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# MINIMISATION OF VIBRATIONS ON A PART OF A PLATE

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*Abstract.* A linear theory describing a behaviour of simply supported rectangular plate with point oscillating force disturbance is used. A property of superposition is used to get a response of plate with two forces. One of these forces is assumed stable localised on the plate's surface and independent, the other one can be relocated in order to minimise vibration's amplitude at the other half of plate's surface in some bandwidth of frequency. Results for some cases of localisation of the independent disturbance are presented.

## 1. Introduction

In this paper we are going to find a possibility of minimisation of vibration's amplitude on some area of a thin rectangular plate. We assume a linear model of simply supported thin rectangular plate as described in [1]. A system response to a two-dimensional forcing function is governed by the following equation:

$$E \cdot I \cdot \left( \frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho \cdot h \cdot \frac{\partial^2 w}{\partial t^2} = -F(x,y) \cdot e^{j\omega t}, \qquad (1)$$

where the plate inertial moment is

$$I = \frac{h^3}{12 \cdot (1 - \nu^2)}; \qquad (2)$$

and a $F(x,y)$ has the units of pressure. Assuming that the plate response can by written as a sum of modes of the free response of the plate vibrating at the forcing frequency we have

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\cdot\sin k_m x\cdot\sin k_n y\cdot e^{j\omega t};\tag{3}$$

where the coefficients $k_m$ and $k_n$ are given by

$$k_m=\frac{m\cdot\pi}{a};\ \text{m=1,2,3,...}\tag{4}$$

$$k_n=\frac{n\cdot\pi}{b};\ \text{n=1,2,3,...}\tag{5}$$

Modal frequencies can by written as

$$\omega_{mn}=\left(\frac{E\cdot I}{\rho\cdot h}\right)^{\frac{1}{2}}\cdot\left[\left(\frac{m\cdot\pi}{a}\right)^2\cdot\left(\frac{n\cdot\pi}{b}\right)^2\right];\tag{6}$$

On substituting the assumed response into equation (1) and using the orthogonality property of the plate mode shapes in the x and y directions we obtain the amplitudes of the plate response of the form

$$W_{mn}=\frac{4}{M\cdot\left(\omega^2-\omega_{mn}^2\right)}\cdot\int_0^a\int_0^b F(x,y)\cdot\sin k_m x\cdot\sin k_n y\cdot dy\cdot dx,\tag{7}$$

where $M$ is the total mass of the plate. If the input forcing function is a point force then Dirac delta function can be used, i.e.

$$f(x,t)=F\cdot\delta(x-x_i)\cdot\delta(y-y_i)\cdot e^{j\omega t}.\tag{8}$$

In this case the integrand in equation (7) has a value only at $x_i$ and $y_i$, and hence amplitudes are given

$$W_{mn}=\frac{4\cdot F\cdot\sin k_m x_i\cdot\sin k_n y_i}{M\left(\omega^2-\omega_{mn}^2\right)};\tag{9}$$

Now, subsi ıting equation (8) into (3) and neglecting an imaginary part we can obtain a value of transverse ( placement of every point of plate surface.

**2. Minimi :ion of vibration amplitudes**

We deal with two point forces acting on the plate. The first .e is an undesirable disturbance with stable location and the second one is vibrating with the same frequency and amplitude as the first one but its location point can be changed. To obtain a response of the plate to these two points forces input a superposition property of a linear model has been used. We have calculated a response of the plate to a first force input and then a response to a second one for t=0. As a total response of the plate we took an algebraic sum of both of these responses. If frequency of input is between $\omega_{1,2}$ and $\omega_{2,1}$ we have found the interesting shape of plate's vibration for some localisation of point inputs (Figure 1). As can be seen from this figure we have obtained a minimisation of vibrations amplitude on ~30% of plate surface. Note that both of inputs are at the opposite site of plate then that on which

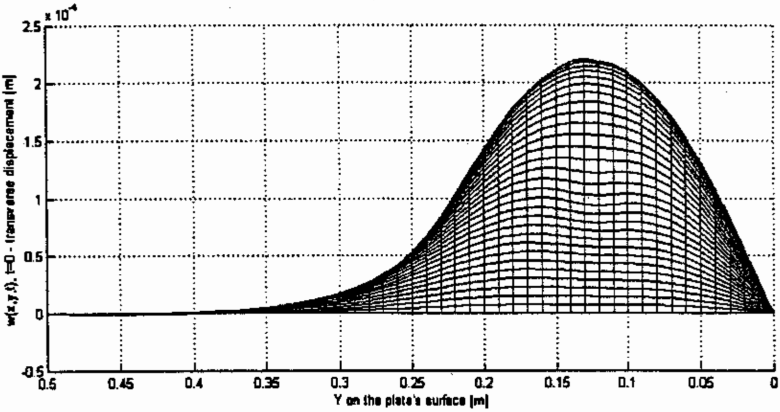


Figure 1. Desired shape of plate's vibration.

the minimisation is taking place. We found it interesting how the second force should be relocated to keep this shape of plate's vibration during increasing or decreasing input frequency.

### 3. Results

The dimensions and physical properties of the plate material are presented in Table 1.

Dimensions and physical properties. Table 1

| | |
|---|---|
| Width (a) [mm] | 260 |
| Length (b) [mm] | 500 |
| Thickness (h) [mm] | 2.26 |
| Density ($\rho$) [kg/m$^3$] | 2700 |
| Young's modulus (E) [GPa] | 70 |
| Poisson ratio (ν) | 0.30 |

For calculations a MATLAB procedure was used to take advantage of it's speed in matrix operations. We have found the required locations of the second input for five different locations of the first input (Table 3) in frequency bandwidth between $\omega_{1,2}$ and $\omega_{2,1}$.

Modal frequencies of the plate. Table 2

| Mode number: | 1 | 2 |
|---|---|---|
| 1 | 102.80 Hz | **168.44 Hz** |
| 2 | **345.55 Hz** | 411.19 Hz |

Amplitudes ( $F_1$ and $F_2$ ) of both inputs are chosen as 20 N/m$^2$.

Locations of first input. Table 3

| Nr: | Location of first input [m]: | |
|---|---|---|
| | $x_1$ | $y_1$ |
| 1. | 0.075 | 0.125 |
| 2. | 0.103 | 0.125 |
| 3. | 0.130 | 0.125 |
| 4. | 0.157 | 0.125 |
| 5. | 0.185 | 0.125 |

In Figure 2 locations of inputs in 170 – 266 Hz frequency bandwidth where minimisation effect exist for this plate are presented. With increasing the frequency the second input should be moved closer to

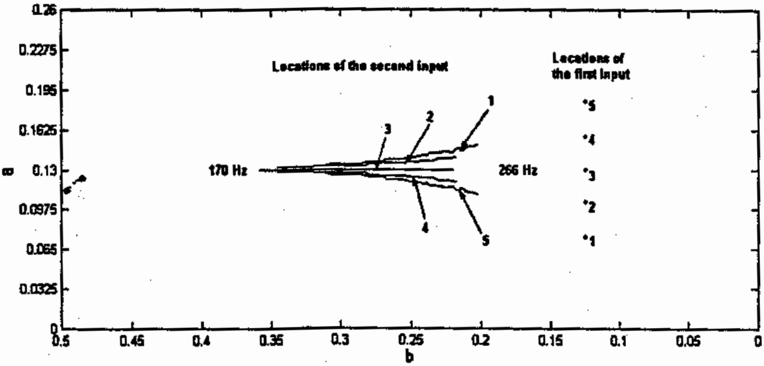


Figure 2. Paths of moving of the second input.

position of the first one and a minimisation takes place on even grater part of plate's area.

Shape of the second input's paths depends strongly on location of the first one and geometry of the plate. Note that those paths are symmetrical on longer symmetry axis of the plate.

## 4. Conclusions

In this paper we have showed a possibility of minimisation of vibration's amplitude on a part of thin rectangular plate's area being under action of two point force inputs. A linear theory has been taken to describe response of the plate and find location of inputs to obtain minimisation of transverse displacement of the plate. The some effect of minimisation takes place in the upper neighbourhood of other (1,n) but the linear theory can be applied only to low modal frequencies, so we have restricted ourselves to (1,2) modal frequency. A shape of the second input's paths depends on geometry of the plate and location of the first one. It is possible to find an appropriate arithmetic relation between location of the second input on the path and the input frequency and use it for an active control of vibrations.


## 5. Acknowledgement

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## 6. References


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