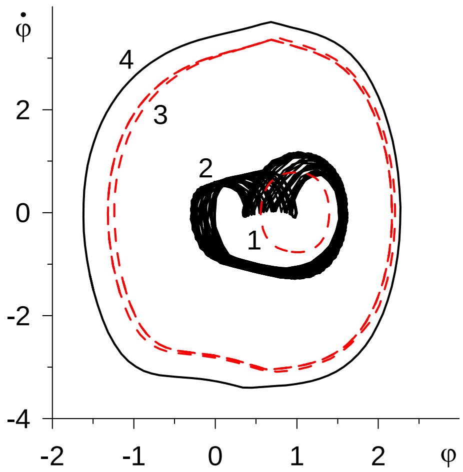

Figure 5 Phase plane of bush motion for $h_1 = 0.5$, $k^w = 0$: curve 1 – $\zeta_k = 3.5$, $\gamma = 0$, curve 2 – $\zeta_k = 3.9$, $\gamma = 0$, curve 3 – $\zeta_k = 3.5$, $\gamma = 1.87$, curves 4 – $\zeta_k = 3.9$, $\gamma = 1.87$.

For $h_1 = 0.5$, $\zeta_k = 3.5$, $\gamma = 0$, $k^w = 0$ regular motion takes place (Figure 5, curve 1). An accout of thermal shaft extension ($\gamma = 1.87$) removes chaotic behawiour of our system (Figure 5, curves 3 and 4). For $\zeta_k = 3.5$ a subharmonic motion with frequency $\omega_0/2$ is obtained (Figure 5, curve 3), whereas for $\zeta_k = 3.9$ periodic motion is exhibited (Figure 5, curve 4).

Owing to an accout of wear ($k^w = 0.01$) and neglection of shaft heat extension ($\gamma = 0$), contact preasure tends to zero, whereas cylinder wear approaches U_* ($p(\tau) \to 0$, $u^w(\tau) \to 1$). The non-dimensional bush wear is presented in Figure 6, curve 1. In addition, in Figure 6, curves 1 and 2 represent time histories of the dimensionless contact pressure.

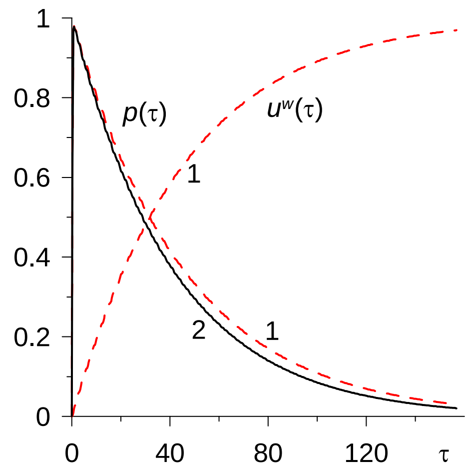

Figure 6 Dimensionless contact pressure $p(\tau)$ and wear $u^w(\tau)$ versus dimensionless time τ : curves 1 – $\zeta_k = 3.5$, $\gamma = 0$, $k^w = 0.01$, $h_1 = 0.5$, curve 2 – $\zeta_k = 3.9$, $\gamma = 0$, $k^w = 0.01$, $h_1 = 0.5$.

A simulteneous account of shaft extension and bush wear yields a finite time of contact between both bodies. For instance for $h_1 = 0.5$, $\zeta_k = 3.9$, $\gamma = 1.87$, $k^w = 0.01$ contact pressure vs. time is exhibited by curve 4 in Figure 7. The dimensionless time contact interval is $\tau_c = 72$. For $\zeta_k = 3.5$ time contact is $\tau_c = 65.8$.

In Figure 8, curves 3 and 4 represent the dependence of non-dimensional wear on the dimensionless time in a genaral case. Curve 3 corresponds to $h_1 = 0.5$, $\zeta_k = 3.5$, $\gamma = 1.87$, $k^w = 0.01$, whereas curve 4 is associated with the following parameters: $h_1 = 0.5$, $\zeta_k = 3.9$, $\gamma = 1.87$, $k^w = 0.01$. Owing to heat shaft extension, the wear of bush is increased on amount of thirty times (see curve 4 and 2 in Figure 8).

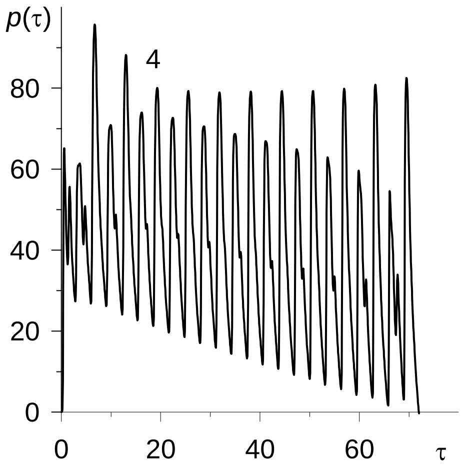

Figure 7 Dimensionless contact pressure $p(\tau)$ versus dimensionless time τ :
curve 4 – $\zeta_k = 3.9$, $\gamma = 1.87$, $k^w = 0.01$, $h_1 = 0.5$.

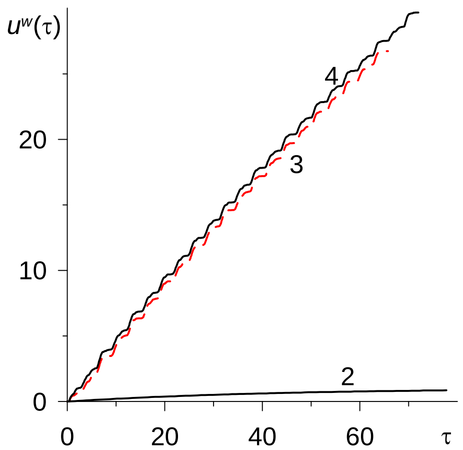

Figure 8 Time history of dimensionless wear $u^w(\tau)$, $h_1 = 0.5$: curve 2 – $\zeta_k = 3.9$, $\gamma = 0$, $k^w = 0.01$, curve 3 – $\zeta_k = 3.5$, $\gamma = 1.87$, $k^w = 0.01$, curve 4 – $\zeta_k = 3.9$, $\gamma = 1.87$, $k^w = 0.01$

5 Conclusion

This paper extends analysis carried out in reference [Awrejcewicz, Pyryev, 2003]. In contrary to the previous results, a novel mechanism of contact between bush and shaft is proposed, a viscous damping is added, and an influence of tribologic factors on both regular and the chaotic dynamics is analysed. The analytical formula of the Mielnikov function of the investigated system has been first formulated, and then numerical analysis of non-linear phenomena is carried out.

Influence of tribological processes on dynamical behaviour of the analysed system in the vicinity of chaos has been illustrated and discussed. An account of bush wear and neglecting of shaft thermal expansion implies that the contact pressure tends to zero, the bush wear approaches the values of the shaft compressing, and bush vibrations are damped.

On the other hand, taking into account shaft thermal extension and neglecting of bush wear results in chaos vanish and occurrence of a regular motion.

In a general case (both shaft thermal extension and bush wear are taken into account), time interval of two bodies contact is bounded. After a lack of contact, the bush stops due to extensive wear process.

References

Andronov, A.A., Witt, A.A. and Khaikin, S.E. (1966). *Theory of Oscillations*. Pergamon Press, Oxford.

Awrejcewicz, J. (1996). Deterministic Oscillations of the Discrete Systems. Warszawa, WNT.

Awrejcewicz, J., Pyryev, Yu. (2003). Influence of Tribological Processes on a Chaotic Motion of a Bush in a Cylinder-Bush System. *Meccanica*. **38**(6), pp. 749–761.

Awrejcewicz, J., Pyryev, Yu. (2002). Thermoelastic contact of a rotating shaft with a rigid bush in conditions of bush wear and stick-slip movements. *International Journal of Engineering Science*, **40**, pp. 1113–1130.

Awrejcewicz, J., Pyryev, Yu. (2004). Contact phenomena in braking and acceleration of bush-shaft system. *Journal of Thermal Stresses*, **27(5)**, pp. 433–454.

Barber, I.R. (1999). Thermoelasticity and contact. *Journal of Thermal Stresses*. **22**, pp. 513-525.

Carslaw, H.S., Jaeger, J.C. (1959). *Conduction of heat in solids*. Oxford, Clarendon Pres.

Martins, J.A.C., Oden, J.T. and Simoes, F.M.F. (1990). A study of static and kinetic friction. *Int. J. Engng. Sci*., **28(1)**, pp. 29-92.

Melnikov, V.K. (1963). On the stability of the center for time periodic perturbations. *Trans. Moscow Math. Soc*. **12(1)**, pp. 3-52.

Neimark, Yu.I. (1978). *Dynamical Systems and Controlled Processes*. Moskva, Nauka.

Nowacki, W. (1986). *Thermoelasticity*. Warszawa, PWN.

Pyryev, Yu., Grylitskiy, D. (1995). Nonlinear transient problem of frictional contact for cylinder with inertia and heat generation. *Reports of NAS of Ukraine*, **9**, pp. 34-47.

Pyryev, Yu., Grylitskiy, D. and Mokryk, R. (1995). Transient temperature field and thermoelastic state of two-layer hollow cylinder under frictional contact. *Applied Mechanics*, **31(1),** pp. 44-50.