HYSTERESIS SIMULATION AND INVESTIGATION OF THE CONTROL PARAMETER PLANES

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In the present work a hysteresis is simulated by means of additional state variables. The analytical models of different types of hysteresis loops allow major and minor loops reproducing and provide a high degree of correspondence with experimental data.

Using an effective algorithm based on analysis of the wandering trajectories [5-9], evolution of chaotic behaviour regions of oscillators with hysteresis is presented in various parametric planes. Substantial influence of a hysteretic dissipation value on the form and location of these regions, and also restraining and generating effects of the hysteretic dissipation on a chaos occurrence are ascertained.

1. Introduction. Hysteresis is caused by very different processes and characterised above all by a manifestation of energy dissipation and memory in a system. The problem of hysteresis simulation occurs in many field of science for mechanical, engineering, physical and biological systems. There are a lot of different phenomenological approaches for hysteresis modelling [1-2]. However, recently a large number of publications are devoted to hysteresis simulation because, it is known, that hysteretic systems is complicated for investigation and various difficulties occur during existent models applying. The question of multi-purpose and generally valid models describing the wide spectrum of hysteretic phenomena is still open. In the present work hysteresis is simulated by means of additional state variables. In particular the behaviour of magnetorheological/electrorheological (MR/ER) fluids (which are known as smart materials and commercially available now) in a damper/absorber [3-4] are simulated. The developed models are effective and contain principally less parameters than, for example, Bouc-Wen or Spencer models.

The models describing systems with hysteresis are discontinuous and contain high nonlinearities with memory dependent properties. The *output* is delayed with respect to the *input* and for every *input* there may be more than one equilibrium states. The chaotic behaviour occurring of the dynamic hysteretic system governed by a coupled differential set is investigated in various parametric planes using methodology described in [5-9]. This methodology already had been successfully applied in particular to predict stick-slip chaos in a weakly forced oscillator with friction [6], in 2-DOF discontinuous systems with friction ([5],[8]) and chaos in other smooth and nonsmooth systems [9].

2. Hysteresis simulation. The energy dissipated in the cycle is simulated with the aid of internal variables

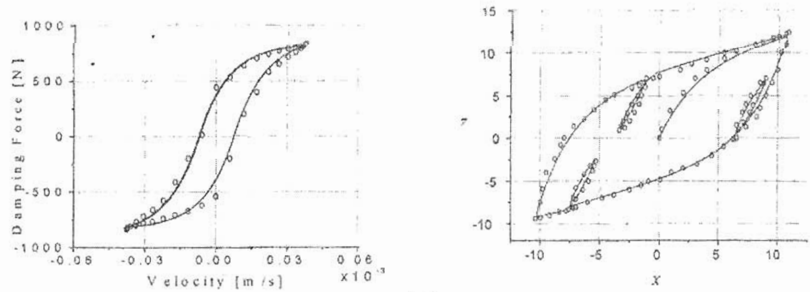
$$z = p(x, y_1, y_2, \ldots, y_N), \quad \dot{y}_i = q(x, \dot{x}, y_i), \qquad i=1, 2, \ldots, N. \tag{1}$$

Here x is an *input* (input signal) and z is an *output* (response) of the hysteretic system, p and q are nonlinear functions of its arguments, y_i (i=1, 2,…, N) are internal variables. Functions p and q are chosen depending on a loop form. The parameters of these functions are determined via a procedure minimizing the criterion function

$$\Phi(c_1,\ldots c_j, \alpha_1,\ldots\alpha_k,\ldots,\beta_1,\ldots,\beta_l) = \sum_i \left(p\left(x(c_1,\ldots c_j, t_i), y_1(\alpha_1,\ldots\alpha_k, t_i),\ldots, y_N(\beta_1,\ldots,\beta_l, t_i)\right) - z_i\right)^2 \tag{2}$$

which characterizes an error between the experimental curve and the calculated one. Here z_i are responses of a hysteretic system, which are known from an experiment and the values $p(x(c_1,\ldots c_j,t_i), y_1(\alpha_1,\ldots\alpha_k,t_i),\ldots,y_N(\beta_1,\ldots,\beta_l,t_i))$ are obtained as result of integration of the system which is described by means of model (1). To minimize the criterion function (2) the method of gradient descent is used. Step-by-step descent to the minimum of the criterion function realises in the opposite direction to the criterion function gradient $\mathbf{grad}\Phi = \left\{\frac{\partial\Phi}{\partial c_1},\ldots,\frac{\partial\Phi}{\partial c_j},\frac{\partial\Phi}{\partial\alpha_1},\ldots,\frac{\partial\Phi}{\partial\alpha_k},\frac{\partial\Phi}{\partial\beta_1},\ldots,\frac{\partial\Phi}{\partial\beta_l}\right\}$.

Applications to different types of hysteresis loops confirmed that models with internal variables are well appropriate to simulate hysteresis. Comparison of simulated loops with the experimental data is presented in fig. 1.



(a) (b)

Fig. 1. Experimental (○ ○ ○) and simulated (——) hysteresis loops (a) for the MR damper filled with MRF-132LD; (b) for the steel rope (force (N) vs. displacement (mm))

Final values of the parameters used in the models (3) and (4) for identification of the experimental data are in Tables 1 and 2 correspondingly.

$$z(t) = y_1(t), \quad \dot{y}_1 = (c_1 - (c_2 + c_3\,\mathrm{sgn}(\dot{x})\mathrm{sgn}(y_1))|y_1|)\dot{x} \qquad (3)$$

$$z(t) = c_4 x(t) + c_5 + y_1(t), \quad \dot{y}_1 = (c_1 - (c_2 + c_3\,\mathrm{sgn}(\dot{x})\mathrm{sgn}(y_1 - z_c))|y_1 - z_c|)\dot{x} \qquad (4)$$

Table 1

c_1	c_2	c_3
70000	80.7208	3.002

Table 2

c_1	c_2	c_3	c_4	c_5
2.22254	0.001023	0.338787	0.387749	1.45286

The model is able to simulate minor loops and presents fast numerical convergence.

3. Investigation of the control parameter planes. For the sake of tracing chaotic and regular dynamics, it is supposed that with the increase of time all trajectories remain in the closed bounded domain of a phase space. To analyse trajectories of the sets (5) and (6), the characteristic vibration amplitudes A_i of components of the motion are introduced $A_i = 1/2\left|\max_{t_1 \le t \le T} x_i(t) - \min_{t_1 \le t \le T} x_i(t)\right|$. Here and below index number i run over three values corresponding to three generalized coordinates x, y, z. Two neighboring initial points $\mathbf{x}^{(0)} = \mathbf{x}(t_0)$ and $\tilde{\mathbf{x}}^{(0)} = \tilde{\mathbf{x}}(t_0)$ ($\mathbf{x} = (x, y, z)^T$ or $\mathbf{x} = (x_1, x_2, x_3)^T$) are chosen in the 3-dimensional parallelepiped $P_{\delta_x,\delta_y,\delta_z}(\mathbf{x}^{(0)})$ such that $\left|x_i^{(0)} - \tilde{x}_i^{(0)}\right| < \delta_i$, where $\delta_i>0$ is small in comparison with A_i. In the case of regular motion it is expected that the $\varepsilon_i>0$ used in inequality $|x_i(t) - \tilde{x}_i(t)| < \varepsilon_i$ is also small in comparison with A_i. The wandering orbits attempt to fill up some bounded domain of the phase space. At instant t_0 the neighboring trajectories diverge exponentially. For some instant t_1 the absolute values of differences $|x_i(t) - \tilde{x}_i(t)|$ can take any values in closed interval $[0, 2A_i]$. An auxiliary parameter α is introduced, $0<\alpha<1$. αA_i is referred to as divergence measures of observable trajectories in the directions of generalized coordinates and with the aid of parameter α one have been chosen, which is *inadmissible* for the case of 'regularity' of the motion. The domains, where a chaotic behavior of considered systems is possible, can be found using following condition:

$$\exists t^* \in [t_1, T] : |x(t^*) - \tilde{x}(t^*)| > \alpha A_x .$$

If this inequality is satisfied in some nodal point of the sampled control parameter space, then such motion is relative to chaotic one (including transient and alternating chaos). The manifold of all such nodal points parameter space set up domains of chaotic behavior of the considered systems.

We succeeded sufficiently accurate to trace regular/unregular responses of the Masing (5) and Bouc-Wen (6) hysteretic models in various control parameter planes (Fig. 2). The figures characterizing the obtained domains and demonstrating various character of motion as chaos, periodic responses, pinched hysteresis and hysteresis loss are obtained. One can observe the *effect of restraining* of the chaotic regions with the increasing of the hysteretic dissipation value in the (Ω, F) plane for Masing oscillator. The chaotic responses of the Bouc-Wen oscillator are not observed right up till δ=0.2 when the influence of the nonlinear terms becomes critical. It demonstrates *generating effect* of the hysteretic dissipation on chaos occurring in hysteretic system which appears after some critical value δ_{cr}.

$$\ddot{x} = -2\mu\dot{x} - (1-\nu)g(x) - \nu z + F\cos\Omega t, \quad \dot{z} = g'\left(\frac{z - z_i}{2}\right)\dot{x}, \quad g(x) = \frac{(1-\delta)x}{\left(1+|x|^n\right)^{1/n}} + \delta x \qquad (5)$$

$$\ddot{x} = -2\mu\dot{x} - \delta x - (1-\delta)z + F\cos\Omega t, \quad \dot{z} = \left[k_z - \left(\gamma + \beta\,\mathrm{sgn}(\dot{x})\,\mathrm{sgn}(z)\right)|z|^n\right]\dot{x}, \qquad (6)$$

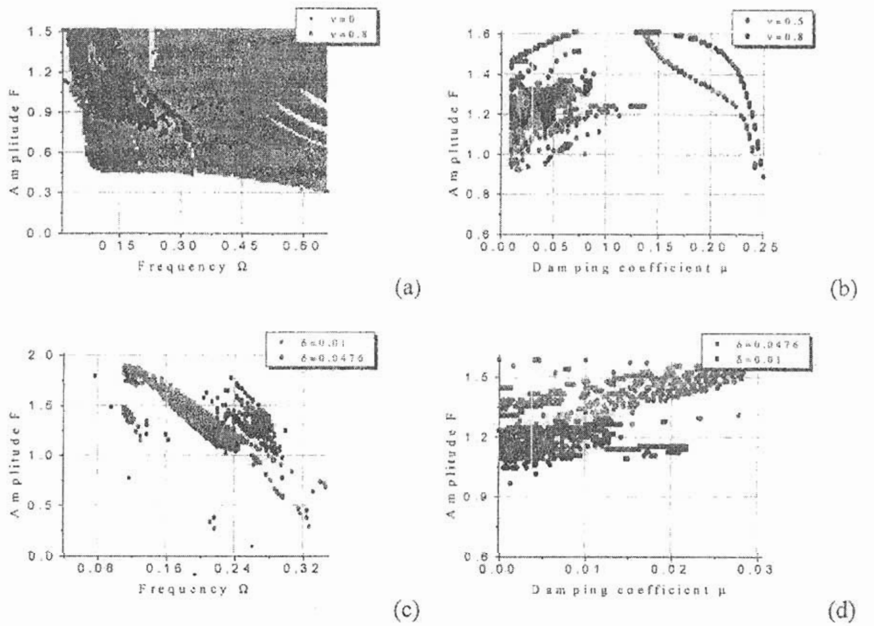


Fig. 2. Evolution of the chaotic regions for the Masing (a), (b) and Bouc-Wen (c), (d) hysteresis models in the (Ω, F)/(μ, F) planes with increasing of the hysteretic dissipation

4. Conclusions. The effective models for hysteresis simulation including minor loops formation are developed; highly non-linear Masing and Bouc-Wen hysteretic models with discontinuous right-hand sides are investigated using an effective approach based on analysis of the wandering trajectories; substantial influence of a hysteretic dissipation value on possibility of chaotic behavior occurring in the systems with hysteresis is shown; a *restraining* and *generating effects* of the hysteretic dissipation on a chaotic behavior occurring are demonstrated.

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