INFLUENCE OF A PRE-CRITICAL BENDING STATE ON THE STABILITY OF A STRINGER SHELL

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A comparison between the experimental and numerical results [1, 2] shows a difference in the obtained critical longitudinal load for the ribbed shells.

The aim of this report is to show that a reason lies on a traditional assumption on the bending pre-critical state.

The tensometry measurement results [2], for the reinforced shells show, that pre-critical state is essentially momentous. In addition, a influence of the momentum factors onto a stress-strain state increases with a simultaneous decrease of a relative lengths of tested particles and with reinforcement stiffness increase.

In this work an non-linear stability analysis is carried out by taking into account a bending critical state and a support discreteness.

a. Any pre-critical solution is composed from both an axially symmetric part being responsible for the membrane state and a boundary conditions as well as from the part corresponding to the reinforcement discreteness.

The following equation governing an axially symmetric state can be obtained from the non-linear equilibrium equations (see [4]):

$$T_{1,\xi}^{(0)} = 0; \quad M_{1,\xi\xi}^{(K)} + T_2^{(K)} R - (\Psi_1^{(K)} T_1^{(0)})_\xi = 0; \tag{1}$$

$$\Psi_1 = -\frac{W_\xi^{(K)}}{R}; \quad \varepsilon_1^{(K)} = \frac{U_\xi^{(K)}}{R} + \frac{1}{2}\left(\frac{W_\xi^{(K)}}{R}\right)^2; \tag{2}$$

$$\varepsilon_2^{(K)} = -\frac{W^{(K)}}{R}; \quad \chi_1^{(K)} = -\frac{W_{1,\xi\xi}^{(K)}}{R^2};$$

where index (0) is related to a membrane state, and index (K) to a state of a boundary effect type.

The relations of material, physical and geometrical properties are reported in reference [2].

A solution to equations (1), (2) is being sought in the following from

$$\begin{aligned} W_a &= W_1^{(K)} + W_2^{(K)} + W^{(0)}, \\ W_1^{(K)} &= e^{-\beta_2 \xi}(A_1 \sin \beta_1 \xi + A_2 \cos \beta_1 \xi), \end{aligned} \tag{3}$$

$$W_2^{(K)} = e^{\beta_2(\xi-l)}[A_3 \sin\beta_1(\xi-l) + A_4 \cos\beta_1(\xi-l)],$$
$$W^{(0)} = -(TB_{12}R)/(B_{11}B_{22} - B_{12}^2), \tag{4}$$

where: $\xi = \frac{\chi}{R}, \quad l = \frac{L}{R},$

$$\beta_i = \sqrt{\frac{R[2\sqrt{(D_{11}B_{11} - K_{11}^2)(B_{11}B_{22} - B_{11}^2)} + (-1)^i (TB_{11}R - 2K_{11}B_{12})]}{4(D_{11}B_{11} - K_{11}^2)}}.$$

The analogical form has also an expression for u_0. We point out, that for

$$\exp(-\beta_2 l)\left(\frac{\beta_1^2 + \beta_2^2}{\beta_1\beta_2}\right) << 1$$

while satisfying the boundary conditions there is no need to include a self-influence of the boundary effects.

The solution component related to a discreteness can be obtained using the averaging procedure [3] and it reads

$$\mathrm{W}^{(g)} = -\frac{4\pi}{a^2N^5}(\gamma_1 u_0 + \alpha_1 W_{0,\xi})_{,\xi\xi\xi} F_4(\eta), \tag{5}$$

where: $a^2 = \frac{h^2}{12R^2}, \quad \alpha_1 = \frac{E_c JN}{2\pi BR^3}, \quad \gamma_1 = \frac{E_c SN}{2\pi BR^2},$

$$F_4(\eta) = \frac{\pi^4}{90} - \frac{\pi^2N^2\eta^2}{12} + \frac{\pi N^3\eta^3}{12} - \frac{N^4\eta^4}{48},$$

N – number of stringers; E_c – elasticity modulus of a stringer material; J, S – inertial and statical moments of the perpendicular cross-section.

For the considered case

$$W^{(g)} = \{e^{-\beta_2\xi}(\theta_1 \sin\beta_1\xi + \theta_2 \cos\beta_1\xi) +$$
$$+ e^{\beta_2(\xi-l)}[\theta_3 \sin\beta_1(\xi-l) + \theta_4 \cos\beta_1(\xi-l)]\}F_4(\eta), \tag{6}$$

where: $\theta_i = -\frac{4\pi}{a^2N^5}[\gamma_1\nu_{12}A_{2i} + (\alpha_1 + \frac{\gamma_1 K_{11}}{B_{11}R})A_{4i}].$

A_{ji} are the matrix coefficient components of the derivatives of relations (4).

In a case of same boundary conditions we have

$$A_1 = -A_3, \quad \theta_1 = -\theta_3, \quad A_2 = A_4, \quad \theta_2 = \theta_4,$$

$$W_1^{(K)}(\xi) = W_2^{(K)}(l - \xi).$$

b. The stability governing equations are obtained from those presented in reference [4]. In order to carry out the asymptotic analysis we introduce the following parameters:

$$\varepsilon_1 = \sqrt{\frac{D_1}{B_2R^2}}, \varepsilon_2 = \frac{D_3}{D_1}, \varepsilon_3 = \frac{D_3}{D_1}, \varepsilon_4 = \frac{B_2}{B_1}, \varepsilon_5 = \frac{B_3}{B_1}, \varepsilon_6 = \frac{K_{11}}{B_{11}R},$$

and the following exponents:

$$\frac{\partial}{\partial \xi} \sim \varepsilon_1^{-\alpha}, \quad \frac{\partial}{\partial \eta} \sim \varepsilon_1^{-\beta}, \quad W \sim \varepsilon_1^{\delta_1} R, \quad U \sim \varepsilon_1^{\delta_2} W, \quad V \sim \varepsilon_1^{\delta_3} W;$$

$$T_{10} \sim \varepsilon_1^{t_1} B_2, \quad T_{20} \sim \varepsilon_1^{t_2} B_2, \quad S_0 \sim \varepsilon_1^{t_3} B_2.$$

The being considered problem is characterized by the following relations between the parameters

$$\varepsilon_1 << 1, \quad \varepsilon_2 \sim \varepsilon_1^2, \quad \varepsilon_3 \sim \varepsilon_1, \quad \varepsilon_4 \sim \varepsilon_5 < 1, \quad \varepsilon_6 \sim \varepsilon_1.$$

In order to estimate terms possessing components of a before critical state and their variations, the following exponent values have been considered.

a) For the boundary related condition they read

$$\alpha = \frac{1}{2}, \quad \beta < \frac{1}{2}, \quad \delta_1 = \beta + \frac{1}{2}, \quad \delta_2 = \frac{1}{2}, \quad \delta_3 = 1 - \beta;$$

b) For a state related to a discrete support one has

$$\alpha = \beta = \frac{1}{2}, \quad \mathrm{a} \sim \varepsilon_1^2, \quad \alpha_1 \sim \varepsilon_1^2, \quad \gamma_1 \sim \varepsilon_1, \quad \varepsilon_1^{-\frac{1}{2}} \leq N \leq\sim \varepsilon_1^{-1};$$

c) For the components appearing in the stability equations we have

$$\alpha = 0, \quad \beta = \frac{1}{2}, \quad \mathrm{t}_1 = 2, \quad \mathrm{t}_2 = 2 + 2\beta, \quad \mathrm{t}_3 = 2 + \beta.$$

In result of an asymptotic estimation of the initial stability governing equations the following simplified equation has been obtained

$$K_{11}\left(\frac{\overline{U}_{\xi\xi\xi}}{R} + \frac{1}{R^2} W^{(K)}_{1\xi\xi\xi} \overline{W}_{1\xi}\right) - \frac{D_{11}}{R^2} \overline{W}_{1\xi\xi\xi\xi} - \frac{D_{22}}{R^2} \overline{W}_{1\eta\eta\eta\eta} + \\ R\overline{T}_2 + \overline{W}_{1\xi\xi} T_1^{(0)} + W^{(K)}_{1\xi\xi} \overline{T}_1 = 0, \tag{7}$$

where: $\overline{T}_1 = \frac{B_{11}}{R} \overline{U}_{1\xi} + \frac{B_{11}}{R^2} W^{(K)}_{1\xi} \overline{W}_{1\xi} - \frac{K_{11}}{R^2} \overline{W}_{1\xi\xi},$

$$\overline{T}_2 = \iint \left(\frac{B_{11}}{R} \overline{U}_{1\xi\xi} + \frac{B_{11}}{R^2} W^{(K)}_{1\xi\xi\xi} \overline{W}_{1\xi} - \frac{K_{11}}{R^2} \overline{W}_{1\xi\xi\xi\xi} \right) d\eta d\eta, \\ \overline{U}_{1\xi} = -\iint \left(\overline{W}_{1\xi\xi} + \frac{1}{R} W^{(g)}_{1\eta\xi\xi} \overline{W}_{1\eta} \right) d\eta d\eta - \frac{1}{R} \int W^{(K)}_{1\xi\xi} \overline{W}_{1\eta} d\eta . \tag{8}$$

and bars are related to the variational components of the initial state.

In Figure 1, in order to illustrate the application of the approximated solutions, the results of critical stress computation corresponding to equation (7) and the results presented in references [2, 5] for a non-momentum pre-critical state are presented. The parameters appear in the figure read

$$T^* = \frac{T_1 R}{2\sqrt{B_1 D_2 (1 - \nu \nu_{21})}}; \quad Z = l^2 \frac{R}{h} \sqrt{\frac{1 - \nu \nu_{12}}{\mu_1}}.$$

The computations have been carried out for a case of "medium" reinforcement due to the classification included in reference [2]. The curves of the top picture correspond to the free support, whereas those of the bottom one correspond to the clamped support.

The obtained results verify our approach. Furthermore, it is expected that an influence of pre-critical momentum state in equation (7) can lead to better approximation of the experimental results.

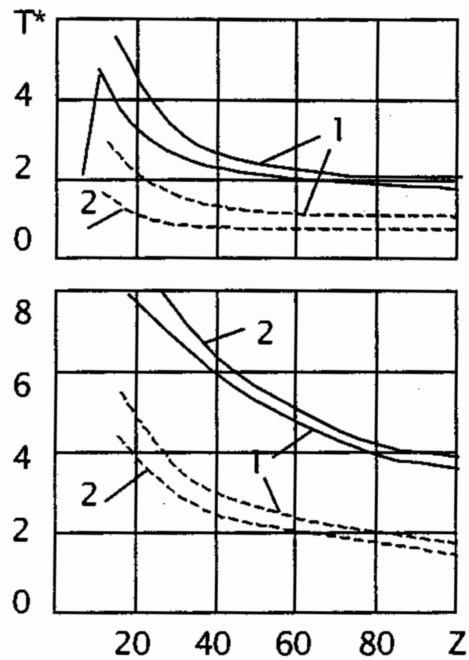

Figure 1. The computational results. The solid (dashed) curves correspond to edge (internal) reinforcement;
1 denote exact, whereas 2 approximate solutions.

In addition, using the Bubnov-Galerkin method the engineering formulas for a critical load computation of the excentrically reinforced shells are obtained (the minimalization procedure along the wave numbers has been carried out using an asymptotical method).

They read for both free:

$$T_1 = \left(\frac{\pi}{l}\right)^2 \left(\frac{D_1}{R^2} + \frac{B_1}{n^2}\right) + \frac{D_2 n^4}{R^2}\left(\frac{l}{\pi}\right)^2 - \frac{2K_{11}}{Rn^2}\left(\frac{\pi}{l}\right)^2;$$

$$n^2 = n_0^2\left(1 - \frac{K_{11}}{4B_{11}R} n_0^2\right), \quad n_0^2 = \frac{\pi}{l}\sqrt[4]{\frac{B_{11}R^2}{D_{22}}};$$

and clamped

$$T_1 = \left(\frac{2\pi}{l}\right)^2 \left(\frac{D_1}{R^2} + \frac{B_1}{n^4}\right) + \frac{3D_2 n^4}{R^2}\left(\frac{l}{2\pi}\right)^2 - \frac{2K_{11}}{Rn^2}\left(\frac{2\pi}{l}\right);$$

$$n^2 = n_0^2\left(1 - \frac{K_{11}}{4B_{11}R} n_0^2\right), \quad n_0^2 = \frac{2\pi}{l}\sqrt[4]{\frac{B_{11}R^2}{3D_{22}}}.$$

supports.

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