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PERTURBATION METHOD FOR CALCULATION OF QUADRILATERAL MEMBRANES

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A construction of an asymptotical solution to a physical problem can be separated into three parts. First, one has to choose small (large) parameters. Then, one of the well developed asymptotical approaches can be applied. Finally, an estimation of validity of the obtained results should be added [1]. Observe that although the first step becomes the most important it is formalized at the very least. Let us begin to find an asymptotic of nontrivial solution. One has to guess initially a kind of sought asymptotics. Observe that this initial guess can not be formalized in practice. Many researchers mention the following properties helping for the initial guess: analogy, experience, physical feeling, intuition [2].

Very often the only method to estimate a validity of a choice of the zero order approximation can be done in a posteriori way by comparing the approximate results with those obtained using either numerical and or experimental results. In addition, it can happen that the successive approximations do not lead to a solution improvement. In principle, it can even lead to conclusion that the applied asymptotic approaches are not appropriate [3]. From this point of view seems to be interesting to consider oscillations of parallelepipedical and trapezoid membranes. The mentioned problems have been solved using various approaches including also perturbation methods. A promising hope to solve the problem in full give numerical results obtained in reference [3]. In the mentioned monograph the recurrent perturbation formulas are given with respect to "small" parameter $\lambda = tg\Delta t$ (see Figures 1a, 2a). It has been shown, among others, that one needs 16 approximations to get a convergence of the

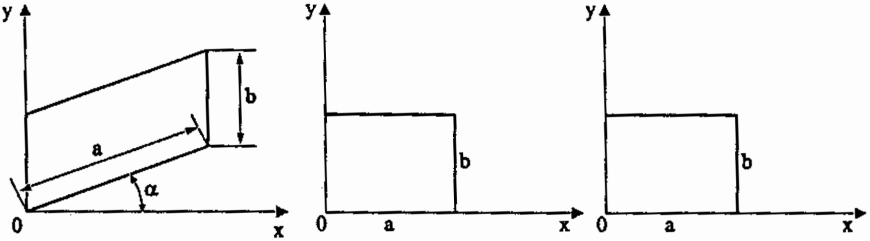

Figure 1. Parallelpiped membrane and different methods of selection of zero order approximation

process for $\lambda \leq 0.21$. Increase λ leads to greater of number of approximations. In fact, the observed property does not allow to get a solution using perturbation technique for large values of λ [3].

Observe that a process of detection higher approximations which not only does not improve a solution but sometimes even leads to uncorrect results is typical for the asymptotic series. However in the discussed case the problem is caused by other reasons which we are going to illustrate and discuss.

The membrane shown in Figure 1b is taken in monograph [3] as the initial one to proceed with an asymptotical analysis. The fundamental frequency of the membrane can be estimated via the following formula:

$$\omega^2 = \frac{\pi^2 a^2 b^2}{a^2 + b^2} \tag{1}$$

In Table 1 the computational results due to the formulae (1) are given. It is seen that the error is large and it increases with increase of λ.

Table 1.

λ	Formula (1)	Error %	Formula (2)	Error %	Numerical solution [3]
0.5	19.74	7	22.21	4	21.32
0.7	19.74	14	24.57	7	22.93
1.5	19.74	46	41.94	13	36.89
2.0	19.74	62	59.22	12	52.63

Let us take another membrane shown in Figure 1c which serves for zero order approximation in the asymptotical procedure. In this case we have:

$$\omega^2 = \frac{\pi^2 a^2 b^2 \cos^2\alpha}{a^2\cos^2\alpha + b^2} \tag{2}$$

The errors related to the formula (2) are essentially smaller in comparison to those generated by formula (1) (see Table 1).

A similar observation holds also with a trapezoidal membrane (see Figure 2a). Analogously, let us take the membranes shown in Figure 2b and Figure 2b as the initial ones. One obtains the following frequence estimation:

$$\omega^2 = \frac{\pi^2 a^2 b^2}{b^2 + 0.25a^2}, \tag{3}$$

and

$$\omega^2 = \frac{\pi^2 a^2 b^2}{b^2 + 0.25a^2\cos^2\alpha}. \tag{4}$$

respectively.

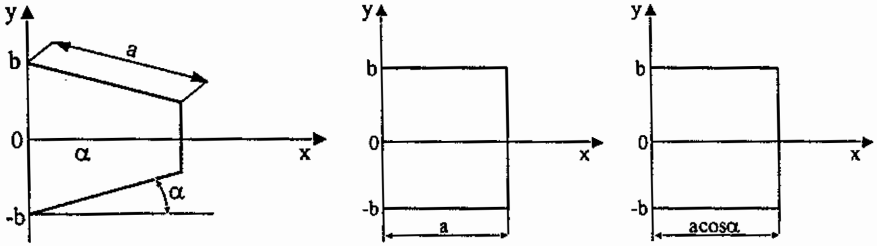

Figure 2. Trapezoid membrane and different methods of selection of zero order approximation

The computational results derived from the formula (3) and (4) are included in Table 2. The obtained results clearly show an advantage of zero order approximation governed by the membrane shown in Figure 2c.

The included results and discussion lead to the following conclusion. It can happen (in fact, very often) that an improvement of an asymptotical solution mainly depends on a proper choice of initial approximation rather than on tedious procedure related to calculation of higher order approximations.

Table 2.

λ	Formula (3)	Error %	Formula (4)	Error %	Numerical solution [3]
0.25	12.3	5	12.9	0.8	13
0.5	12.3	15	14.81	2	14.5
0.75	12.3	30	17.89	2	17.5
1	12.3	50	22.21	9	24.67

Reference

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