AWREJCEWICZ Jan, PYRYEV Yuriy
Technical University of Lodz,
Department of Automatics and Biomechanics (K-16)
90-924 Łódź, Stefanowskiego 1/15, Poland
awrejcew@p.lodz.pl, jupyrjew@p.lodz.pl

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FRICTIONAL AUTO-VIBRATIONS IN A CONTACT THERMOELASTIC PROBLEM IN WEAR CONDITIONS

Friction and wear are complex processes which influence each other making up a sole diverse process of a friction unit work [1]. Consider contact and wear of one-dimensional model of the thermo-elastic contact of a body with a surrounding medium. Assume, that this body is represented by a rectangular block $-L < X < L$, $-a/2 < Y < a/2$, $-b/2 < Z < b/2$ (Fig.1). The block has the mass M subject to the force $F = F_* h_F(t)$ and moves vertically along rigid walls in direction z_1 of the rectangular co-ordinates $0x_1y_1z_1$. The distance between rigid walls is always equal to initial block thickness $2L$. Also assume that according to Coulomb's law the friction force is proportional to normal part of reaction and kinematical coefficient of friction f. The coefficient of friction is assumed to be a nonlinear function of the sliding velocity [2]. It is assumed that the heat conduction between the block and the rigid walls obeys Newton's law. The ends of the block $X = \pm L$ are maintained at temperature $T_0 h_T(t)$ ($h_T(t) \to 1$, $t \to \infty$) and the sides $Y = \pm a/2$, $Z = \pm b/2$ are thermally insulated. It causes a parallelepiped heat extension in the direction of $0x_1$, and the body starts to contact with walls. In the result of this process a frictional contact on the parallelepiped sides $X = \pm L$ occurs.

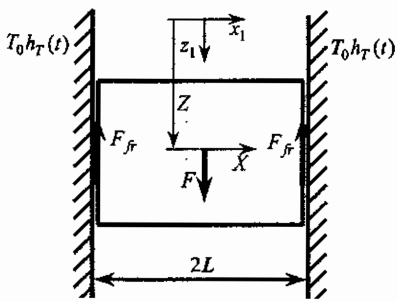

Figure 1 A model of the problem

In the considered case, the studied problem is governed by dynamics of the block mass centre

$$m\ddot{Z}(t) = F_* h_F(t) - 2f(\dot{Z})P(t), \tag{1}$$

and equations of the stress theory for an isotropic body

$$\frac{\partial}{\partial X}\left[\frac{\partial}{\partial X}U(X,t)-\alpha\frac{1+\nu}{1-\nu}T(X,t)\right]=0,\quad \frac{\partial^2}{\partial t^2}T(X,t)=\frac{1}{k}\frac{\partial}{\partial t}T(X,t), \tag{2}$$

with the attached mechanical

$$U(-L,t)=-U^w(t),\ U(L,t)=U^w(t), \tag{3}$$

heat

$$\mp K\frac{\partial}{\partial X}T(\mp L,t)+\alpha_T\left[T(\mp L)-T_0h_T(t)\right]=f(\dot{Z})\dot{Z}(t)P(t), \tag{4}$$

and initial

$$T(X,0)=0,\ -L<X<L,\ Z(0)=0,\ \dot{Z}(0)=0 \tag{5}$$

conditions. Normal stresses occurred in block are defined through

$$\sigma_{XX}(X,t)=\frac{E}{1-2\nu}\left[\frac{1-\nu}{1+\nu}\frac{\partial}{\partial X}U(X,t)+\alpha T(X,t)\right], \tag{6}$$

where K, k are the thermal conductivity and thermal diffusivity of the block material respectively; E, ν, α are Young's modulus, Poisson's ratio and the coefficient of thermal expansion respectively for the material of the block; $1/\alpha_T$ is the contact resistance; $P(t)=-\sigma_{XX}(\pm L,t)$ is pressure; $\dot{Z}(t)$ is sliding velocity; $m=M/ab$.

To determine the amount of wear at the right and left ends during sliding the Archard's law of wear is used:

$$\dot{U}^w(t)=K_w\dot{Z}(t)P(t), \tag{7}$$

where K_w is known as the wear coefficient.

Let us introduce the following similarity coefficients

$$t_*=L^2/k\ [\text{s}],\ \nu_*=k/L\ [\text{m/s}],\ P_*=T_0E\alpha/(1-2\nu)\ [\text{N/m}^2], \tag{8}$$

and the following non-dimensional parameters

$$x=\frac{X}{L},\ \tau=\frac{t}{t_*},\ z=\frac{Z}{L},\ u^w=\frac{2(1-\nu)U^w}{T_*\alpha L(1+\nu)},\ p=\frac{P}{P_*},\ \theta=\frac{T}{T_0},\ \varepsilon_1=\frac{2P_*t_*^2}{mL}, \tag{9}$$

$$\gamma=\frac{E\alpha k}{(1-2\nu)K},\ k^w=\frac{K^wE(1-\nu)}{(1+\nu)(1-2\nu)},\ Bi=\frac{L\alpha_T}{K},\ m_0=\frac{F_*}{2P_*},\ F(\dot{z})=f(\nu_*\dot{z}). \tag{10}$$

From the mathematical viewpoint our problem is reduced to solution of the following dimensionless form of the non-linear system of integral and differential equations:

$$p(\tau)=Bi\int_0^\tau \dot{h}_T(\xi)G_p(\tau-\xi)d\xi+\gamma\int_0^\tau F(\dot{z})\dot{z}(\xi)p(\xi)\dot{G}_p(\tau-\xi)d\xi-u^w(\tau), \tag{11}$$

$$\dot{z}(\tau)=\varepsilon_1\left[m_0\int_0^\tau h_F(\xi)d\xi-\int_0^\tau F(\dot{z})p(\xi)d\xi\right],\ \ \dot{u}^w(\tau)=k^w\dot{z}(\tau)p(\tau), \qquad (12)$$

which yields the non-dimensional pressure $p(\tau)$ and velocity $\dot{z}(\tau)$. The temperature is defined through the following formula

$$\theta(x,\tau)=Bi\int_0^\tau \dot{h}_T(\xi)G_\theta(x,\tau-\xi)d\xi+\gamma\int_0^\tau f(\dot{z})\dot{z}(\xi)p(\xi)\dot{G}_\theta(x,\tau-\xi)d\xi\,, \qquad (13)$$

$$\left\{G_p(\tau),\ G_\theta(1,\tau)\right\}=\frac{1}{Bi}-\sum_{m=1}^{\infty}\frac{\left\{2Bi,\ 2\mu_m^2\right\}}{\mu_m^2[Bi(Bi+1)+\mu_m^2]}e^{-\mu_m^2\tau}\,, \qquad (14)$$

and μ_m are the roots of the following characteristic equation $\operatorname{tg}\mu_m=Bi/\mu_m$, $m=1,2,...$

The numerical analysis of the problem is performed using the Runge-Kutta method.

In this case the steel made parallelepiped plate with $L=0.01\,\mathrm{m}$, $T_0=5^\circ C$, $z^\circ=\dot{z}^\circ=0$ and with non-constant friction coefficient is studied. In order to confirm the given conclusions, numerical analysis is carried out for $Bi=5$ (critical value $\varepsilon_1\approx\tilde{\varepsilon}=586.5$), and the computational results are carried out for a few values of the parameter $\varepsilon_1=400;\ \ 800$.

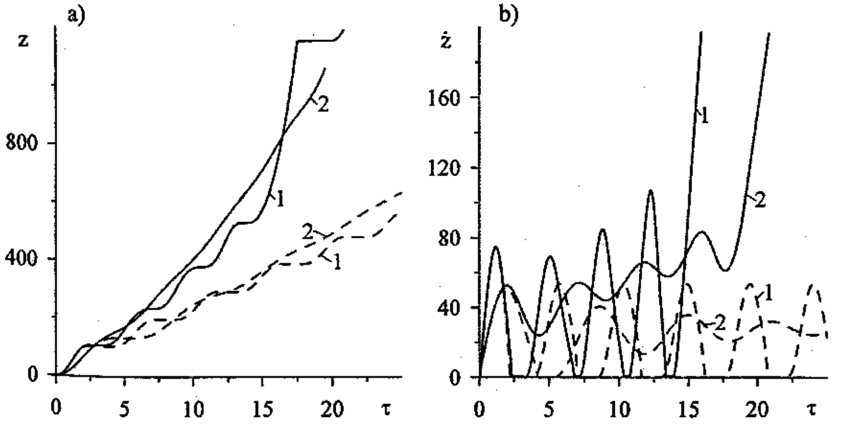

Figure 2 Time history of non-dimensional displacement (a) and body's velocity (b) for various values of ε_1, solid curves $k^w=0.001$, dashed curves $k^w=0$ (curves 1: $\varepsilon_1=800$; curves 2: $\varepsilon_1=400$).

Also the dependence of non-dimensional body velocity $\dot{z}(\tau)$ (friction force, contact pressure $p(\tau)$, contact temperature) on the non-dimensional time τ is analysed.

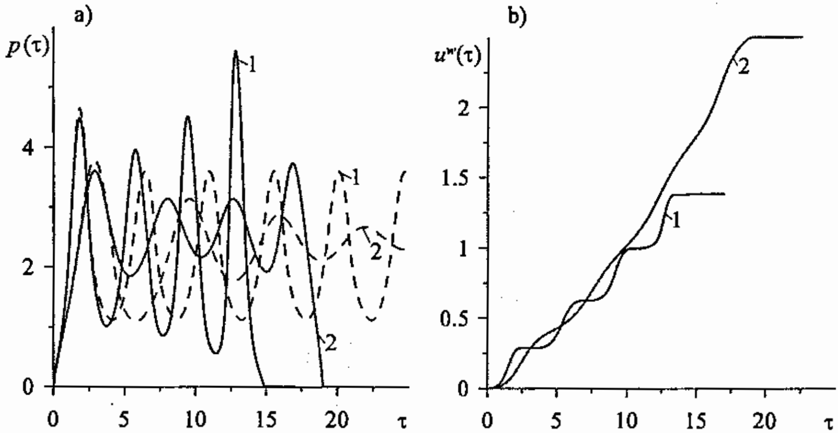

Figure 3 Time history of non-dimensional contact pressure (a) and wear (b) for various values of ε_1, solid curves $k^w = 0.001$, dashed curves $k^w = 0$ (curves 1: $\varepsilon_1 = 800$; curves 2: $\varepsilon_1 = 400$).

For $\varepsilon_1 = 800$, $k^w = 0$ the stationary solution is unstable, and the stick-slip periodic orbit appears with the expected period $T = 4.18$ (frictional auto-vibrations [3]). For $k^w = 0.001$ wear occurs and the sliding contact is lost.

References

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Acknowledgements

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