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NUMERICAL STUDY OF SOME MODELS OF A SURFACE GRINDER FEED DRIVE SYSTEM INCLUDING FRICTION FORCE AND FORCED VIBRATIONS

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Abstract: We present a special class of mechanical systems with friction modelling surface grinder feed drive systems. We recall the general frame for the study of such models. Results of existence and uniqueness are given. Then numerical results obtained via different analytical expressions of the friction force are presented.

1. Introduction

An analysis of the vibrations of material system relies the most often on the studying of the solutions of a differential equation of vibrations. Generally, the difficult thing is to find a mathematical (analytical) expression of the solution leading to the exact solution at least in a numerical point of view (i.e. leading to an approximation with an arbitrary high accuracy if exact mathematical expression can not be obtained). In many cases such an expression is unknown and numerical schemes have to be used. Generally, the following groups of methods used in order to analyse the vibrations [8] can be distinguished: methods based on expressions of the exact solutions, the topological methods, approached trace and trace-analytical methods, approached analytical methods, analogical methods, experimental methods.

That analysis can be realized when the exact solution of the system of differential equations describing the motion is explicitly known. Those exact solutions are well known in the case of systems of linear ordinary differential equations. In the case of numerical methods, approximated solutions of the system of differential equations with initial conditions is constructed recursively, point after point. Such processes provides the solution with a high accuracy but does not permit a priori qualitative or quantitative study of the effect of the different parameters: only a lot of simulations lead to conclusions. Nevertheless in many cases, numerical methods are the only ones that can be used. Even in the case of non smooth systems (including friction or impacts), it is difficult to generalize known methods or to build appropriate methods [6], [7]. Even if sophisticated methods are available to solve the models based on differential inclusions, identification of the paramcters of the system from experiments could be a difficult task. This is why, models have to be carefully studied in order to keep only the needed and simplest terms in the models. Here, we first present the physical models in the Section 2. Exact or approximate expressions of the friction force are proposed. The exact model does not possess exact analytical solutions. Numerical methods are needed. The Section 3 is devoted to existence and uniqueness results. The mathematical frame is recalled and can be used in the Section 4 in order to build numerical schemes. Then in the Section 5, numerical results are provided. Several models are tested: especially we consider a simplified linear model of the friction force and we compare the obtained results of the exact model and the approximated model.

2. The physical models

A physical model of surface grinder feed-drive system takes into consideration. The table of the machine tool is moved along direct contact cast iron slideways and powered by a fluid drive. This motion is realised for small linear velocity of the table, which correspond with the conditions of creep-feed grinding. The physical model [1], [2] of that system is presented in the Figure 1.

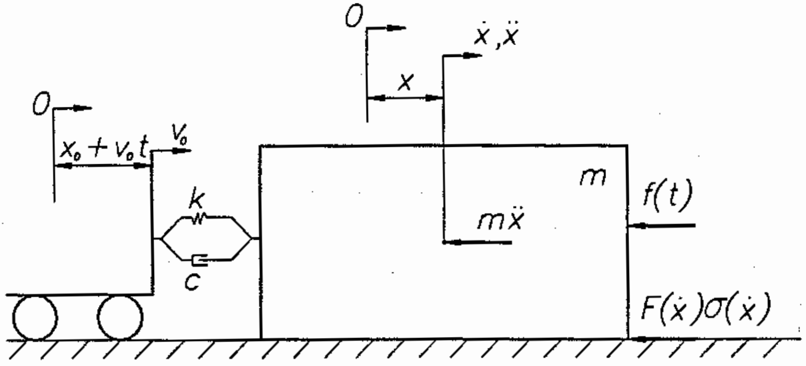

Fig. 1. Physical model of surface grinder feed-drive system.

An experimental characteristic curve of sliding friction is presented in the Figure 2.

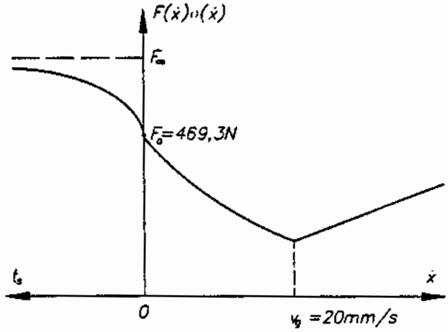

Fig. 2. Experimental characteristic of sliding force friction.

The physical model of the surface grinder feed-drive system is described with mathematical models. We propose two mathematical models for the present analysis. The first one is written in the form:

$$m\ddot{x} + f(x,\dot{x},t) + F(\dot{x})\sigma(\dot{x}) + f(t) \ni 0, \tag{1}$$

and the second one in the form:

$$m\ddot{x} + f(x,\dot{x},t) + \alpha x^3 + F(\dot{x})\sigma(\dot{x}) + f(t) \ni 0, \tag{2}$$

where the smooth linear part of the model is expressed by:

$f(x,\dot{x},t) = c\dot{x} + kx$,

and the friction force is given by the formula: $F(\dot{x})\sigma(\dot{x})$.

This formula is first detailed in the form:

1) - $F(\dot{x})\sigma(\dot{x}) = \dfrac{F_0}{1 + A_0 B\dot{x}}\operatorname{sgn}(\dot{x})$ circumscribing decreasing part of friction function $F(\dot{x})\sigma(\dot{x})$

on [0, v_g], with constants $A_0 = 0.23$, $B = 1.033$, and maximal velocity v_g. Then a linear approximation could be given for the decreasing part of friction function $F(\dot{x})\sigma(\dot{x})$ on [0, v_g]:

2) - $F(\dot{x})\sigma(\dot{x}) = F_0(1 - A_0 B\dot{x})\operatorname{sgn}(\dot{x})$.

The external solicitation is an harmonic force given by: $f(t) = f\cos(\nu t + \varphi)$.

c is the damping coefficient of power unit, k is the stiffness of power unit.

αx^3 provides a non linear model of the Duffing type.

3. Existence and uniqueness results

We observe that all the models introduced in section 2, can be subsumed under one form which is conveniently described in the language of maximal monotone operators [3], [5].

In general case we can write: $F(z) = G(z) + A(z)$ (3)

with $G(z)$ Lipschitz continuous function so that:

$$\begin{cases} z \le 0 & G(z) = F(-z) + 2A_0 \\ z \ge 0 & G(z) = F(z) \end{cases} \tag{4}$$

and $A(z) = \begin{cases} 0 & \text{if } z \succ 0 \\ [-2A_0, 0] & \text{if } z = 0 \\ -2A_0 & \text{if } z \prec 0 \end{cases}$ is the maximal monotone graph.

So, problem (1) is equivalent to: $m\ddot{x} + f(x, \dot{x}, t) + G(\dot{x}) + A(\dot{x}) \ni 0$ (5)

We can write:

$$\begin{cases} m\ddot{x}_2 + f(x_2, x_1, t) + G(x_1) + A(x_1) \ni 0 \\ \dot{x}_2 = x_1 \end{cases} \tag{6}$$

i.e. $\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} f(x_2, x_1, t) + G(x_1) \\ -x_1 \end{pmatrix} + \begin{pmatrix} A(x_1) \\ 0 \end{pmatrix} \ni \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (7)

So: $\begin{cases} \dot{x}_1 + \dfrac{1}{m}[f(x_2, x_1, t) + G(x_1) + A(x_1)] \ni 0 \\ \dot{x}_2 - x_1 \ni 0 \end{cases}$ (8)

it means: $\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and

$$\mathbf{X} + \mathbf{F}(\mathbf{X}, t) + \mathbf{A}(\mathbf{X}) \ni \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{9}$$

with $\mathbf{F} : (\mathbf{X}, t) \to \begin{pmatrix} \dfrac{1}{m}[f(x_2, x_1, t) + G(x_1)] \\ -x_1 \end{pmatrix}$ (10)

4. The numerical schemes

The mathematical tools introduced in the Section 3 permit us to build numerical schemes. Let us first write the model in the form:

$$\begin{cases} m\ddot{x}_2 + f(x_2, x_1, t) + G(x_1) + A(x_1) \\ \dot{x}_2 = x_1 \end{cases} \tag{11}$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{m} f(x_2, x_1, t) + \frac{1}{m} G(x_1) + \frac{1}{m} A(x_1) \\ -x_1 \end{pmatrix} \ni \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

These equations are denoted:

$$\dot{\mathbf{X}} + \mathbf{F}(\mathbf{X}, t) + \mathbf{A}(\mathbf{X}) \ni 0 \tag{13}$$

and the first numerical scheme is obtained with:

$$\frac{\mathbf{X}_{n+1}-\mathbf{X}_n}{\Delta t}+\mathbf{F}(\mathbf{X}_n,t_n)+\mathbf{A}(\mathbf{X})\ni\begin{pmatrix}0\\0\end{pmatrix} \tag{14}$$

so that: $\mathbf{X}_{n+1}=(\mathbf{I}+\Delta t\mathbf{A})^{-1}[\mathbf{X}_n-\Delta t\mathbf{F}(\mathbf{X}_n,t_n)]$ (15)

with: $\mathbf{F}(\mathbf{X},t)=\begin{pmatrix}F_1(\mathbf{X},t)\\F_2(\mathbf{X},t)\end{pmatrix}$ and the maximal monotone operator $\mathbf{A}(\mathbf{X})=\begin{bmatrix}A(x_1)\\0\end{bmatrix}$

The calculation of $(\mathbf{I}+\Delta t\mathbf{A})^{-1}(\mathbf{Z})$ is easy according to the figure 4 a and 4 b:

Then we have:

$$(\mathbf{I}+\Delta t\mathbf{A})\begin{pmatrix}y_1\\y_2\end{pmatrix}=\begin{pmatrix}z_1\\z_2\end{pmatrix}\Leftrightarrow\begin{cases}y_2=z_2\\y_1=(\mathbf{Id}_{\mathbf{R}}+\Delta t\mathbf{A})^{-1}(z_1)\end{cases} \tag{16}$$

Hence: $y_1=(\mathbf{I}+\Delta t\mathbf{A})^{-1}(z_1)$ (17)

a) b)

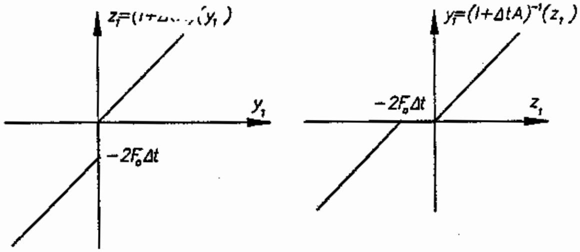

Figure 4 a: The graph (I + Δt A); b: Inversion of the graph (I + Δt A).

Then, introducing $\dot{x}=v$, the numerical scheme of Euler (with implicit term only inside the maximal monotone graph: semi-implicit Euler scheme) becomes:

$$\begin{cases}m\dfrac{v_{n+1}-v_n}{\Delta t}+f(x_n,v_n,t_n)+G(v_n)+A(v_{n+1})\ni 0\\ \dfrac{x_{n+1}-x_n}{\Delta t}=v_n\end{cases} \tag{18}$$

$$\begin{cases}x_{n+1}=x_n+\Delta t v_n \qquad \text{(explicit Euler)}\\ \left(\mathbf{Id}_{\mathbf{R}}+\dfrac{\Delta t}{m}A\right)(v_{n+1})=v_n-\dfrac{\Delta t}{m}f(x_n,v_n,t_n)-\dfrac{\Delta t}{m}G(v_n)\end{cases} \tag{19}$$

$$x_{n+1}=x_n+\Delta t v_n \tag{20}$$

$$\text{aux}=v_n-\frac{\Delta t}{m}f(x_n,v_n,t_n)-\frac{\Delta t}{m}G(v_n) \tag{21}$$

$$\text{if aux} \le \frac{-2F_0\Delta t}{m} \text{ then } v_{n+1} = \text{aux} + \frac{2F_0\Delta t}{m} \tag{22}$$

$$\text{if } \frac{-2F_0\Delta t}{m} \prec \text{aux} \le 0 \text{ then } v_{n+1} = 0 \tag{23}$$

$$\text{if aux} \succ 0 \text{ then } v_{n+1} = \text{aux} \tag{24}$$

For this numerical scheme we are able to prove the convergence [4] of the discrete functions defined by v_n, x_n to the exact solution with order 1 when Δt tends to 0. The main advantage of this scheme is the following one:

- it is not necessary to approximate the time of "phase change" (stick, slip, etc..)
- when $\Delta t \to 0$, the approximate solution converge to the exact one together with approximated "phase change" times.

Another numerical scheme can be built using classical approximations (from mathematical or physical point of view). It is given by second order approximation of the acceleration by

$$w_{n+1} = \frac{x_{n+1} - x_{n-1}}{2\Delta t} \tag{25}$$

$$m\left[2\frac{w_{n-1}}{\Delta t} + 2\frac{x_{n-1} - x_n}{\Delta t^2}\right] + f\left(x_n, \frac{x_n - x_{n-1}}{\Delta t}, t_n\right) + G\left(\frac{x_n - x_{n-1}}{\Delta t}\right) + A(w_{n+1}) \ni 0 \tag{26}$$

that provides:

$$\left(Id + \frac{\Delta t}{2m}A\right)(w_{n+1}) = -\frac{x_{n-1} - x_n}{m\Delta t} - \frac{\Delta t}{2m} f\left(x_n, \frac{x_n - x_{n-1}}{\Delta t}, t_n\right) - \frac{\Delta t}{2m} G\left(\frac{x_n - x_{n-1}}{\Delta t}\right) \tag{27}$$

so:

$$\text{if aux1} = -\frac{x_{n-1} - x_n}{m\Delta t} - \frac{\Delta t}{2m} f\left(x_n, \frac{x_n - x_{n+1}}{\Delta t}, t_n\right) - \frac{\Delta t}{2m} G\left(\frac{x_n - x_{n-1}}{\Delta t}\right) \tag{28}$$

we have:

$$\text{if aux1} \le \frac{-F_0\Delta t}{m} \text{ then } \begin{cases} w_{n+1} = \text{aux1} + \dfrac{F_0\Delta t}{m} \\ x_{n+1} = x_{n-1} + 2\Delta t w_{n+1} \end{cases} \tag{29}$$

$$\text{if } \frac{-F_0\Delta t}{m} \prec \text{aux1} \le 0 \text{ then } \begin{cases} w_{n+1} = 0 \\ x_{n+1} = x_{n-1} \end{cases} \tag{30}$$

$$\text{if aux1} \succ 0 \text{ then } \begin{cases} w_{n+1} = \text{aux1} \\ x_{n+1} = x_{n-1} + 2\Delta t\,\text{aux1} \end{cases} \tag{31}$$

We consider the second order initial problem (1) in the form (5) and we build the numerical scheme:

h: time step x_0, $\dot{x}_0 = \frac{x_1 - x_0}{h}$ given

$$m\frac{x_{n+1}-2x_n+x_{n-1}}{h^2}+f\left(x_n,\frac{x_n-x_{n-1}}{h},t_n\right)+G\left(\frac{x_n-x_{n-1}}{h}\right)+A\left(\frac{x_{n+1}-x_{n-1}}{2h}\right)\ni 0 \quad (32)$$

with explicit: $\dot{x}(t_n)\cong\frac{x(t_n)-x(t_{n-1})}{h}$ if $t_n=t_{n-1}+h$ (33)

So we have with $v_{n+1}=\frac{x_{n+1}-x_{n-1}}{2h}$ and

$$2m\left\{\frac{x_{n+1}-x_{n-1}}{2h}+\frac{2(x_{n-1}-x_n)}{2h}\right\}+hf\left(x_n,\frac{x_n-x_{n-1}}{h},t\right)+hG\left(\frac{x_n-x_{n-1}}{h}\right)+hA\left(\frac{x_{n+1}-x_{n-1}}{2h}\right)\ni 0. \quad (34)$$

We solve

$$(2m+hA)(v_{n+1})+2m\left[\frac{2(x_{n-1}-x_n)}{2h}\right]+hf\left(x_n,\frac{x_n-x_{n-1}}{h},t\right)+hG\left(\frac{x_n-x_{n-1}}{h}\right)\ni 0, \quad (35)$$

It is equal to $v_{n+1}=(2m+hA)^{-1}(s_n)$ (36)

We obtain the detailed solution:

$$\begin{aligned}
&\text{if } z\prec -2A_0h && \text{then } (2m+hA)^{-1}(z)=\frac{1}{2m}(z+2A_0h);\\
&\text{if } z\in[-2A_0h,0] && \text{then } (2m+hA)^{-1}(z)=0; \quad (37)\\
&\text{if } z\succ 0 && \text{then } (2m+hA)^{-1}(z)=\frac{1}{2m}z.
\end{aligned}$$

In the case of this second numerical scheme, no detailed proof have been made for convergence. But we expect this scheme to be convergent and to have again order 1 because of its similarity with the previous semi-implicit Euler scheme.

5. The numerical simulations

The numerical simulations were performed for two models describing by the equations (1) and (2). We have obtained two numerical schemes for each model (1) and (2) with friction formula: exact $\frac{A_0}{1+A_0B\dot{x}}$ and approximate $A_0(1-A_0B\dot{x})$. Each equation was solved in two ways by using the explicit and implicit scheme. Examples of the curves obtained are presented in the Figures 5 to 8. The Figure 5 shows the curves of displacement and velocity for exact formula friction with the first scheme (semi-implicit Euler scheme). We can denote t_0=15s for first value 0 of the velocity. In the Figure 6 we can see stick – slip damping vibrations for exact formula friction with semi implicit Euler scheme. The Figure 7 presents the results of the simulations for approximate formula friction with second scheme. It is impossible to find exact analytical solution for the model described with the equation (2) including αx^3 – Duffing function. The solution of numerical scheme shown in the Figure 5 is according to the analytical solution of problem (1), so in first step, we can apply this numerical scheme to solve problem (2). The result of the simulation for approximate formula friction with implicit is presented in the Figure 8.

6. Conclusions

We examined two types of models including exact and approximate formula friction. We used classical results in order to prove existence and uniqueness of solutions to this class of problems.

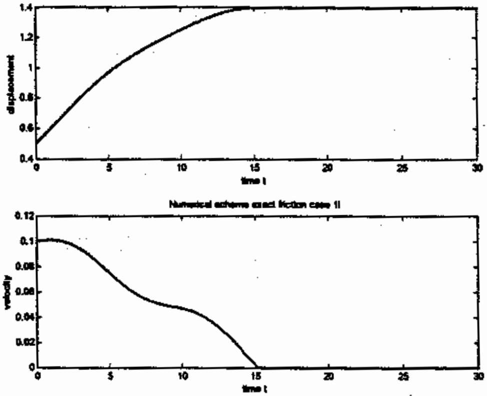

Fig. 5. The curves of displacement and velocity for exact formula friction with explicit by c= 0.01, k=1, f=1, v=0.66, φ=0, x_0=0.5, v_0=0.1, t_0=0, n_{final}=10000, t_{max}=30

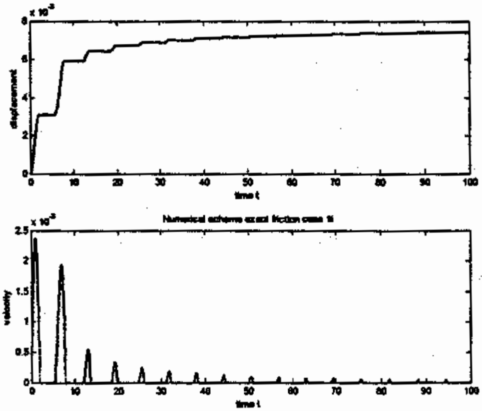

Fig. 6. The curves of displacement and velocity for exact formula friction with explicit by c= 0.1, k=1, f=1, v=1, φ=0, x_0=0, v_0=0, t_0= 0, n_{final}=10000, t_{max}=100

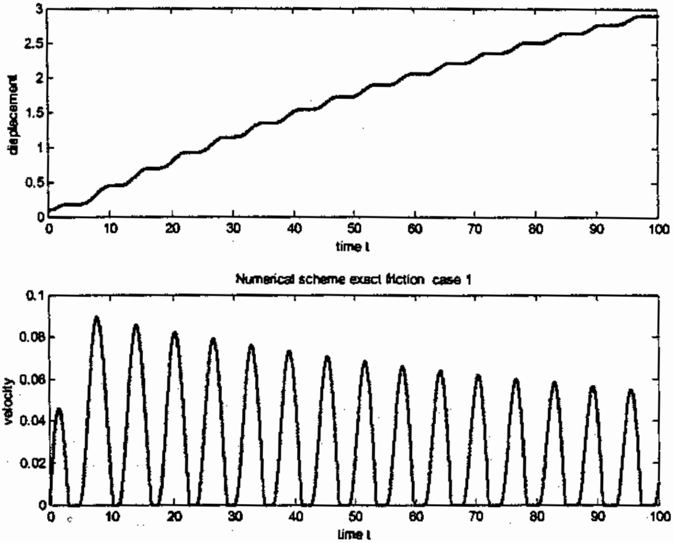

Fig. 7. The curves of displacement and velocity for approximate formula friction with implicit by

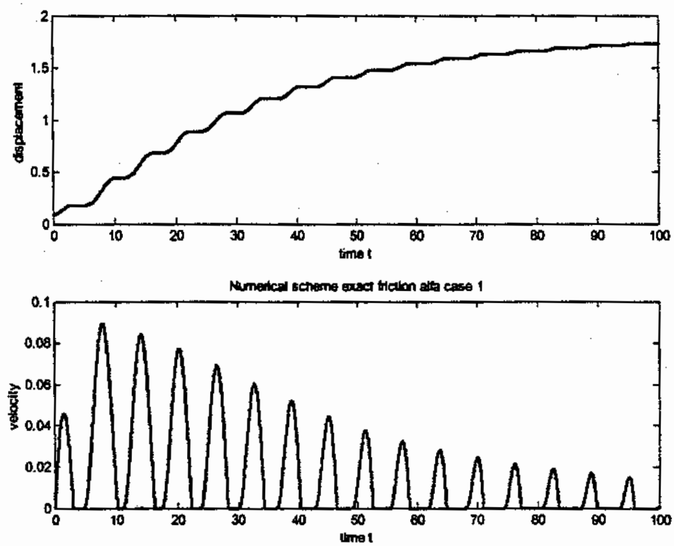

Fig. 8. The curves of displacement and velocity for approximate formula friction with implicit by $c= 0.1$, $k=1$, $f=10$, $\nu=1$, $\varphi=0$, $\alpha=1$, $x_0=0.1$, $v_0=0$, $t_0= 0$, $n_{final}=10000$, $t_{max}=100$

We have built numerical schemes with explicit and implicit which approximate these solutions. For the usual physical value of velocities occurring in the surface grinder process, we can see from numerical results that the linear approximation could provide enough accuracy. So. it will be interested toprepare the calculation of exact analytical solutions of model (1) and linear approximation of friction force. In the case of the model (2), the numerical solutions are valid, but only analytical approximations of the solutions could be obtained with different value of α, and weak non linearities.

The comparison between analytical approximations and exact numerical results will decide if one can keep only analytical resolution of the problem.

7. References

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