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NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF SIMPLE NON-LINEAR SYSTEM MODELLING A GIRLING DUO-SERVO BRAKE MECHANISM

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ABSTRACT

Our analysis is focused on modeling of friction phenomena exhibited by a brake like duo-servo system. The occurred in girling duo-servo brake an frictional mechanism can be modeled approximately by two degrees-of-freedom mechanical system, where a normal force varies during displacement of a block. The Coulomb friction is used after numerical analysis, and a normal force pressing the sliding body to a belt depending on an angle bar motion of the analyzed system is applied. In addition, the numerical analysis is supported by investigation of the real laboratory object modeling the feedback reinforcement of friction forces acting on the block.

1 INTRODUCTION

A nature of sliding components with an occurrence of intermittent stick and slip leads to unpredictable behaviors. These problems are exhibited in many industrial applications including bearings, disc brake systems, electric motor drives, rail mass transit systems, and machine tool piece systems [16]. Understanding the stick-slip phenomena and its possible elimination are important for applications requiring high precision motion [2].

The simplest models describe friction as a function of the sliding bodies velocity difference. Such models like Coulomb's friction one are called static models. In fact, Coulomb's dry friction laws simplify a very complex behavior which involves mechanical, plastic, and chemical processes [26]. Experimentally observed differences in application of Coulomb's law are often found [1], [14]. Computer simulations of mechanical systems with friction are difficult because of the strongly nonlinear behavior of the friction force near zero velocity, and the lack of universally considered friction model. For rigid bodies with dry friction, the classical Coulomb's law of friction is usually applied in engineering contact problems exactly because of its simplicity. It can explain several phenomena associated with friction and it is commonly used for friction compensation [13].

The problem of modeling a friction force is not solved, because the physics and dynamical effects are not sufficiently understood. There are two main theoretical approaches to model dry friction interfaces: the macro-slip and micro-slip approaches [12], [26]. In the micro-slip approach, a relatively detailed analysis of the friction interfaces should be made. In this case investigations can provide accurate results only when preload between interfaces is very high. In the macro-slip approach, the entire surface is assumed to be either sliding or sticking. The force necessary to keep sliding at a constant velocity depends on the sliding velocity of the contact surfaces. With this respect, smooth and non-smooth velocity depended friction laws have been applied in the references [19], [21], [25].

There is a lack of works which take into account problems of experimentally observed velocity depended friction force models. The paper [10] deals with measurement of dry friction. A tribometer was developed to identify both sticking and sliding friction coefficients. The so called Stribeck-curve has been determined for any material in contact zone. Similarly, a multi degrees-of-freedom model of friction was investigated in [9], where an experimentally observed friction characteristic expresses the kinetic friction force as a function of relative velocity of motion. The experimental investigations of vibrations of the system composed of a steel-polyester pair confirmed that the friction static force increases both with increasing time of adhesion and with growing force. Additionally, kinetic friction force depends also on the sign of acceleration.

In spite of the fact that there are many papers devoted to analysis of regular and chaotic dynamics of mechanical systems with friction it seems that up to now not all possible non-linear phenomena have been properly understood or even detected and explained [5]-[7], [11], [14], [17], [20], [21]. Although this paper is devoted to numerical and experimental investigations, but the problem is expected to be attacked also from an analytical point of view. The stick-slip chaos has been predicted analytically using the Melnikov technique by Awrejcewicz and Holicke [8], but such a prediction for two degrees-of-freedom system is in general more complicated. Even if this problem will be solved it will contain only special type of nonlinear terms, and it will be valid only for special systems. Therefore, in this paper we have focused on numerical simulations, which do not include the mentioned drawbacks.

Self-excited system with friction analyzed in this work requires a special suitable algorithm to omit the problems occurred during integration of equations of motion with the sign function, which causes sudden jumps and yields errors which are not required. In what follows the Hénon method [3], [15] is applied, which is particularly adjustable for obtaining a uniformly suitable solutions of non-smooth systems (here with friction).

It is extremely difficult (perhaps even impossible) to build a general friction model including all possible accompanying processes. Moreover, it seems to be pointless, since only some of the above mentioned processes dominate in a specific object of study. The difficulties involved with explaining numerous effects of friction that appear during confrontations of mechanical (geometric), molecular (adhesive), mechano-molecular and energy theories with experiments created the need to model friction with the use of simple dynamic systems for analysis of friction-induced processes [18], [27]. With the above circumstances taken into consideration, this work describes friction modelling based on a relatively simple model in which vital dynamic processes occurring in a few typical mechanical brake systems are retained [24].

Diverse characteristics of friction can be observed in real dynamic systems. Many models do not require taking friction into account, although numerous systems are based on utilising friction – hence omitting the friction-induced effects is impossible. Friction reveals its dominating nature in friction clutches [22], belt drive systems [23], as well as in the brake systems in which friction force between a wheel's brake drum and brake blocks causes braking of a vehicle . The most widely applied brakes are the ones which are assembled in the wheels of vehicles with certain brake mechanisms [23], **Błąd! Nie można odnaleźć źródła odsyłacza.**.

2 THE ANALYZED BRAKE MECHANISM

Let us examine a theoretical brake mechanism illustrated by the diagram in Fig. 1, which may help to comprehend an analogy to the mechanical system being considered within this dissertation. Mass m influenced by friction force T induced by movement of a belt at constant velocity v, moves in direction x and affects spring k_1. The force that is induced through the relocation of mass m, affects the leg of an angle bar turning it through an angle of ϕ in relation to point s. The turn results in compression of spring k_2 in direction y and an increase of the pressure force affecting the belt. It clearly shows that the model is constructed in such a way so that friction force T is dependent on the force exerted by mass m on the belt. Obviously, when the force in spring k_1 is larger than friction force T, then a loss of adhesion occurs and mass m moves in direction $-x$. At that time, spring k_1 expands and if its length exceeds its free length, then the perpendicular arm of the angle bar is "pulled" in direction $-x$ and the angle bar is turned in direction ϕ. The horizontal arm of the angle bar expands spring k_2, which decreases the pressure force exerted by the spring on mass m oscillating on the belt. The coupling repeats in the system throughout the entire friction process as long as the belt's linear velocity v is not equal to zero.

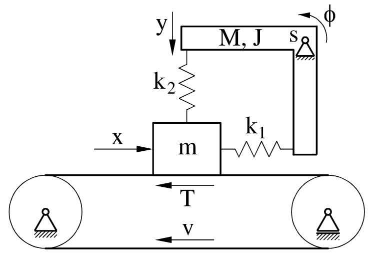

Figure 1. Theoretical model of a brake mechanism.

The fundamental element that needs utmost attention while estimating the similarity of the described system (the model) to a brake mechanism (a real object) is the coupling of the mass m transfer on the belt induced by friction (Fig. 1) with the normal pressure force affecting the belt. A model of a brake mechanism with intensified braking force is shown in Fig. 2.

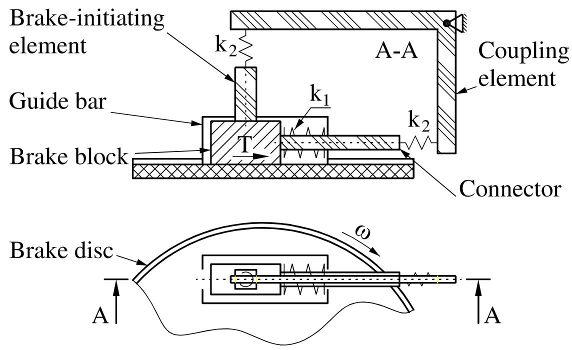

Figure 2. Model of a brake mechanism with intensified braking force.

As it is shown in Fig. 2, when the element that initiates braking (the pressure on the brake pedal) starts to press the brake block against the disc, then friction force occurs between the block and the disc of a wheel. In effect, the brake block moves in direction x, the coupling element moves or turns, and by an increase of the pressure force intensifies braking. When the initial pressure decreases (the release of the brake lever), friction force between the brake block lining and the brake disc also decreases and return spring k_1 may pull the brake block back to its initial position. As it can be noticed, coupling of the friction force with the pressure force may function as power-assistance to the braking system. That is why the pressure force on the brake lever may be significantly lowered during braking. The described mechanical coupling can also be obtained with the use of a hydraulic oil system with a pump.

The brake block lining has low susceptibility and it can be modelled with a belt when the experiment's conditions are satisfied (see: Fig. 1). Additionally, it is possible to choose the materials which the brake disc and mass m are made of. According to the conducted investigation, there is a similarity between functioning of the dynamic systems presented in Figs. 1 and 2. The friction model, assumed for the system in Fig. 1 approximately corresponds to the brake block lining's friction against a car brake drum (or a disc). Assuming that the same materials are chosen, the model enables to investigate the phenomena that result from friction and wear of rubbing surfaces.

The method of friction modelling that includes the relation between the friction force and the rubbing bodies' relative velocity, and between the friction force and the changes of the normal force (the pressure force of the frictional pairs) may also be applied to the study of friction-induced dynamic phenomena in the braking system presented in Fig. 3.

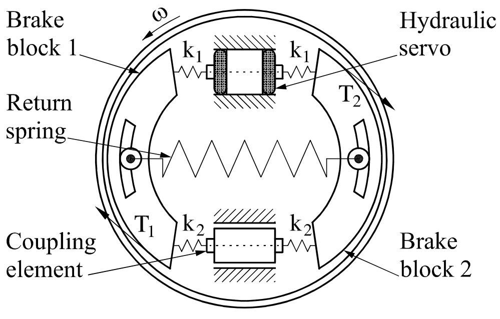

Figure 3. Simplified girling duo-servo brake.

The brake mechanism diagrammatically shown in Fig. 3 is assembled in a popular type of drum brake, a.k.a. "a duo-servo" (see Fig. 4). When the hydraulic servo initiates braking, the brake blocks *1* and *2* are drawn aside and pressed against the inner surface of the drum. As a result, friction forces T_1 and T_2 are exerted between the blocks' linings and the drum and the wheel stops.

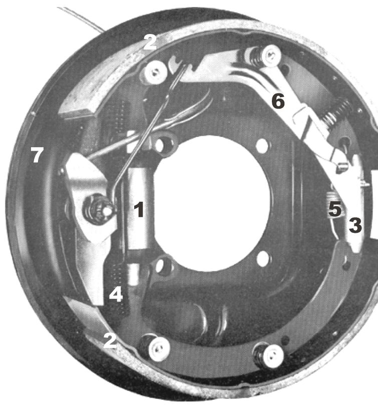

Figure 4. Girling duo-servo brake: 1-hydraulic cylinder, 2-brake shoes with friction linings, 3-couple element, 4-auxiliary long spring, 5- auxiliary short spring, 6-hand-brake mechanism, 7-body.

A careful analysis of the mechanism shown in Fig. 4 reveals certain type of coupling. Brake block *1* (also called a "backward" block) takes over a larger part of the friction force at the initial stage of braking, whereas brake block *2* (called a "concurrent" block) impedes with a weaker force. However, the coupling element (the angle bar in the system in Fig. 1) combines the circumferential motion of brake block *1* with the motion of brake block *2* and the pressure force of the latter on the drum's inner surface increases.

The ratio of the braking forces exerted by brake blocks *1* and *2* is about 2:4. Brake blocks *1* and *2* are connected by the return spring in such a way that enables them to return to their initial position as soon as the braking process is over. In practice, there are several types of braking mechanisms that function in a similar way (Fig. 4).

The purpose of the considerations presented above is to show that a simple self-excited system (Fig. 1) with a changeable pressure force on the belt may function as a starting point for analyses of friction in brake systems represented by drum brakes.

3 THE MODELED SYSTEM

The study and prevention of self-excited vibration of systems with friction is very important in industry and there is a need of friction pair modeling that could correctly describe kinetic and static friction forces change between two moveable surfaces.

A further developed model can govern dynamics of the girling duo-servo brake mechanism [4] described in section above. Therefore, the schematically illustrated in Fig. 5 2-DOF dynamical system is analyzed numerically and investigated experimentally.

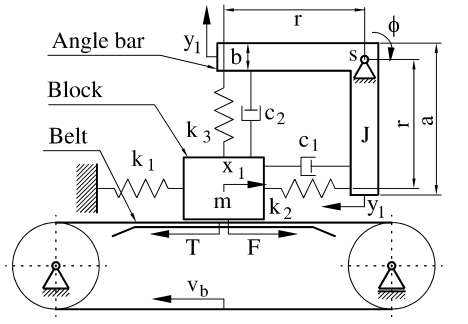

Figure 5. The analyzed 2-DOF system.

The self-excited system presented in Fig. 5 is equivalent to a real experimental rig in which *block* mass m is moving on the *belt* in x_1 direction, and where the *angle body* represented by moment mass of inertia J is rotating around point s with respect to angle direction ϕ. The analyzed system consists of the following parts: two bodies are coupled by linear springs k_2 and k_3; block on the belt is additionally coupled to fixed base using linear spring k_1; angle body is excited only by spring forces; there is not extra mechanical actuators; rotational motion of the angle body is damped using virtual actuators characterizing air resistance and they are marked by constants c_1 and c_2; damping of the block is neglected; it is assumed that angle of rotation of the angle body is small and it is within interval [+5, -5] degrees (in this case a rotation is equivalent to linear displacement y_1 of legs a of the angle body); belt is moving with constant velocity v_b and there is not deformation of the belt in a contact zone.

Non-dimensional equations governing dynamics of our investigated system have the following form

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 - \alpha_1^{-1}\left[\eta_1\left(x_2 + y_2\right) - y_1 - T\right],$$

$$\dot{y}_1 = y_2, \qquad (1)$$
$$\dot{y}_2 = \alpha_2^{-1}\left(-\beta_3 y_1 - \eta_{12} y_2 - x_1 - \eta_1 x_2\right),$$

where: x_2, y_2, are velocities of the block and angle body, respectively; $v_{rel} = x_2 - v_b$ is a relative velocity between bodies of the investigated system; $\alpha_1 = \frac{\omega^2 m}{k_2}$, $\alpha_2 = \frac{\omega^2 J}{k_2 r^2}$, $\beta_1 = \frac{k_1 + k_2}{k_2}$, $\beta_2 = \frac{\mu_0 k_3}{k_2}$, $\beta_3 = \frac{k_2 + k_3}{k_2}$, $\eta_1 = \frac{c_1 \omega}{k_2}$, $\eta_2 = \frac{c_2 \omega \mu_0}{k_2}$, $\eta_{12} = \frac{\omega(c_1 + c_2)}{k_2}$ are remaining parameters; ω is a periodicity of mass m. Friction force is described in the following manner

$$T = \begin{cases} \mathrm{sgn}(v_{rel}) T_+ & \text{if} \quad v_{rel} > 0, \\ \mathrm{sgn}(v_{rel}) T_- & \text{if} \quad v_{rel} < 0, \\ |T_s| & \text{if} \quad v_{rel} = 0. \end{cases} \qquad (2)$$

T_+ and T_- friction force characteristics are described by linearly and exponentially decaying functions, respectively.

4 EXPERIMENTAL INVESTIGATIONS

In this section, the laboratory rig designed for observations and experimental research of friction effects including friction force measurement is described. Photos of the rig are presented in Fig. 6.

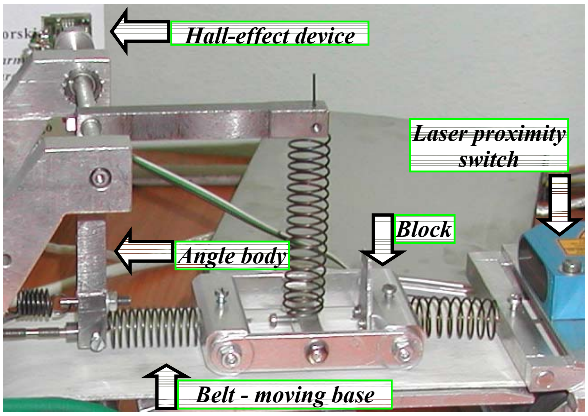

Figure 6. The laboratory rig in a particular view (vibratory subsystem).

The general view, component parts and some connectors like coil springs correspond to those schematically indicated elements presented in Fig. 5.

Displacement of the block and angle of rotation are measured using laser proximity switch and Hall-effect device, which guaranties a non-sticking method of the measurement. Both of them provide linear dependency of the measured quantity versus analogue voltage output. Measurement instruments connected through PCI computer card to LabView software allows to perform dynamic acquisition of the two measured signals. Disturbances of whole construction, noise in electrical circuits, and another additional maintenances have significant influence on accuracy of any measured signals. Therefore, some signals are filtered digitally (elliptic topology) and a real differentiation preventing high peaks formation is applied.

Appropriate transformed equations of motion (see: Eq. (1)) can be used for friction force calculation after real time measurement of state variables of the investigated system. Characteristic of friction force versus relative velocity between belt and block for positive and negative velocities of the belt are shown in Fig. 7.

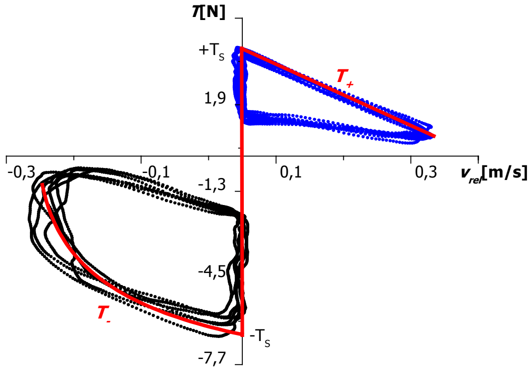

Figure 7. Friction force characteristics for positive (T_+) and negative (T_-) relative velocities.

One may observe that zones occupied by closed functions of friction model differ significantly. It is a regularity, since angle body causes reinforcement of friction force for positive velocity of moving. Owing to these considerations, T_+ and T_- friction force characteristics are described by linearly and exponentially (of second order) decaying functions, respectively.

In the case of T_+ branch, the equation of friction force dependence describing friction force model for positive relative velocity has the following form:

$$T_+ = T_s - |v_{rel}| \frac{T_s - T_{min}}{v_{rel,max}}, \qquad (3)$$

where: T_s is a static friction force, $v_{rel,\ max}$ is a maximum positive relative velocity. The T_- branch can be described by a second order exponentially decay function describing friction force model for negative relative velocity of the following form:

$$T_- = T_s + A_1 \exp\left(-\frac{|v_{rel}| - v_{rel,min}}{t_1}\right) + A_2\left(-\frac{|v_{rel}| - v_{rel,min}}{t_2}\right), \qquad (4)$$

where: $v_{rel,\ min}$ is a maximum negative velocity, A_1, A_2, t_1, t_2 are the constant values. The main multivalued function describing friction force changes (red line in Fig. 7) occurring in our investigated 2-DOF system with a variable normal force is determined from the Eq. (2).

5 EXPERIMENTAL MEASUREMENT

Results of executed measurements are obtained following the methodology described in Sec. 4. The examples of time characteristics of state variables are shown in Fig. 8.

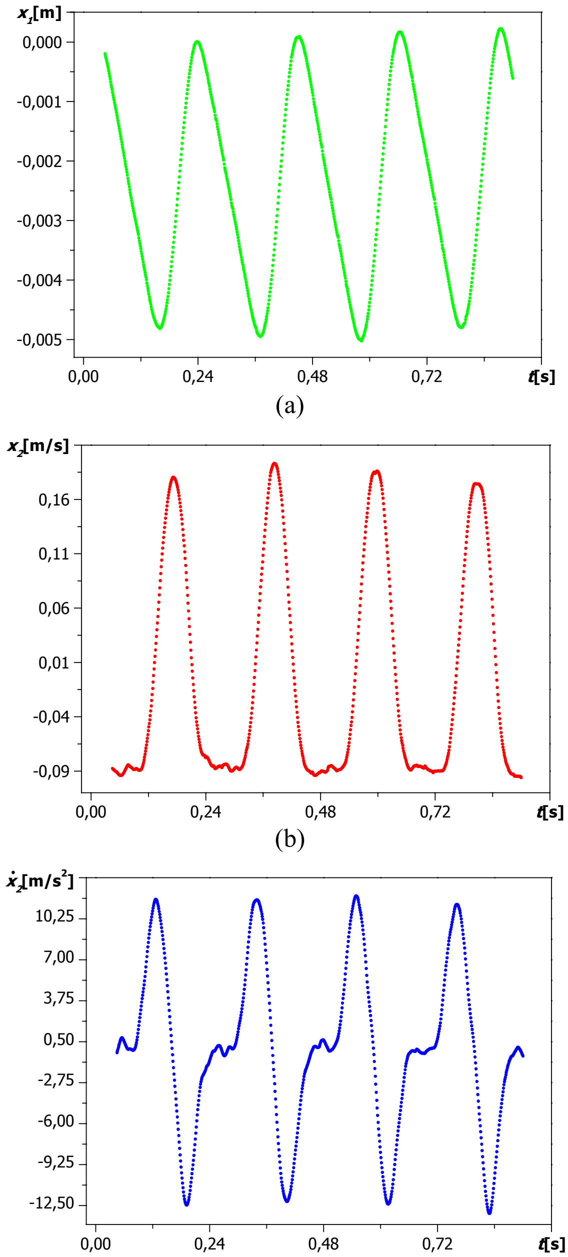

Figure 8. Real time histories of (a) displacement x_1, (b) velocity x_2, and (c) acceleration $\dot{x}_2$ of the block for velocity of the belt $v_b = -0.13$ m/s.

A characteristic positive slope (slip phase of block) of the time history and a negative slope (stick phase of block) can be observed in Fig. 8a. A time dependency of velocity of the block is presented in Fig. 8b. There are some time intervals between nodes, where velocity is almost constant and equal to the belt velocity. This situation happens if the considered block is in a stick phase. Otherwise, stick phase can be observed on a time history graph of acceleration, where some of intervals (at zero value of velocity) are parallel to t-axis.

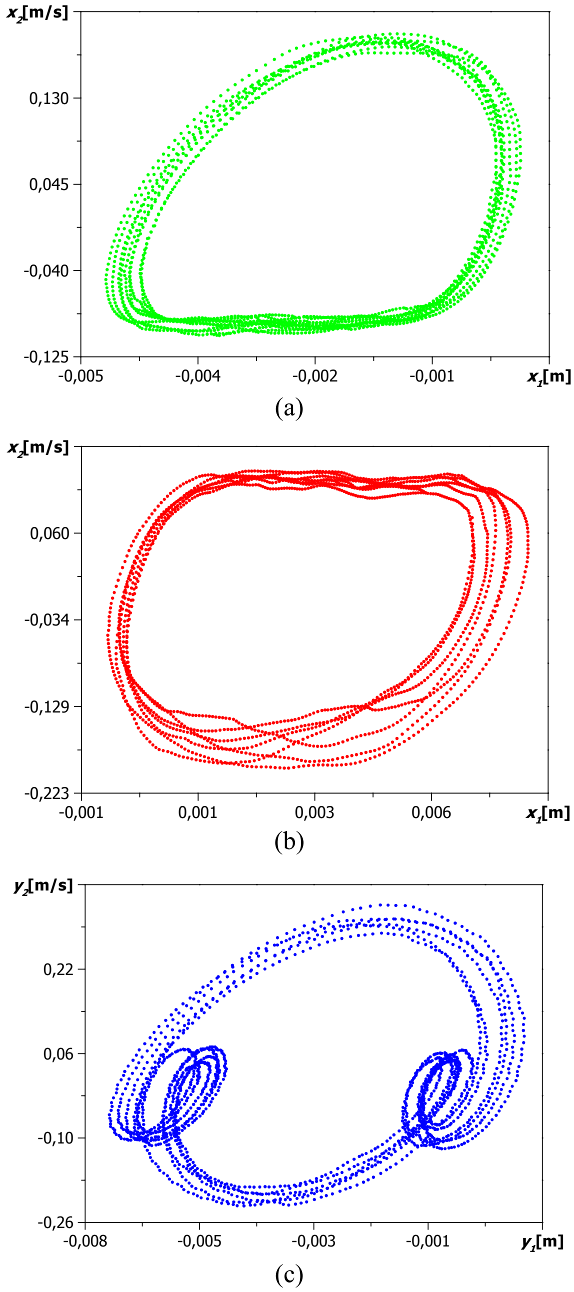

Figure 9. Phase planes of (a) block for $v_b < 0$, (b) $v_b > 0$, and phase plane of (c) angle body for $v_b > 0$.

The phase planes give opportunity to explore more comprehensively dynamics of the investigated system. The very known shapes of phase curves usually visible in the case of stick-slip motion are presented in Fig. 9a-b. The stick (almost straight lines) and slip (arcs connecting ends of straight lines) phases can be easily observed.

6 NUMERICAL ANALYSIS

Friction force model given by Eq. (2) is transformed to the non-dimensional one, and then a numerical analysis based on T_+ and T_- friction force characteristics is carried out. Parameters of both models are obtained by both measurement and identification: T_s=3.63, T_{min}=0.86, $v_{rel,,max}$=0.27 (T_+ branch); T_s=-5.94, T_{min}=-1.42, $-v_{rel,,max}$=0.28, A_1=3.23453, A_2=2.87362, t_1=0.0342, t_2=0.30529 (T_- branch). Numerical analysis with implementation of introduced friction force dependency have yielded the results presented in Fig. 10.

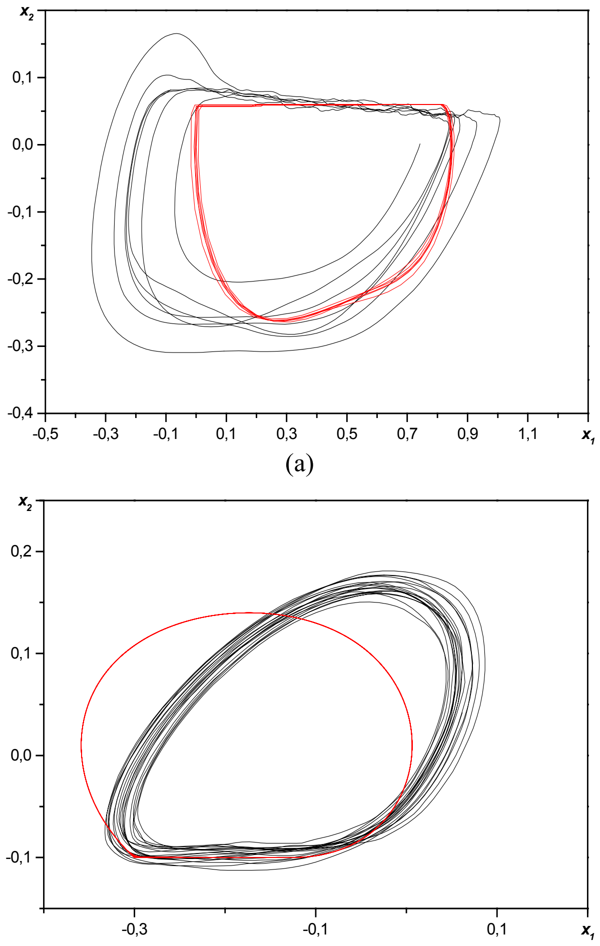

Figure 10. Verification of the results obtained via computer simulation (red line) with experimental measurement of state variables (black line) of the analyzed system: friction force model: (a) T_-, (b) T_+; parameters: α_1=1.87, α_2= 0.72, β_1=2.87, β_2=1.62, β_3= 2.47, $\eta_1=\eta_2=\eta_{12}$=0, v_b=0.06, μ_0=1.1; initial conditions: t_0=400, t_k=5000, $x_1(0)$=0, $x_2(0)$=0, $y_1(0)=y_2(0)$=0.

The numerical trajectory (red line) illustrated in Fig. 10a is satisfactory close to its experimental counterpart recorded for the investigated dynamical system. The sticking velocity is almost the same, but in sliding phase some distinguishable differences are observed. T_- friction force model can be used after analysis of friction effects occurring in systems, where the normal force acting between cooperated surfaces can be varied.

A significant difference between being considered trajectories presented in Fig. 10b is visible, but the sticking phase is still coinciding for a positive relative velocity. A comparison of the results with those illustrated in Fig. 10b provide very useful information, namely that our analyzed system is non-symmetric.

8 CONCLUDING REMARKS

The numerical analysis is supported by investigation of the real laboratory object modeling the feedback reinforcement of friction force (model of T_- branch) and without the feedback (model of T_+ branch). The numerical solution (red curve) obtained using T_- branch model did not prove a transition, which can be observed in our experimental measurement (black curve). The sticking velocity is almost the same, but in sliding phase some distinguishable differences are observed.

T_- friction force model is suggested to be used after analysis of friction effects occurring in systems, where the normal force acting between cooperated surfaces is fluctuated. The application of T_+ branch friction force model leads to rapid entries on stick phase and a rather smooth backslides from it (see red line in Fig. 10b).

To conclude, a new idea for the friction pair modeling using both laboratory equipment and numerical simulations is proposed allowing for observation and control of friction force. The experimental data are compared with those obtained via numerical simulations showing a good agreement.

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