DYNAMICS OF A BUSH-SHAFT SYSTEM WITH AN ACCOUNT OF TRIBOLOGIC PROCESSES

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In this paper, both method of analysis and model of contact bush-shaft systems exhibiting heat generation and wear due to friction, are presented [1-3]. From the mathematical point of view, the considered problem is reduced to analysis of ordinary differential equations governing velocities change of the contacting bodies, and to the integral Volterra type equation governing contact pressure behaviour. The latter one is derived with a help of Laplace transformation. The following theoretical background has been used during analysis: perturbation methods, Melnikov techniques [4], Laplace transformations, theory of integral equations and various variants of numerical analysis.

1. Introduction. It should be emphasized that in bibliography devoted to this research, either tribological processes occurring on the contact surfaces are not accounted, or inertial effects are neglected. In other words, both mentioned processes are treated separately. In this work both elements of complex contact behaviour are simultaneously included into consideration, which allows for a proper modelling of the real contact system dynamics. Analytical and numerical analyses are carried out in a wide aspect through investigation of various types of nonlinearities, dampings and excitations applied to the analysed system. A Duffing type elastic nonlinearity, a nonlinear density of the frictional energy stream, a nonlinear friction dependence versus velocity and a nonlinear contact temperature characteristic, as well as nonlinear character of a wear are accounted, among others.

2. The analyzed system and mathematical problem formulation. Consider thermoelastic contact of a solid isotropic circular shaft (cylinder) of radius R_1 with a cylindrical tube-like rigid bush of external radius R_2, which is fitted to the cylinder according to the expression $U_* h_U(t)$. The internal bush radius is: $R_1 - U_*$ ($U_*/R_1 << 1$) (Fig. 1). The bush is linked with the housing by springs and the damper with viscous coefficient c.

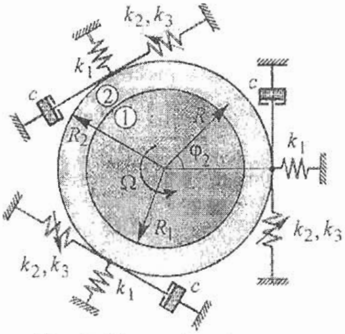

Fig. 1. The analyzed system

We assume, that the bush is a perfect rigid body, and that radial springs have the stiffness coefficient k_1, whereas tangent springs are characterized by non-linear stiffness k_2 and k_3 of Duffing type. In addition, the bush is subjected to a damping force action in tangent direction.

The cylinder rotates with a such angular velocity $\Omega(t) = \Omega_* \omega_1(t)$, that the centrifugal forces may be neglected. We assume that the angular speed of the shaft rotation changes in accordance with $\omega_1 = \omega_* + \zeta_k \sin \omega' t$. We assume that between bush and shaft dry friction appears defined by the function $F_t(V_r)$, where V_r is a relative velocity between the two given bodies $V_r = \Omega R_1 - \dot{\varphi}_2 R_1$. B_2 denotes the mass moment of inertia. We assume also that in accordance with the Amontos assumption the friction force reads: $F_t = f(V_r)N(t)$ ($f(V_r)$ is the kinetic friction coefficient).

The friction force F_t yields heat generated by friction on the contact surface $R = R_1$, and wear U^w of the bush occurs. Observe that the frictional work is transformed to heat energy. Let the shaft temperature, denoted by $T_1(r,t)$, be initially equal to T_{ot}. It is further assumed that the bush transfers heat ideally, and that between both shaft and bush the Newton's heat exchange occurs and that the bush has constant temperature T_{ot}.

Vibrations of the bush being in thermoelastic contact with the rotating shaft are governed by the following non-dimensional equation:

$$\ddot{\varphi}(\tau) + 2h\dot{\varphi}(\tau) - \varphi(\tau) + b\varphi^3(\tau) = \varepsilon F(\omega_1 - \dot{\varphi})p(\tau), \; 0 < \tau < \infty, \tag{1}$$

with the initial condition $\varphi(0) = \varphi^\circ$, $\dot{\varphi}(0) = \omega^\circ$, where the non-dimensional contact pressure is defined through solutions to the equation

$$p(\tau) = h_U(\tau) - u^w(\tau) + 2\gamma\tilde{\omega}\int_0^\tau \dot{G}_p(\tau - \xi)F(\omega_1 - \dot{\varphi})p(\xi)(\omega_1 - \dot{\varphi})d\xi \,. \tag{2}$$

The bush wear $u^w(\tau)$ and the shaft temperature $\theta(r,\tau)$ are defined through the following equations

$$u^w(\tau) = k^w \int_0^\tau |\omega_1 - \dot{\varphi}(\tau)| p(\tau) d\tau \,, \; 0 < \tau < \tau_c \,, \tag{3}$$

$$\theta(r,\tau) = \gamma\tilde{\omega} \int_0^\tau \dot{G}_\theta(r, \tau - \xi) F(\omega_1 - \dot{\varphi}) p(\xi)(\omega_1 - \dot{\varphi}) d\xi \tag{4}$$

where:

$$\{G_p(\tau), G_\theta(1,\tau)\} = \frac{\{0.5, 1\}}{Bi\tilde{\omega}} - \sum_{m=1}^{\infty} \frac{\{2Bi, 2\mu_m^2\}}{\mu_m^2 \tilde{\omega}(Bi^2 + \mu_m^2)} e^{-\mu_m^2 \tilde{\omega}\tau} \,, \tag{5}$$

μ_m $(m = 1,2,3,...)$ are the roots of characteristic equation $BiJ_0(\mu) - \mu J_1(\mu) = 0$.

In equations (1)-(5) the following non-dimensional quantities are introduced:

$$\tau = \frac{t}{t_*} \,,\; r = \frac{R}{R_1} \,,\; p = \frac{P}{P_*} \,,\; \theta = \frac{T_1 - T_{0t}}{T_*} \,,\; \varphi(\tau) = \varphi_2(t_*\tau) \,,\; u^w = \frac{U^w}{U_*} \,,\; \varepsilon = \frac{P_* t_*^2 2\pi R_1^2}{B_2} \,,$$

$$h = \frac{cR_2^2}{2B_2 t_*} \,,\; k^w = \frac{P_* K^w R_1}{U_*} \,,\; \gamma = \frac{(1-\eta)E_1\alpha_1 R_1^2}{\lambda_1(1-2\nu)t_*} \,,\; Bi = \frac{\alpha_T R_1}{\lambda_1} \,,\; \tau_c = \frac{t_c}{t_*} \,,\; \tilde{\omega} = \frac{t_* a_1}{R_1^2} \,,$$

$$\omega_0 = \omega' t_* \,,\; h_U(\tau) = h_U(t_*\tau) \,,\; F(\omega_1 - \dot{\varphi}) = f(V_*(\omega_1 - \dot{\varphi})) \,,$$

$$b = \left(k_3 R_2^4 - (2/3)k_2 R_2^2 + (l_1 + R_2)R_2\left((l_0/l_1)\left(1 + 3(R_2/l_1) + 3(R_2/l_1)^2\right) - 1\right)k_1/6\right)\left(t_*^2/B_2\right),$$

where

$$V_* = \frac{R_1}{t_*} \,,\; t_* = \sqrt{\frac{B_2}{k_* R_2^2}} \,,\; k_* = k_1\left(\frac{l_0}{l_1} - 1\right)\left(1 + \frac{l_1}{R_2}\right) - k_2 \,,\; T_* = \frac{U_*}{\alpha_1(1+\nu_1)R_1} \,,\; P_* = \frac{\alpha_1 E_1 T_*}{(1-2\nu_1)} \,,$$

and l_0 is the un-stretched spring length, l_1 is the length of the compressed spring for $\varphi_2 = 0$, ($k_* > 0$), E_1 is the elasticity modulus, ν_1 is the Poisson coefficient, α_1 is the coefficient of thermal expansion of the shaft, α_T is heat transfer coefficient, a_1 is thermal diffusivity, λ_1 is the heat transfer coefficient, $\varphi_2(t)$ is the angle of bush rotation, K^w is the wear coefficient, η is denotes the part of heat energy associated with wear $\eta \in [0,1]$, t_c is time of contact ($0 < t < t_c$, $P(t) > 0$).

Notice that the stated problem is modeled by the both nonlinear differential equation (1) and integral equation (2) governing rotational velocity $\dot{\varphi}(\tau)$ and contact pressure $p(\tau)$. Temperature and wear is defined through equations (4) and (3).

3. Melnikov's method. A particular case of our problem is further studied ($\gamma = 0$,

$k^w = 0$). The dependence of kinematic friction on relative velocity is approximated through the function $F(y) = F_0 \operatorname{sgn}(y) - \alpha y + \beta y^3$. In this case The Melnikov function reads (see [2,4])

$$M(\tau_0) = -\int_{-\infty}^{+\infty} y_0(t)\left[F_0 \operatorname{sgn}(\omega_r) - \alpha\omega_r + \beta\omega_r^3 - h_1 y_0(t)\right]dt = I(\tau_0) + J(\tau_0), \tag{6}$$

where $\omega_r(t) = \omega_* + \zeta_k \sin(\omega_0(t + \tau_0)) - y_0(t)$,

$$J(\tau_0) = 2C + 2\zeta_k\sqrt{A^2 + B^2}\sin(\omega_0\tau_0 + \varphi_0) +$$
$$6\beta\zeta_k^2\left(I_{220}\cos^2\omega_0\tau_0 + I_{202}\sin^2\omega_0\tau_0 - 2\omega_* I_{111}\sin\omega_0\tau_0\cos\omega_0\tau_0\right) +$$
$$2\beta\zeta_k^3\left(-I_{130}\cos^3\omega_0\tau_0 - 3I_{112}\sin^2\omega_0\tau_0\cos\omega_0\tau_0\right),$$

$$A = (\alpha - 3\beta\omega_*^2)I_{110} - 3\beta I_{310},\ B = 6\beta\omega_* I_{201},\ C = \beta I_{400} - (\alpha - h_1 - 3\beta\omega_*^2)I_{200},$$

$$\varphi_0 = \arctan(A/B),\ h_1 = \frac{2h}{\varepsilon},\ I_{200} = \frac{2}{3b},\ I_{400} = \frac{8}{35b^2},\ I_{201} = \frac{\pi\omega_0(2 - \omega_0^2)}{6b\sinh(\pi\omega_0/2)},$$

$$I_{110} = -\frac{\pi\omega_0}{\sqrt{2b}\cosh(\pi\omega_0/2)},\ I_{112} = \frac{\pi\omega_0\cosh(\pi\omega_0/2)}{\sqrt{2b}(1 - 2\cosh(\pi\omega_0))},\ I_{111} = -\frac{\pi\omega_0}{\sqrt{2b}\cosh(\pi\omega_0)},$$

$$I_{220} = \frac{\pi\omega_0(2\omega_0^2 - 1) + \sinh(\pi\omega_0)}{3b\sinh(\pi\omega_0)},\ I_{202} = \frac{\pi\omega_0(1 - 2\omega_0^2) + \sinh(\pi\omega_0)}{3b\sinh(\pi\omega_0)},$$

$$I_{310} = \frac{\omega_0(11 + 10\omega_0^2 - \omega_0^4)}{120b\sqrt{2b}}\left\{\psi\left(\frac{1 - i\omega_0}{4}\right) - \psi\left(\frac{3 - i\omega_0}{4}\right) + \psi\left(\frac{1 + i\omega_0}{4}\right) - \psi\left(\frac{3 + i\omega_0}{4}\right)\right\},$$

$$I_{130} = -\frac{3\pi\omega_0}{8\sqrt{2b}}\left\{\cot\left(\frac{\pi(1 - i\omega_0)}{4}\right) + \cot\left(\frac{3\pi(1 - i\omega_0)}{4}\right) -\right.$$
$$\left.\cot\left(\frac{\pi(3 - i\omega_0)}{4}\right) - \cot\left(\frac{\pi(1 - 3i\omega_0)}{4}\right)\right\},\quad \psi(z) = \frac{\Gamma'(z)}{\Gamma(z)},$$

$$y_0(\tau) = -\sqrt{\frac{2}{b}}\frac{\sinh(\tau)}{\cosh^2(\tau)}. \tag{7}$$

In equation (6) the term $I(\tau_0)$ is defined by the formula

$$I(\tau_0) = -F_0\int_{-\infty}^{+\infty} y_0(t)\operatorname{sgn}(\omega_r)dt = 2F_0\sqrt{\frac{2}{b}}\sum_m \frac{\operatorname{sgn}(\omega_r'(t_m))}{\cosh t_m}, \tag{8}$$

where t_m are the roots of the equation

$$\omega_r(t_m) = \omega_* + \zeta_k\sin(\omega_0(t_m + \tau_0)) - y_0(t_m) = 0, \tag{9}$$

and $\omega_r'(t) = \zeta_k\omega_0\cos(\omega_0(t + \tau_0)) - x_0(t) + bx_0^3(t)$.

If the Melnikov function (6) changes sign, then chaos may occur.

5. Conclusions. This paper extends analysis carried out in reference [2]. In contrary to the previous results, a novel mechanism of contact between bush and shaft is proposed, a viscous damping is added, and an influence of tribologic factors as well as chaotic dynamics is analyzed. The analytical formula of the Mielnikov function of the investigated system has been first formulated, and then numerical analysis of non-linear phenomena is carried out.

It has been shown that (owing to wear) chaos vanishes, since there is a lack of contact between both bodies. Owing to heat generation through friction, either chaos vanishes or thermal instability appears.

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