SOLITONS EXHIBITED BY THE VON KÁRMÁN EQUATIONS

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Introduction

One dimensional stationary waves appearing in one dimensional continuous systems (communication lines), or flat waves (for instance, solitons on water governed by Kortweg de Vries equation) belong to well studied. On the other hand, it is clear that solitons created on water, on a falling down fluid layer, and ionic-sound solitons in plasma should depend on two spatial co-ordinates. A simple model is given by Kadonitsev and Petrashvilli [1], which is generalization of the Kortweg de Vries equation.

As it has been mentioned in reference [2], the particularity of Jupiter atmosphere is expressed by its great red glow, the so called two dimensional Rossby soliton. The Rossby waves in a linear approximation correspond to waves appearing in rotating atmosphere, and the main reason of its existence depends on a change (with a latitude) of a horizontal projection of the Coriolis force. For a medium with dissipation the Chochleva-Zabolotskij equation plays a representative role to describe the occurring waves [3]. However, in a case of thin flexible plates within the kinematic Kirchhoff's model the solitons are not detected yet and we are going to illustrate and analyse this behaviour.

Forced oscillations of flexible plates with a longitudinal, time dependent load, acting on one plate side are investigated in this report. Regular (harmonic, subharmonic and quasi-periodic) and irregular (chaotic) oscillations appear depending on the system parameters as well as initial and boundary conditions. In order to achieve highly reliable results, an effective algorithm has been applied to convert a problem of finding solutions to the hybrid type partial differential equations (the von Kármán form) to that of the ordinary differential equations (ODEs) and algebraic equations (AEs). The obtained equations are solved using finite difference method with the approximations $0(h^4)$ in respect to the spatial co-ordinates. The ODEs are solved using the Runge-Kutta fourth order method, whereas the AEs are solved using either the Gauss or relaxation methods. The analysis and identification of spatial-temporal oscillations is carried out by investigation of the series $w_{ij}(t)$, $w_{t,ij}(t)$, phase portraits $w_{t,ij}(w_{ij})$ and $w_{tt,ij}(w_{t,ij}, w_{ij})$ and the mode portraits in the planes $w_{x,ij}(w_{ij})$, $w_{y,ij}(w_{ij})$ and in the space $w_{xx}(w_{x,ij}, w_{ij})$, FFT as well as the Poincaré sections and pseudo-sections.

Equations, Initial and Boundary Conditions

The known equations governing dynamics of flexible isotropic plates are taken as the mathematical model. The plate material is elastic and both Hook's and Kirchhoff's hypotheses are valid. A plate is thin and the hypothesis on an average deflection holds. The known von Kármán equations [4] satisfy the listed hypotheses, which have the form

$$\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{\partial w}{\partial t}=-\frac{1}{12(1-\nu^2)}\nabla^2\nabla^2 w+L(w,F)-P_x\frac{\partial^2 w}{\partial x^2}+q,$$

$$\nabla^2\nabla^2 F=-\frac{1}{2}L(w,w), \qquad (1)$$

where $w(x, y, t)$ is a deflection function along z co-ordinate oriented toward the Earth centre, and $F(x, y, t)$ is the Airy's function.

The following relations between dimensional and non-dimensional quantities bold

$$x=a\bar{x},\ y=b\bar{y},\ w=2H\bar{w},\ \lambda=\frac{a}{b},\ t=t_0\bar{t},\ \varepsilon=(2H)\bar{\varepsilon},$$

$$P_x=\frac{E(2H)^3}{b^2}\overline{P_x},\ F=E(2H)^3\bar{F},\ q=\frac{E(2H)^4}{a^2b^2}\bar{q}.$$

The equations (1) are already transformed to the non-dimensional form (variables with bars are non-dimensional and they are omitted in (1)).

The following notation is used: $P_x(y, t)$- longitudinal load along Ox axis; 2H - plate thickness; a, b - plate dimensions; ε- damping; E - Young modulus; ν- Poisson coefficient (during calculations ν=0.3 has been taken); a low left corner of a plate serves for a co-ordinate system Oxy. The plate volume $G\in\{x,y\,|\,0\le x\le 1,\ 0\le y\le 1\},\ -H\le z\le H$. The Airy's function satisfies the following relations:

$$T_{xx}=\frac{\partial^2 F}{\partial y^2}-P_x,\ \ T_{yy}=\frac{\partial^2 F}{\partial x},\ \ T_{xy}=\frac{\partial^2 F}{\partial x\partial y}, \qquad (2)$$

where: T_{xx}, T_{yy} and T_{xy} are the stresses occurring in the middle plate surface, and $L(w, F)$ is the known non-linear operator.

Loosely clamped edge on unstretched in tangential plane ribs is used as the boundary condition:

$$w=\frac{\partial w}{\partial x}=F=\frac{\partial^2 F}{\partial x^2}=0 \quad \text{for} \quad x=0;1,$$

$$w=\frac{\partial w}{\partial y}=F=\frac{\partial^2 F}{\partial y^2}=0 \quad \text{for} \quad y=0;1. \qquad (3)$$

The initial conditions must satisfy the boundary conditions (3) and they read

$$w|_{t=0}=A\sin^2\pi x\sin^2\pi y,\ \ A-\text{const},$$

$$\left.\frac{\partial w}{\partial t}\right|_{t=0}=0. \qquad (4)$$

Results

The method of finite differences with the spatial approximation of $0(h^4)$ order is applied to reduce a dimension of the PDEs by a projection to ODEs. This approach is used to study standing and moving waves in space occupied by a thin plate subjected to one sided periodic longitudinal load action.

The boundary and initial conditions (3) and (4) are applied, and the mesh space $\overline{G}_h$ has been fully used without symmetry conditions. The spatial step of h=1/16 and the time step $\Delta t = 2 \cdot 10^{-4}$ have been taken.

We take P_0 as control parameter, and the other parameters are fixed: $\omega = 10.47$, $\lambda = \varepsilon = 1$, $\nu = 0.3$. The deflections w(0.5, 0.5, t), phase portraits, Poincaré maps and FFT are analysed for different values of P_0. Beginning with $P_0 \geq 16$, the analysed mechanical system is in a chaotic state, and the series of transitions between symmetric and non-symmetric oscillation forms are observed.

Further, our attention will be focused on interval $P_0 \in [18.95; 19.25]$. In this interval an oscillation jump is observed, which results in a change of spatial-time dynamical state configuration, and an occurrence of standing and moving waves. For P_0=19 the fundamental characteristics are reported in Figure 1. The plate oscillations before a jump are chaotic, which indicate the presented characteristics. The bending moving waves and maximal deflections for two self perpendicular symmetry axes are observed. The described behaviour, referred to as moving waves, reach a standing wave, and then a jump to another deflection level occurs, which is indicated by all characteristics given in Figure 1. In the plate centre the mentioned discontinuity effect is observed in the phase and modal portraits. Space phase and modal portraits and their projections into the planes show that chaos occurs (see broadband character of the power spectrum). But a collapse of the standing waves beginning from t=10.04; P_0=9 does not appear, as it has been observed for many other parameters. A new type of wave selforganization occurs, i.e. the standing wave. This wave type does not change in time and is practically stable over $t \in [10.04; 100]$. This observation leads to conclusion, that within a chaos state a new selforganized behaviour appears called two-dimensional standing wave (soliton).

For P_0>19.25 the solitons as well jump phenomena do not appear. The travelling waves and regular oscillations are observed instead for $t \in [48; 100]$.

Conclusions

The von Kármán form of equations governing dynamics of flexible plates harmonically excited are analysed. The system is very interesting from a point of view of non-linear dynamical systems because it possesses an infinite dimension. A special difference algorithm is used to reduce the von Kármán partial differential equations to a set of ordinary differential and algebraic equations.

This subtle numerical technique applied allowed to detect both chaotic and selforganized soliton states.

References

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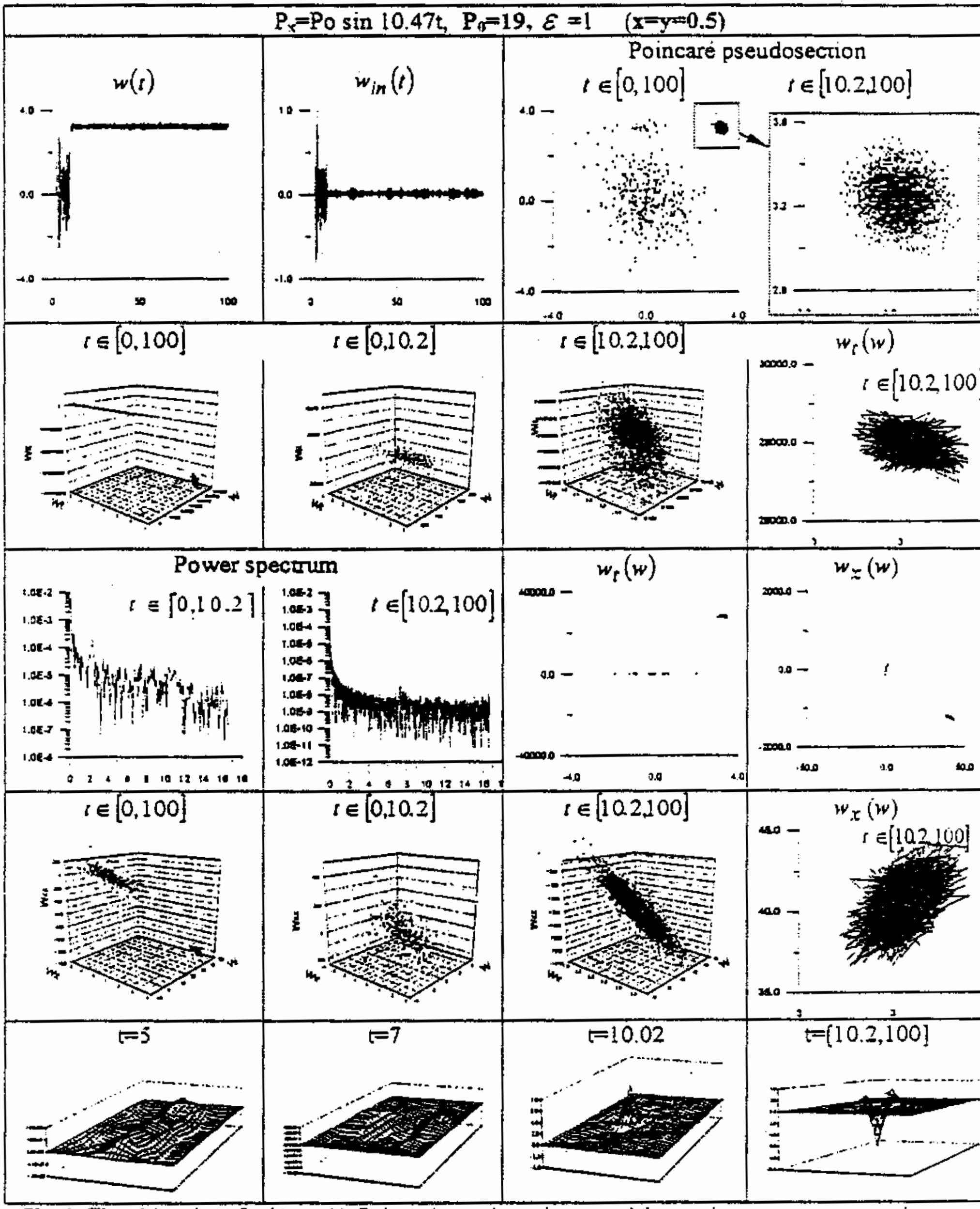

Fig. 1. Time histories of w(t), $w_{in}(t)$, Poincaré pseudosections, modal portraits, power spectra and spatial configurations for the given time moments and intervals ($w_{in}(t)=\int_0^1\int_0^1 w(x,y,t)dxdy$).