THE METHOD OF DYNAMIC CHARACTERISTICS DETERMINING OF MULTIDIMENSIONAL MECHANICAL OBJECTS WITH RESPECT TO VIBRATION CONTROL

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Introduction

In many cases we encounter vibratory systems consist of mechanical objects in many points supported by elastic elements. The dynamic analysis of such systems, the problems of force or displacement vibro-isolation, or problems of passive or active vibrations control can be formulated and solved by the method of composing of the dynamic characteristics of particular subsystems [1, 2].

The matrix of complex factors of spectral transmittance defined by formula (1) constitutes the dynamic characteristics of multidimensional vibratory object for a given input function frequency:

$$G_{ij}(i\omega) = \frac{y_i}{x_j} = A_{ij}(\omega)\exp(i\varphi_{ij}(\omega)), \qquad i^2 = -1, \tag{1}$$

where : x_j - j-th complex entry (force); y_i – i-th complex exit (displacement); $A_{ij}(\omega)$ – the modulus of spectral transmittance, $\varphi_{ij}(\omega)$ – argument of spectral transmittance.
The vibration of the system shown in Fig. 1 can be dynamically excited by the forces originating in the object during its work (e.g. centrifugal inertia forces from unbalanced whirling elements) or by kinematic vibrations of the background.

Assuming that the object in the system from Fig. 1(a) generates not well known harmonic input function during its working time (the so-called active object), the block scheme of the considered vibratory system can be presented in the form shown in Fig. 1(b),

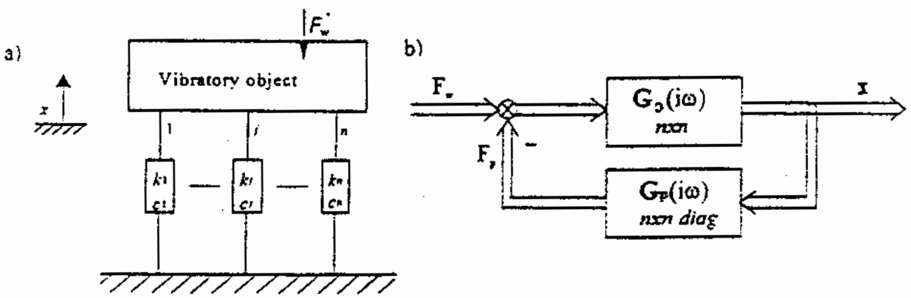

Fig. 1. The scheme (a) and the block scheme (b) of the vibratory system.

where: x – n-dimensional vector of complex displacements of object supporting points. F_w - n-dimensional vector of complex forces in supporting points originating from the input function generated by the object, $G_0(i\omega)$ – square (n x n) matrix of spectral transmittances of the object treated as free in space, $G_p(i\omega)$ – diagonal (n x n) matrix of spectral transmittances of the elastic support elements.

In the cases of massless, elastic–suppressing (k_j, c_j) supporting elements, the elements of spectral transmittances matrix are expressed in the following way

$$G_{j,jp}(i\omega) = k_j + ic_j\omega . \tag{2}$$

The vector of displacement of supporting points of the vibratory objects reads

$$x = (I + G_0 G_p)^{-1} G_0 F_w , \tag{3}$$

where I is the elementary matrix. It is essential, however, to determine previously the matrix G_0 and the vector F_w. This is why the experimental analytical method was suggested to determining this dynamic characteristic of real, both stiff and flexible, passive (not generating input function) as well as active (generating input function) objects.

Passive object

It consists of the passive object O and its massless supporting elements of the known stiffness k_j' and damping c_j'. The known external harmonic input function F_z influences the object in the supporting points.

The system in Fig. 2 can be divided in the supporting points into 2 subsystems: the object and the supporting elements (Fig. 3). The following conditions of continuity and balance for separation points describe the mutual dynamic influence of subsystems:

$$x = x_0 = x_p , \qquad F_o = -F_p . \tag{4}$$

The displacement of the object in the separation points is governed by the equation

$$x_0 = G_0 (F_0 + F_z) , \tag{5}$$

where G_0 is the sought spectral transmittances matrix of the object treated as free in space.

The displacement of supporting elements k_j', c_j' (Fig. 3) in the separation points has the form

$$x_p = G_p^{-1} F_p , \tag{6}$$

where G_p is the known diagonal matrix of spectral transmittance of the supporting elements with the coefficients $G_{j,jp}(i\omega)=k_j'+ic_j'\omega$.

Having taken into account the conditions of (4) and the equation (6) in (5) we get

$$(I + G_0 G_p) x = G_0 F_z . \tag{7}$$

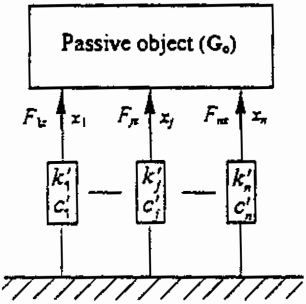

Fig. 2. The scheme of the system to determine the dynamic characteristics of the passive object

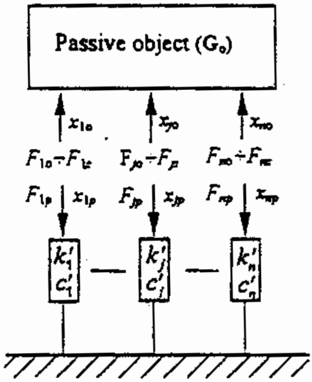

Fig. 3. The system presented in Fig. 2 after separation into two subsystems.

Having transformed the expression (7) we get the being sought matrix

$$G_o = G_u(I - G_p G_u)^{-1}, \tag{8}$$

where G_u is the matrix of spectral transmittances of the object from Fig. 2 which can be experimentally determined on this system, and at the same time $G_{i,jz} = x_i / F_{jz}$.

Active objects

The system shown in Fig. 4 (and after separation in Fig. 5) has been studied.

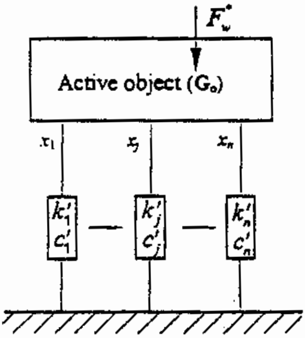

Fig. 4. The scheme of the system to determine the dynamic characteristics of the active object.

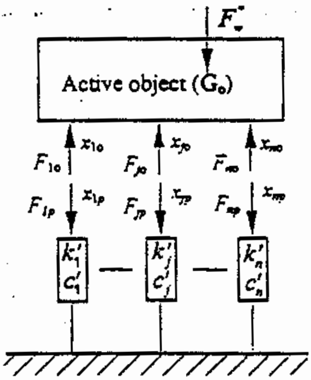

Fig. 5. The system presented in Fig.4 after separation into subsystems.

The matrix of force distribution of F_w into particular supporting points from the forces F_w^* generated in the object must be additionally determined for the active object. The conditions of continuity and balance have the form (4). The displacement of subsystems separation points equals to

$$x_0 = G_0 F_0 + x_w, \tag{9}$$

where x_w is the vector of displacement of separation points of the object coming from the inner forces generated by the object. The displacements of the separation points of supporting elements are expressed by (6). Taking into account (4) and (6) in (9) we get

$$x_w = (I + G_0 G_p) x_u, \tag{10}$$

Multiplying both sides of (10) (from the left side) by matrix G_0^{-1} we obtain the expression for the sought vector of force separation

$$F_w = (G_0^{-1} + G_p) x_u, \tag{11}$$

where x_u is the vector of supporting points displacements in the system from Fig. 4 caused by harmonic input function F_w^* generated by the object.

Conclusions

The method presented in the paper allows for determination of the dynamic characteristics of flexible or stiff passive G and active F objects on the basis of direct experimental measurements carried out on real objects and computer calculations. The determined characteristics can be used for dynamical analysis of the system with such objects, for the solution of vibro-isolation questions and the problems of passive or active vibrations control of these objects.

The presented method for determination of the dynamical characteristics of multidimensional elastically supported mechanical object will be further applied with respect to the proposed scheme (see Fig. 6).

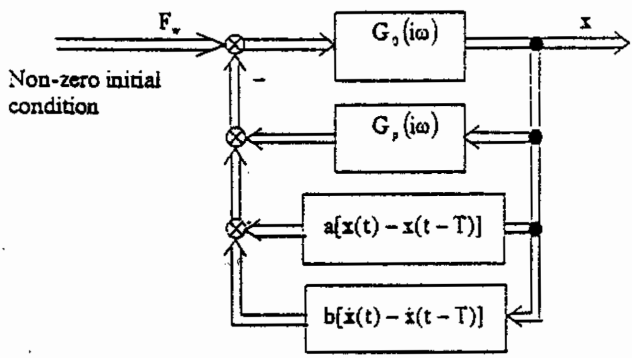

Fig. 6. The block scheme of a system to active vibrations control of an object $G_0(i\omega)$.

References

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2. G. T. S. Done and A. D. Hughes (1976) Reducing vibration by structural modification, Vertica, vol. 1, 31-38.