



The method of friction modelling that includes the relation between the friction force and the rubbing bodies' relative velocity, and between the friction force and the changes of the normal force (the pressure force of the frictional pairs) may also be applied to the study of friction-induced dynamic phenomena in the braking system presented in Fig. 3.

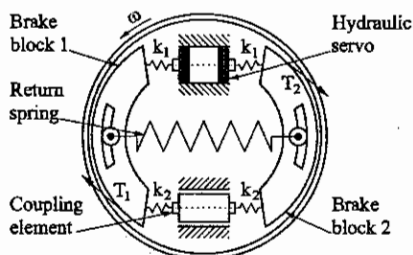


Figure 3. Simplified girling duo-servo brake.

The brake mechanism diagrammatically shown in Fig. 3 is assembled in a popular type of drum brake, a.k.a. "a duo-servo" (see Fig. 4). When the hydraulic servo initiates braking, the brake blocks 1 and 2 arc drawn aside and pressed against the inner surface of the drum. As a result, friction forces  $T_1$  and  $T_2$  are exerted between the blocks' linings and the drum and the wheel stops.

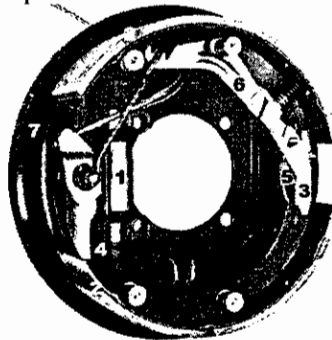


Figure 4. Girling duo-servo brake: 1-hydraulic cylinder, 2-brake shoes with friction linings, 3-couple element, 4-auxiliary long spring, 5-auxiliary short spring, 6-hand-brake mechanism, 7-body.

An analysis of the brake shown in Fig. 4 reveals certain type of coupling. Brake block 1 takes over a larger part of the friction force at the initial stage of braking, whereas brake block 2 impedes with a weaker force. However, the coupling element (the angle bar in the system in Fig. 1) combines the circumferential motion of brake block 1 with

the motion of brake block 2 and the pressure force of the latter on the drum's inner surface increases.

The ratio of the braking forces exerted by brake blocks 1 and 2 is about 2:4. Brake blocks 1 and 2 are connected by the return spring in such a way that enables them to return to their initial position as soon as the braking process is over. In practice, there are several types of braking mechanisms that function in a similar way (Fig. 4).

The purpose of the considerations presented above is to show that a simple self-excited system (Fig. 1) with a changeable pressure force on the belt may function as a starting point for analyses of friction in brake systems represented by girling duo-servo brakes. The occurred there frictional mechanism is modelled by two degrees-of-freedom mechanical system.

A friction between two sliding bodies is modelled by two different friction coefficients, and a normal force pressing the sliding body to the belt depends on the angle bar motion. The system dynamics are governed by the following non-dimensional set of ordinary differential equations [2]

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -x_1 - \frac{\eta_1}{\alpha_1}(x_2 + y_2) - \frac{1}{\alpha_1}y_1 - \operatorname{sgn} v_w \frac{1 - \beta_2 y_1 - \eta_2 y_2}{\alpha_1} \mu_i, \quad i=1,2, \\ \dot{y}_1 &= y_2, \\ \dot{y}_2 &= \frac{1}{\alpha_2}(-\beta_3 y_1 - \eta_{12} y_2 - x_1 - \eta_1 x_2), \end{aligned} \quad (1)$$

where:  $\mu_i = \frac{1}{1 + \delta |v_w|}$ ,  $\mu_2 = \frac{\mu_G}{\mu_0} + \left(1 - \frac{\mu_G}{\mu_0}\right) \exp \frac{-c|v_w|}{\mu_0 - \mu_G}$ , friction coefficients models;

$v_w = x_2 - v$ , is the non-dimensional relative velocity;  $v = \frac{v_p \sqrt{(k_1 + k_2)m}}{\mu_0 mg}$ ,  $\alpha_1 = \frac{\omega^2 m}{k_2}$ ,

$\alpha_2 = \frac{\omega^2 J}{k_2 r^2}$ ,  $\gamma_1 = \frac{\delta \mu_0 mg}{\sqrt{(k_1 + k_2)m}}$ ,  $\gamma_2 = \frac{c \mu_0 mg}{\sqrt{(k_1 + k_2)m}}$ ,  $\beta_1 = \frac{k_1 + k_2}{k_2}$ ,  $\beta_2 = \frac{\mu_0 k_3}{k_2}$ ,

$\beta_3 = \frac{k_2 + k_3}{k_2}$ ,  $J = \frac{M(a^2 + b^2)}{3}$ ,  $\omega = \sqrt{(k_1 + k_2)/m}$ ,  $\eta_1 = \frac{c_1 \omega}{k_2}$ ,  $\eta_1 = \frac{c_1 \omega}{k_2}$ ,  $\eta_2 = \frac{c_2 \omega \mu_0}{k_2}$ ,

$\eta_{12} = \frac{\omega(c_1 + c_2)}{k_2}$ . Dimensional parameters of the system are as follows:  $\delta, c$  - constant

values of the friction model;  $\mu_0, \mu_G$  - static and dynamic friction coefficients, respectively;  $v_p$  - velocity of the belt;  $m$  - mass of the sliding body;  $M$  - mass of the angle bar;  $a, b$  - dimensions of the angle bar;  $k_i$  ( $i=1, \dots, 3$ ) - stiffness coefficients;  $c_i$  ( $i=1, 2$ ) - damping coefficients.

A novel numerical scheme on a basis of the Hénon approach [3] is applied, which is very suitable for investigations of non-smooth dynamical systems. Many interesting dynamical nonlinear behaviours have been reported and analysed, including stick-slip

periodic, quasi-periodic and chaotic orbits. In addition, the numerical analysis has been supported by investigation of the real laboratory object modelling the feedback reinforcement of friction forces acting on the brake shoes.

To conclude, the new idea for a friction pair modelling using both laboratory equipment and numerical simulations is proposed allowing for observation and control of friction force. The experimental data have been compared with those obtained via numerical simulations showing a good agreement.

### **Acknowledgement**

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### **References**

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