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Continuous modeling of discrete chain using two-points Padé approximation

The worm-like motion systems can be modelled in form of straight chains of interconnected mass points [1]. Dynamics of mass chain can be considered from a point of view of continuous approach, but microstructural effect should be taken into account. Two main continuum approaches can be distinguished. First, each microstructural component can be modelled separately by means of a classical continuum. Then, the microstructure is considered as an inhomogeneous classical continuum. Second, a homogeneous continuum can be used and the influence of the microstructure can be taken into account through an averaging procedure. Here three strategies exist: phenomenological, statistical and homogenization approach.

Note that independently of the strategy employed, a constitutive relation of the homogenized material becomes non-local. It means that integral relations between stresses and strains should be derived, where stress in a point is not only related to strain in the same point, but it also depends instantly of strains magnitude in the neighbouring points.

The applicability of continuum models in dynamics is restricted to limited ranges of frequencies. For high frequencies, these models either become unstable or presume an infinite speed of the energy propagation. From a physical point of view, the inapplicability of homogeneous continuum models at high frequencies is fully justified, since inhomogeneity and anisotropy of microstructural materials cannot be ignored, especially when wavelength size approaches a span of the microstructural irregularity.

In this paper new approach (in order to obtain explicit relations between micrtostructural properties and macrostructural parameters) is used. It is firstly based on asymptotic construction of two limiting cases, and then two-point Padé approximation is applied.

We describe our idea using simple example, i.e. a linear 1D mass chain goverened by the following equation (1)

$$m\sigma_{jtt}(t) = c(\sigma_{j+1} - 2\sigma_j + \sigma_{j-1}), \quad \sigma_0(t) = -f(t), \quad \sigma_n(t) = 0, \quad j = 0,1,\ldots,n-1. \qquad (1)$$

Usually, for large values of n the following continuous approximation to the above discrete problem is applied:

$$m\sigma_{tt}(x,t) = ch^2\sigma_{xx}(x,t),\ \sigma(0,t) = 0,\quad \sigma(l,t) = 0,\ \ l = (n+1)h. \tag{2}$$

However, this continuous model does not take into account a splash effect and other features of discrete media.

The intermediate continuous model accounting more than one term of Taylor series of difference operator $\sin^2(-\frac{iH}{2}\frac{\partial}{\partial x})$ has the following form

$$m\frac{\partial^2\sigma}{\partial y^2} = ch^2\left(\frac{\partial^2\sigma}{\partial x^2} + \frac{h^4}{12}\frac{\partial^4\sigma}{\partial x^4} + \frac{h^6}{360}\frac{\partial^6\sigma}{\partial x^6}\right),\ \sigma = \sigma_{xx} = \sigma_{xxxx} = 0 \quad \text{for} \quad x = 0, l,$$

and can not overcome above mentioned problems.

A construction of intermediate continuous models is based on the development of a difference operator into the Taylor series. However, more effective results can be obtained using PA. The quasi-continuum model is constructed using one point PA owing to the following equation

$$m\left(1 - \frac{h^2}{12}\frac{\partial^2}{\partial x^2}\right)\sigma_{tt} - ch^2\sigma_{xx} = 0,\ \sigma(0,t) = 0,\quad \sigma(l,t) = 0.$$

On the other hand, more rigorous mathematical model can be used applying two-point PA for the difference operator [2]. Hence, a pseudo-continuous approximation is governed by the equation

$$m\left(1 - \alpha^2h^2\frac{\partial^2}{\partial x^2}\right)\sigma_{tt} - ch^2\sigma_{xx} = 0,\ \sigma(0,t) = 0,\quad \sigma(l,t) = 0,\ \alpha^2 = 0.25 - \pi^{-2}.$$

This equation governs the splash effect and other phenomena caused by discreteness effects in the best way in comparison with other approximations.

We also describe generalization of our approach to 2D and 3D cases, for a chain with unequal elements, and for a nonlinear case.

References:

[1] K.Zimmermann, I.Zeidis, Worm-like locomotion as a problem of nonlinear dynamics, Proc. 7th Conf. Dynamical Systems – Theory and Appl., Łódź , Poland, vol. 1 (2003), 293-300.

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