CONTROL OF STRUCTURES

C.-H. Lamarque[(1)], J.Awrejcewicz[(2)], G.Kacprzak[(3)]
Technical University of Łódź
[(1)] Department of Automatics and Biomechanics (K-16)
1/15 Stefanowskiego St., 90-924 Łódź, Poland
[(2)] Faculty of Civil Engineering
6 Politechniki St, 93-590 Łódź, Poland
[(3)] Ecole Nationale des Travaux Publics de l'Etat
1 rue Audin, F69 518 Vaulx-en-Velin, France
e-mail: claude.lamarque@entpe.fr, awrejcew@ck-sg.p.lodz.pl, kacprzak@poczta.onet.pl .

Introduction

In today's architecture an attempt to construct very high buildings is observed. The potential and usually unpredictable external loads caused by winds or earthquakes can be very dangerous to the inhabitants. Therefore, a very important question appears: how to prevent the inhabitants against the mentioned loads?

A commonly used protection relies on a so-called passive control of buildings. However, their advantages are rather limited because they can not be adapted to changes of the mentioned loads. Hence, the active control is used (see Roberti (1994) and Inman (1989)), which is the subject of this report.

Active control[1-2]

Let us consider a building modelled by a discrete system with n degrees of freedom, which is governed by the following second order differential equations (ODEs)

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = D_1 f(t) + B_1 u(t). \tag{1}$$

The $n \times 1$ vector y(t) represents displacements of the building floors in a previously established co-ordinate system. M, C *and* K are $n \times n$ order mass, damping and stiffness matrices correspondingly. The matrix B_1 represents the influence of external excitation $f(t)$. The introduced control $u(t)$ can be presented in the following form:

$$u(t) = F_1 y(t) + F_2 \dot{y}(t) + F_3 f(t), \tag{2}$$

where F_i , i=1,2,3, are the amplification matrices. Substituting (2) into (1) we get:

$$M\ddot{y}(t) + (C - B_1F_2)\dot{y}(t) + (K - B_1F_1)y(t) = (D_1 + B_1F_3)f(t) \tag{3}$$

It is seen from (3) that the control modifies parameters of the construction, i.e. its stiffness and damping in order to achieve a suitable response of a building to the excitation $f(t)$.

Algorithm of control

The equations (1) are transformed to the first order ODEs

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + Dz(t), \tag{4}$$

where

$$x(t) = [y(t), \dot{y}(t)]^T \tag{5}$$

is the $2n$ vector of a structure state, and

$$z(t) = [f(t), \dot{f}(t)]^T \tag{6}$$

is the $1 \times n$ vector of an external excitation. The $2n \times 2n$ matrix A represents the structure parameters, and it reads:

$$A = \begin{pmatrix} 0_n & I_n \\ -M^{-1}K & -M^{-1}C \end{pmatrix} \tag{7}$$

The $2n \times m$ matrix

$$B^T = \begin{pmatrix} 0 & M^{-1}B_1 \end{pmatrix} \tag{8}$$

defines control positions, whereas the $2n \times p$ matrix D

$$D^T = \begin{pmatrix} 0 & M^{-1}D_1 \end{pmatrix} \tag{9}$$

defines an influence of the external excitation on the system behaviour. Finally, 0 and I_n are the $n \times n$ zero and unit matrices, correspondingly.

Optimal control

In order to achieve the optimal control a sought vector u(t) is found through a minimisation condition applied to the following functional:

$$J = \int_{t_0}^{t_f} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt. \tag{10}$$

By a proper choice of the equilibrium matrices Q and R during the action of excitations $[t_0, t_f]$ one can find a suitable compromise between the effective control and the smallest energy dissipation.

Numerical results and conclusions

In order to investigate a control of an object, which consists of a building and a soil a two degrees of freedom approximation will be used. In order to control the building 2 (modelled as the material point) a force generator situated between two masses is applied (Fig.1).

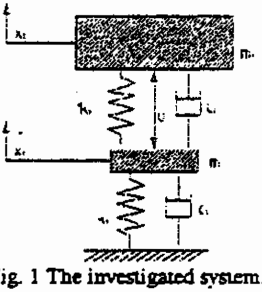

Fig. 1 The investigated system.

The system is governed by the following ODEs:

$$\begin{cases} m_1\ddot{x}_1(t) = -u(t) + k_1(f(t) - x_1(t)) + c_1(\dot{f}(t) - \dot{x}_1(t)), \\ m_2\ddot{x}_2(t) = u(t). \end{cases} \tag{11}$$

Let the control law will be governed by the equation:

$$u(t) = -R^{-1}B^T(K_x x(t) + K_z z(t)) = F_x x(t) + F_z z(t), \tag{12}$$

where the matrix K_x is the solution to a Riccati equation, and K_z is the solution to a Lyapunov type equation. They are defined by the following equations:

$$\begin{cases} K_x A + A^T K_x + Q - K_x B R^{-1} B^T K_x = -\dot{K}_x, & K_x(t_f) = \theta, \\ K_x D + K_z A_z + A^T K_z + S - K_x B R^{-1} B^T K_z = -\dot{K}_z, & K_z(t_f) = 0. \end{cases} \tag{13}$$

For t_f large enough (t_f □ ∝), the matrices A . B. Q and R can be treated as the constant ones, and therefore also the matrices K_x and K_z approach the tables with constant values. The mentioned situation corresponds to the stationary case. In order to testify the stationary property of the system (13) we apply the force $f = f_1 \cos(\omega t)\, e^{-(\lambda/2)t}$, and the following system parameters are fixed :

$m_1 = 1500\ kg,\ m_2 = 11000\ kg,\ k_1 = 975000\ N/m,\ k_2 = 530000\ N/m,$

$c_1 = 10800\ Ns/m,\ c_2 = 15000\ Ns/m,\ \lambda = 10,\ \varpi = 1,\ f_1 = 20N$

$r = 10^{-10.3},\ q_1 = 10,\ q_2 = 1,$

and the matrices Q and R are defined in the following way:

$$R = (r),\ Q = \begin{pmatrix} q_1 + q_2 & -q_2 & 0 & 0 \\ -q_2 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

In this case the control takes the form of

$$u(t) = f_1 x_1(t) + f_2 x_2(t) + f_3 \dot{x}_1(t) + f_4 \dot{x}_2(t) + f_5 z(t) + f_6 \dot{z}(t), \tag{15}$$

where the matrices F_x and F_z components are shown in Fig. 2.

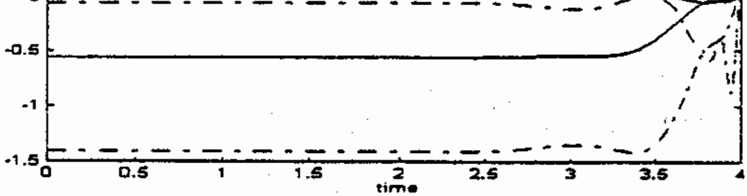

Fig. 2 Components f_i, i=1,..6 of the amplification matrices F_x and F_z

In order to compare the passive and active (with the control forces) control the value $u(t)$ has been computed using (15). It is obvious that acceleration plays the key point on the building inhabitants' behaviour.

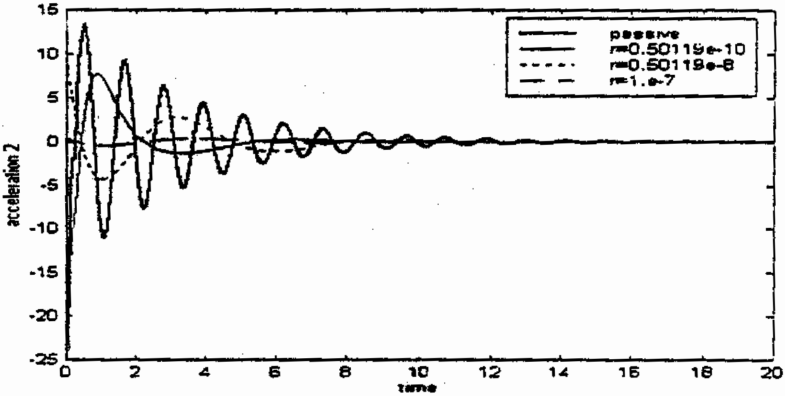

Fig. 3 Acceleration versus time for the passive and active control.

Figure 3 illustrates that the application of an active control is more suitable in comparison to classical passive control.

References

1. Roberti, V. (1994) Ph.D. Thesis, *Controle de structures: theories et applications*. l'Ecole Centrale de Lyon.
2. Inman, D. J. (1989) *Vibrations: with control, measurement and stability*, Prentice-Hall International, New Jersey.