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STICK-SLIP MOTIONS IN A CONTACT THERMOELASTIC PROBLEM

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Abstract. The solution of a contact thermoelastic problem on the interaction of inertial bodies involving both friction and heating is presented. It is assumed that the friction coefficient depends on the relative velocity between the contacting bodies. A stability of the stationary solution is studied. A computation of contact parameters during heating of the bodies is performed. The possibility of the existence of frictional auto-vibrations is shown.

1. Introduction

There are many examples in a literature focused on analysis of autonomous systems exhibiting regular non-linear self-excited vibrations [1-4, 6-8]. Vibrations of the mechanical system modelling woodpecker behaviour have been analysed in [8] and also studied in the monograph [3]. On the other hand, vibrations of the so-called "Olędzki's slider" have been studied first in the reference [6]. Note that two mentioned models are associated with "stick-slip" vibration behaviour.

In spite of simplicity of the introduced models, an important information is obtained. Namely, it is shown that the static self-braking cinematic pairs can initiate movement, when vibration appears.

Consider now our new proposed model, which does not have any elastic part, but which can exhibit self-excited stick-slip vibrations (Figure 1). For simplicity, it will be further referred as the "frog-slider" system.

2. Mathematical description of the problem

Let us consider a one-dimensional model of the thermo-elastic contact of a body with a surrounding medium. Assume, that this body is represented by a rectangular plate ($b_1 \times b_2 \times 2L$) (Fig.1). The plate together with a "frog" has the mass M_1 subject to the force $F = F_* h_F(t)$ and moves vertically along walls in direction z_1 of the rectangular co-ordinates $0x_1y_1z_1$. In the initial instant the body is situated in the distance Z_0 and possesses the velocity $\dot{Z}_0$. The distance between walls is always equal to initial plate thickness $2L$. The plate moves with non-constant velocity $\dot{Z}(t)$.

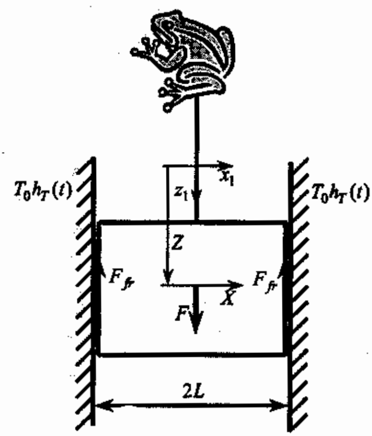

Figure 1 The model of the problem

It is assumed that the heat transfer between the plate and the walls is ideal and the Newton assumptions hold. In the initial instant the temperature is governed by the formula $T_0 h_T(t)$ ($h_T(t) \to 1$, $t \to \infty$). It causes a parallelepiped heat extension in the direction of $0x_1$, and the body starts to contact with walls. In the result of this process a frictional contact on the parallelepiped sides $X = \pm L$ occurs. The simple frictional model is applied further, i.e. a friction force F_{fr} is a product of a normal reaction force $N(t)$ and a friction coefficient. It means, that $F_{fr} = f(\dot{Z}) N(t)$ is the friction force defining a resistance of two sliding bodies movement. Here, contrary to the assumption made in the reference [7], the kinematic friction coefficient $f(\dot{Z})$ depends on the relative velocity of the sliding bodies [2].

The friction force $\sigma_{XZ}(X,t)$ per unit contact surface $X = -L$, $X = L$ generates a heat. According to the Ling [5] assumptions, the friction forces work is transmitted into a heat energy.

Note that the non-contacting plate surfaces are heating isolated and have the dimensions of $L/b_1 << 1$, $L/b_2 << 1$, which stands in agreement with assumption of our one-dimensional modelling.

In what follows the problem is reduced to determination of the mass plate centre displacement $Z(t)$, plate velocity $\dot{Z}(t)$, contact pressure $P(t) = N(t)/b_1 b_2 = -\sigma_{XX}(-L,t) = -\sigma_{XX}(L,t)$; plate temperature $T_1(X,t)$, and displacement $U(X,t)$ in the X axis direction.

In the considered case, the studied problem is governed by dynamics of the plate mass centre

$$m\ddot{Z}(t) = F_* h_F(t) - 2f(\dot{Z})P(t), \tag{1}$$

and equations of the heat stress theory for an isotropic body

$$\frac{\partial}{\partial X}\left[\frac{\partial}{\partial X}U(X,t) - \alpha_1 \frac{1+\nu_1}{1-\nu_1}T(X,t)\right] = 0, \quad \frac{\partial^2}{\partial X^2}T(X,t) = \frac{1}{a_1}\frac{\partial}{\partial t}T(X,t), \quad X \in (-L, L), \tag{2}$$

with the attached mechanical

$$U(-L,t) = 0, \; U(L,t) = 0, \tag{3}$$

heat

$$\mp\lambda_1 \frac{\partial T_1(\mp L,t)}{\partial X} + \alpha_T\left(T_1(\mp L,t) - T_0 h_T(t)\right) = f(\dot{Z})\dot{Z}(t)P(t), \tag{4}$$

and initial

$$T(X,0) = 0, \; X \in (-L, L), \; Z(0) = Z_0, \; \dot{Z}(0) = 0 \tag{5}$$

conditions. Normal stresses occurred in plate are defined through

$$\sigma_{XX}(X,t) = \frac{E_1}{1-2\nu_1}\left[\frac{1-\nu_1}{1+\nu_1}\frac{\partial U}{\partial X} + \alpha_1 T_1\right]. \tag{6}$$

In the above, the following notation is applied: E_1 – elasticity modulus, ν_1, λ_1, a_1, α_1, α_T are Poisson's ratio, thermal conductivity, thermal diffusivity, thermal expansion an heat transfer coefficients, respectively; $m = M_1/b_1 b_2$, whereas $P(t) = N(t)/b_1 b_2$ denotes the contact pressure.

Integration of equation (2) with an account of (8) and the boundary conditions (4) yields the contact pressure $P(t) = -\sigma_{XX}(-L,t) = -\sigma_{XX}(L,t)$:

$$P(t) = \frac{E_1 \alpha_1 T_0}{1-2\nu_1}\frac{1}{2L}\int_{-L}^{L} T_1(\xi,t)d\xi. \tag{7}$$

Let us introduce the following similarity coefficients

$$t_* = L^2/a_1 \text{ [s]}, \; v_* = a_1/L \text{ [m/s]}, \; P_* = T_0 E_1 \alpha_1/(1-2\nu_1) \text{ [N/m}^2\text{]}, \tag{8}$$

and the following non-dimensional parameters

$$x=\frac{X}{L},\ \tau=\frac{t}{t_*},\ z=\frac{Z}{L},\ p=\frac{P}{P_*},\ \theta=\frac{T_1}{T_0},\ \varepsilon_1=\frac{2P_*t_*^2}{mL},\tag{9}$$

$$\gamma=\frac{E_1\alpha_1 a_1}{(1-2\nu_1)\lambda_1},\ Bi=\frac{L\alpha_T}{\lambda_1},\ m_0=\frac{F_*}{2P_*},\ F(\dot z)=f(\mathrm{v}_*\dot z).\tag{10}$$

Applying an the Laplace transformation, the following system of equations is obtained:

$$p(\tau)=Bi\int_0^\tau \dot h_T(\xi)G_p(\tau-\xi)d\xi+\gamma\int_0^\tau F(\dot z)\dot z(\xi)p(\xi)\dot G_p(\tau-\xi)d\xi,\tag{11}$$

$$\ddot z(\tau)=\varepsilon_1\left[m_0h_F(\tau)-F(\dot z)p(\tau)\right],\tag{12}$$

which yields the non-dimensional pressure $p(\tau)$ and velocity $\dot z(\tau)$. The temperature is defined through the following formula

$$\theta(x,\tau)=Bi\int_0^\tau \dot h_T(\xi)G_\theta(x,\tau-\xi)d\xi+\gamma\int_0^\tau F(\dot z)\dot z(\xi)p(\xi)\dot G_\theta(x,\tau-\xi)d\xi,\tag{13}$$

where:

$$\left\{G_p(\tau),\ G_\theta(1,\tau)\right\}=\frac{1}{Bi}-\sum_{m=1}^{\infty}\frac{\left\{2Bi,\ 2\mu_m^2\right\}}{\mu_m^2[Bi(Bi+1)+\mu_m^2]}e^{-\mu_m^2\tau},\tag{14}$$

and μ_m are the roots of the following characteristic equation: $\mathrm{tg}\mu_m=Bi/\mu_m$, $m=1,2,\ldots$.

A stationary solution to the problem reads:

$$p_{st}=\frac{1}{1-\mathrm{v}},\ \theta_{st}=\frac{1}{1-\mathrm{v}},\ \mathrm{v}=F(\mathrm{v}_{st})\frac{\mathrm{v}_{st}\gamma}{Bi},\tag{15}$$

where v_{st} is the solution of the non-linear equation

$$F(\mathrm{v}_{st})=\frac{m_0}{1+\gamma m_0\mathrm{v}_{st}/Bi}.\tag{16}$$

Graphical solution of equation (16) is presented in Figure 2 for various parameters m_0 and Bi. Recall that for steel $\gamma=1.87$.

In this case the steel made parallelepiped plate ($\alpha_2=14\cdot10^{-6}\,{}^\circ\mathrm{C}^{-1}$, $\lambda_1=21W/(\mathrm{m}\cdot{}^\circ\mathrm{C}^{-1})$, $\mathrm{v}_1=0.3$, $a_1=5.9\cdot10^{-6}\mathrm{m}^2/s$, $E_1=19\cdot10^{10}\mathrm{Pa}$) with $L=0.01$ m, $T_0=5^\circ C$, $z^\circ=\dot z^\circ=0$ and with non-constant friction coefficient is studied. One gets $t_*=16.95s$, $\mathrm{v}_*=0.59\cdot10^{-3}$ m/s, $P_*=3.3\cdot10^7$ Pa. The function $F(\dot z)=f(\mathrm{v}_*\dot z)$ is defined through the formula [2].

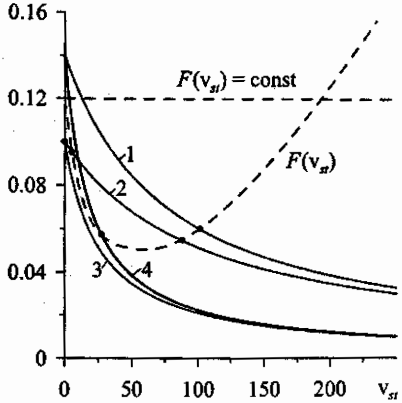

Figure 2 Graphical solution of equation (16)
(solid curves: 1 – $m_0 = 0.14$, $Bi = 20$; 2 – $m_0 = 0.1$, $Bi = 20$; 3 – $m_0 = 0.1$, $Bi = 5$;
4 – $m_0 = 0.14$, $Bi = 5$; dashed curve corresponds to $F(\mathrm{v}_{st})$).

In the fourth case ($m_0 = 0.14$, $Bi = 5$) we have one solution of the form: $\mathrm{v}_{st}^2 = 27.8$, $p_{st}^2 = \theta_{st}^2 = 2.45$. It is unstable if the parameter ε_1 is larger than its critical value ($\varepsilon_1 \geq \tilde{\varepsilon}$).

In order to confirm the given conclusions, numerical analysis is carried out for the fourth case for $Bi = 5$ (now $\varepsilon_1 \approx \tilde{\varepsilon} = 586.5$), and the computational results are shown in Figures 3 for a few values of the parameter $\varepsilon_1 = 400;\ 586.5;\ 800$. In Figure 3a, the dependence of non-dimensional body velocity $\dot{z}(\tau)$ on the non-dimensional time τ is shown.

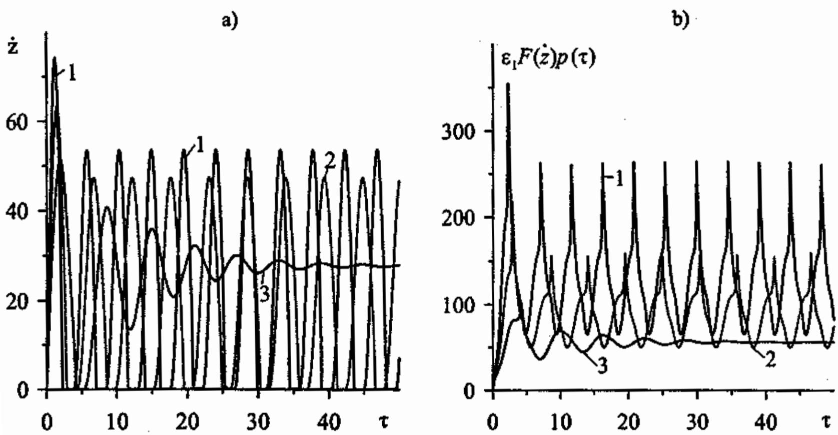

Figure 3 Time history of non-dimensional body's velocity (a) and friction force (b) for various values of ε_1 (curve 1: $\varepsilon_1 = 800$; curve 2: $\varepsilon_1 = 586.5$; curve 3: $\varepsilon_1 = 400$).

4. Conclusions

It this work the results devoted to a novel problem of the so called "frog-slider" mechanical system exhibiting frictional thermoelastic contact of a moving body subject to both non-constant friction coefficients are presented and discussed. It is worth noticing that in the case of non-constant friction coefficient, the self-excited vibration can appear in our system without an elastic part (stiffness). The last phenomenon is caused by the body heating while accelerating, the friction increase, and than the braking and cooling of the system. The characteristic changes of both displacement (Figure 3) and velocity of the analysed system motivated us to use the expression: "frog-slider" system.

5. References

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