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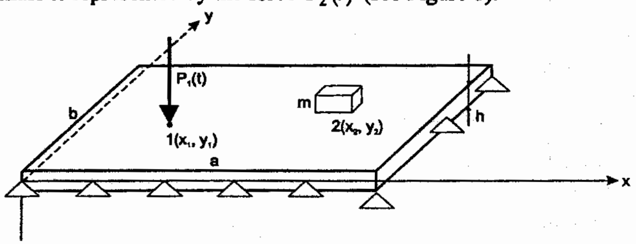
# SUPPRESSION OF VIBRATIONS IN SOME CHOSEN POINTS OF A PLATE USING A SUPPLEMENTED MASS

In this work a passive control of vibration level in some chosen points of a flexible continuous systems is studied using example of plate vibrations [1, 2]. In what follows a structural modification is applied in the form of the additive lumped system represented by a mass. Using the mentioned technique one may monitor and control modes of plates vibrations and modes positions, and hence one may achieve a total vibration suppression in some chosen points. We consider a rectangular and isotropic plate $(a\times b)$ with its thickness $h$, simply supported and subject to harmonic load $P_1(t)$. A dynamical action of the added mass is represented by the force $P_2(t)$ (see Figure 1).




The governing equation reads

$$D\Delta\Delta u(x,y,t)=g(x,y,t)-\rho\frac{\partial^2 u(x,y,t)}{\partial t^2}, \tag{1}$$

where:

$$D=\frac{Eh^3}{12\left(1-\nu^2\right)},$$

$$g(x,y,t)=\frac{P_1(t)}{\varepsilon^2}-\frac{P_2(t)}{\varepsilon^2}.$$

Transversal plate vibrations are sought in a form of the following product

$$u(x,y,t)=v(x,y)f(t). \tag{2}$$

In our case we have

$$v(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{3}$$

where $m$, $n$ are half-waved numbers in directions $x$ and $y$, respectively, and

$f(t) = e^{i\omega t}$.

Plate deflection arrow reads

$$A_{mn} = \frac{16\left(P_1(t) f_{mn1} - P_2(t) f_{mn2}\right)}{D\left(\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 f(t) + \frac{\rho}{D}\frac{d^2 f(t)}{dt^2}\right)ab}, \tag{5}$$

where:

$$f_{mn1} = \sin\left(\frac{m\pi x_1}{a}\right)\sin\left(\frac{n\pi y_1}{b}\right), \tag{6}$$

$$f_{mn2} = \sin\left(\frac{m\pi x_2}{a}\right)\sin\left(\frac{n\pi y_2}{b}\right).$$

In the above $x_1, y_1, x_2, y_2$ are coordinates of action of $P_1(t)$ and $m$ (force $P_2(t)$), respectively.

Elimination of vibration in the chosen plate point $A(x_A, y_A)$ takes place if $v(x_A, y_A) = 0$, i.e. when

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} f_{mnA} = 0. \tag{7}$$

The last condition is realized, when

$$\frac{P_1(t)}{P_2(t)} = \text{const}, \tag{8}$$

where: $P_2(t) = -m\omega^2 f(t)$.

Finally, the mass value required for vibration elimination in the chosen point $A(x_A, y_A)$ is found from the formula

$$m = \frac{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_{mn1} f_{mnA}}{\beta_{mn}}}{16\omega^2\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_{mn1} f_{mnA}}{\beta_{mn}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_{mn2}^2}{\beta_{mn}} - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_{mn2} f_{mnA}}{\beta_{mn}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{f_{mn1} f_{mn2}}{\beta_{mn}}\right)} \tag{9}$$

**References**


1. Wodzicki W., Awrejcewicz J., Passive control of the vibration modes of the beam systems, *Proceedings of the 70 years birthday and 45 years of the scientific activity of Prof. Dr hab. Józef Giergiel and the 5th School on Modal Analysis*, Kraków, December 12-14, 2000 (Ed. T. Uhl), 5-14, in Polish.
2. Wodzicki W., Awrejcewicz J., The method of vibration control in the points of continuous flexible systems, *Journal of Systems Analysis Modelling Simulation*, 43(3), 2003, 361-369.