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SYMMETRIC OSCILLATIONS AND TRANSITION TO CHAOS IN A FREELY SUPPORTED SINUSOIDALLY EXCITED FLEXIBLE SHELL

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Summary. In this contribution transition to chaos via symmetric oscillations in a freely supported and sinusoidally excited flexible shell is analysed

1. Introduction

In this contribution dynamics of flexible shells is analysed. The discrete approximation of a continuous system is carried out using a finite difference approximation with an error of $O(H^4)$. The obtained set of ordinary differential (ODE) and algebraic (AE) equations is solved using either the Runge-Kutta method of fourth order and the Gauss method, respectively.

2. Governing equations

The following Karman's equation is formulated

$$\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{\partial w}{\partial t}=-\frac{1}{12(1-\nu^2)}\nabla^2\nabla^2 w+$$
$$+\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 F}{\partial y^2}-P_x\right)-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}+q, \qquad (1)$$
$$\nabla^2\nabla^2 F=-\frac{1}{2}L(w,w),$$

which holds for an isotropic plate. The following notation is used: $w(x, y, t)$ - the plate deflection along z co-ordinate directed to the Earth centre, $q(x, y, t)$ - transversal load function $(q = 0)$, $P_x(y, t)$ - a longitudinal load function along ox axis. The co-ordinate origin lies in left below corner of the plate, whereas the ox and oy axes are attached to the plate sides. The plate volume

$$G \in \{x, y \mid \quad 0 \le x \le a, \ 0 \le y \le b\}, \ -h \le z \le h,$$

where $2h$ is the plate thickness. In equations (1) the following Airy's function is introduced

$$T_{xx} = \frac{\partial^2 F}{\partial y^2} - P_x, \quad T_{yy} = \frac{\partial^2 F}{\partial x^2}, \quad T_{xy} = \frac{\partial^2 F}{\partial x \partial y}, \tag{2}$$

where T_{xx}, T_{yy}, T_{xy} are the stresses in average plate surface (ε denotes damping and ν is the Poisson coefficient).

The system of equation (1) is transformed to the non-dimensional form (the non-dimensional quantities possess the bars) via relations

$$x = a\bar{x}, \quad y = b\bar{y}, \quad w = 2h\bar{w}, \quad \lambda = \frac{a}{b}, \quad t = t_0 t, \quad \varepsilon = (2h)\bar{\varepsilon},$$
$$P_x = \frac{E(2h)^3}{b^2}\bar{P}_x, \quad F = E(2h)^3\bar{F}, \quad q = \frac{E(2h)^4}{a^2 b^2}\bar{q},$$

where: E - Young's modulus.

To the system (1) the following boundary conditions are attached (free support on flexible unstretched ribs)

$$\begin{aligned} w = M_x = N_x = \varepsilon_x = 0 \quad & for \quad x = 0;\ 1, \\ w = M_y = N_y = \varepsilon_y = 0 \quad & for \quad y = 0;\ 1, \end{aligned} \tag{3}$$

which can be expressed via w and F in the form

$$\begin{aligned} w = \frac{\partial^2 w}{\partial x^2} = F = \frac{\partial^2 F}{\partial x^2} = 0 \quad & for \quad x = 0;\ 1, \\ w = \frac{\partial^2 w}{\partial y^2} = F = \frac{\partial^2 F}{\partial y^2} = 0 \quad & for \quad y = 0;\ 1. \end{aligned} \tag{4}$$

The following initial conditions are used

$$\begin{aligned} & w\big|_{t=0} = A \sin \pi x \sin \pi y, \quad (A = 1 \cdot 10^{-3}), \\ & \dot{w}\big|_{t=0} = 0. \end{aligned} \tag{5}$$

We should emphasise that both boundary and initial conditions are symmetric, and additionally a symmetry condition is applied to the form of plate oscillations (axis symmetry). It means that only 1/4 of a square is considered ($0 \le x \le 0.5;\ \ 0 \le y \le 0.5$). Therefore, other points oscillations are detected using a symmetry condition.

3. Reliability of the obtained results

In order to check a reliability of numerical computations the following tests have been carried out. A plate has been subjected to excitation $P_x = P_0 \sin \omega t (\omega = 5.72,\ \ P_0 = 6)$ and the 1/4 plate surfaces has been divided using the space step *H=1/8* and *H=1/10*. The AE system has been solved using the Gauss method. Because a high accuracy has been obtained in both cases therefore further the steps *H=1/8* in space and $\Delta t = 2 \cdot 10^{-4}$ in time have been used.

In order to analyse a structure of multifrequency and chaotic oscillations and their bifurcations we need to apply various approaches (observation of oscillations in all points of the plate, phase portraits, Fast Fourier Transform, Poincaré maps, bifurcation diagrams, and so on).

Contrary to the problems occurred in radiophysics and electronics, where mainly timing chaos is investigated, in a field of continuous mechanical systems a key point of research is oriented on spatial chaos. The last one can be solved relatively easy if the continuous system is homogeneous because of *x* and *y*. It means that *x* and *y* do not appear explicitly in the differential equations. If we treat a spatial co-ordinate as a time we get an autonomous system. In this case a problem from mathematical point of view is similar to that of timing chaos. However, in theory of plates with finite dimension this can not be achieved.

In this work we consider the so called "modal portraits": $w_x^{\cdot}(w), w_y^{\cdot}(w), w_{xx}^{\cdot\cdot}(w), w_{yy}^{\cdot\cdot}(w)$, $w_{xx}^{\cdot\cdot}(w_x^{\cdot}), w_{yy}^{\cdot\cdot}(w_y^{\cdot})$ for different time points for an arbitrary point of the plate. In addition, a cross section of this portraits is used every period of excitation. Both modal and phase portraits are analysed which allows to analyse a spatial chaos and its influence on timing chaos.

4. Analysis

We analyse a transition from a steady state to chaos using the following control vector $\{P_0, \omega\}$, where $4.72 \le \omega \le 6.72$ (Fig. 1a). A transition to chaos will be described using $\omega = 5.72$ (Fig. 1b), which corresponds to a frequency of linear plate oscillation ($\varepsilon = 1$, and $P_0(\omega)$ is shown in Fig. 1). In addition, the average shell deflection is presented in Fig. 1c.

In Table 1 a relation between frequencies versus amplitudes P_0 (P_0 = *4; 7; 7,5; 8; 8,25; 8,5; 9; 10; 15; 20*) is given. Analysis of the Table leads to conclusion that for $3.5 \le P_0 \le 7$ (Fig. 2) the

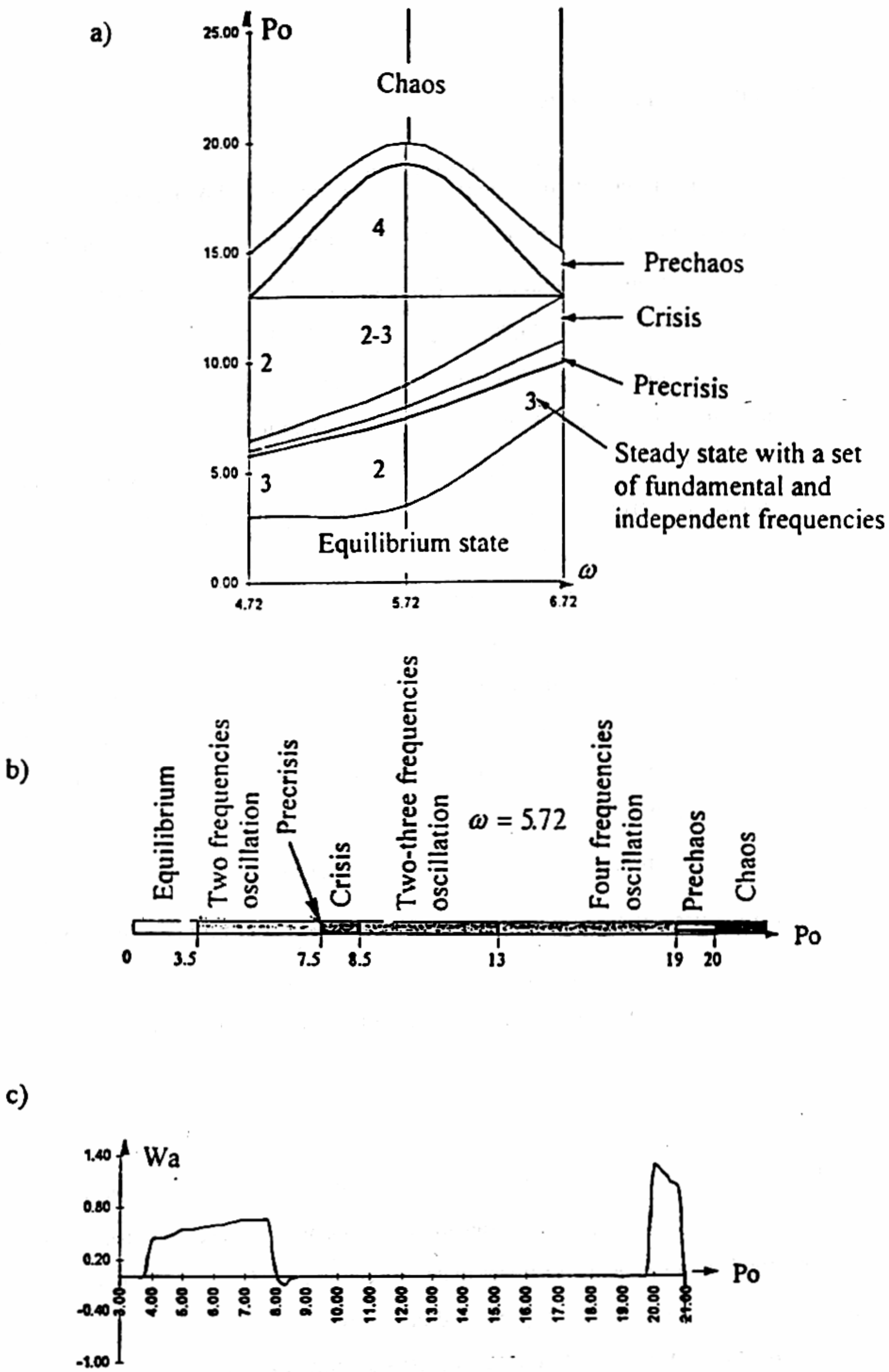

Fig. 1. Different steady states in two (a) and one (b) dimensional parameter space and average plate deflection versus P_0 (c)

Table 1.

Px=Po sin 5.72 t											
№ ω	ω	Po									
		4	7	7.5	8	8.25	8.5	9	10	15	20
1.	9.45E-14										×
2.	0.00555	×	×	×	×		×				
3.	0.0111							×			
4.	0.02219										
5.	0.45499			$\omega_{14}-\omega_8$				⊗	⊗	⊗	
6.	0.8323										
7.	0.88778										
8.	0.90998	⊗	⊗	⊗					$2\omega_5$		⊗
9.	0.91553						×				
10.	0.99321					×					
11.	0.99876										
12.	1.04869				×						
13.	1.31503						×				
14.	1.36497			⊗				$3\omega_5$	$3\omega_5$	$3\omega_5$	
15.	1.81996	$2\omega_8$	$2\omega_8$	$2\omega_8$					$4\omega_5$		$2\omega_8$
16.	1.82551						×				
17.	1.84215					×					
18.	1.95867				×						
19.	2.27495			$\omega_{14}+\omega_8$				$5\omega_5$	$5\omega_5$	$5\omega_5$	
20.	2.72439										
21.	2.72994			$2\omega_{14}$					$6\omega_5$		$3\omega_8$
22.	3.17938									⊗	
23.	3.63437										×
24.	4.08936									$\omega_{22}+2\omega_5$	
25.	4.54434										×
26.	4.99933							×			
27.	5.00488								×		
28.	5.45432										×
29.	6.81929									×	
30.	7.27428										×
31.	7.72927									×	
32.	8.18426										×

⊗ - fundamental frequency, × - independent frequency, $N\omega_k$ - synchronization

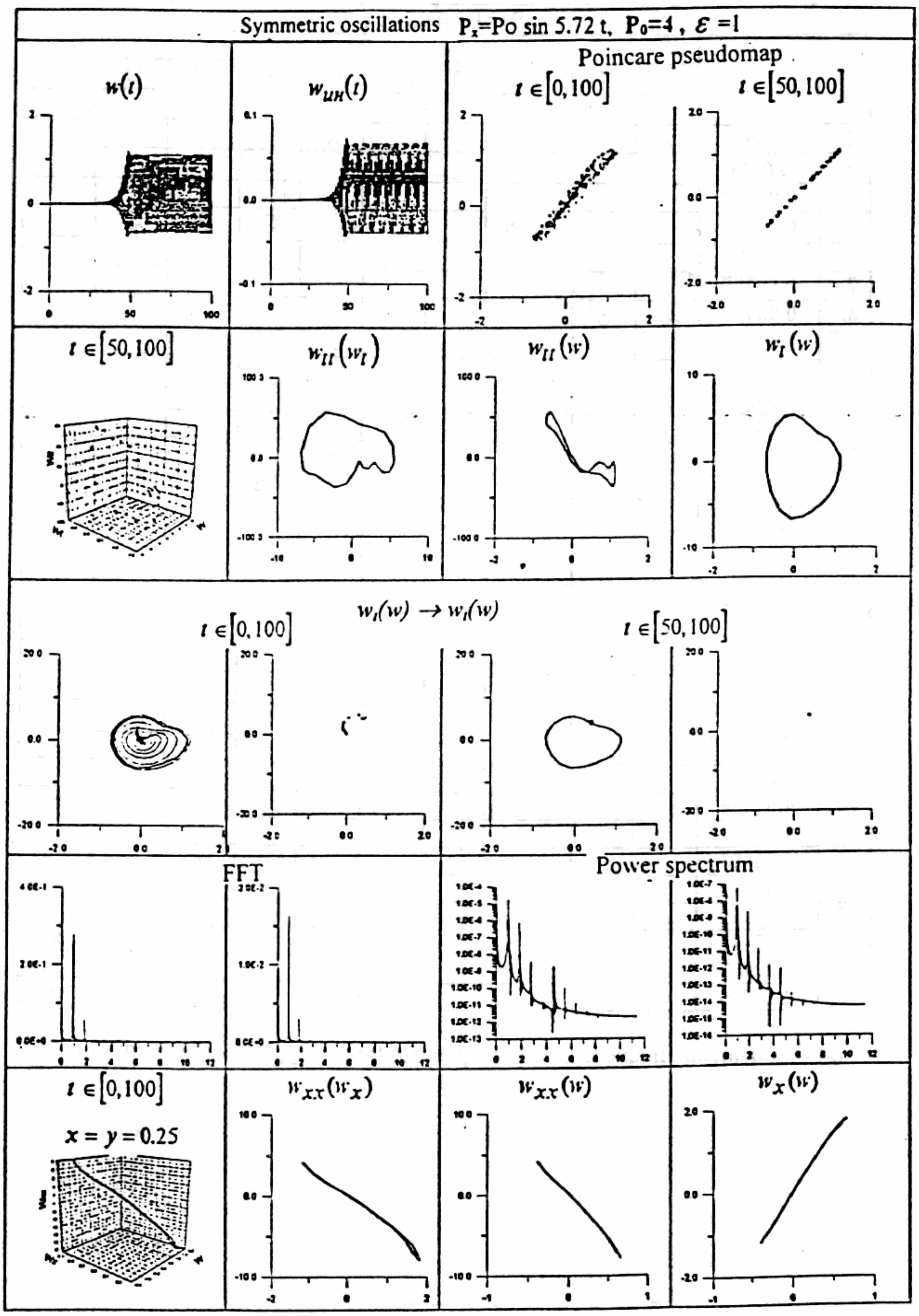

Fig. 2. Time histories, Poincare maps and pseudomaps, phase and modal portraits, Fast Fourier Transform (FFT) and power spectrum of non-symmetric plate oscillations.

oscillations include two frequencies ω_2 and ω_8. An internal synchronization phenomenon is observed $\omega_{15} = 2\omega_8$ (as it is pointed out by Andronov et al. [1], "a unique rhythm of common existence occurs"). In the interval of $7 < \omega \le 7.9$ the oscillations occur on three independent frequencies ω_2, ω_8 and ω_{14} : $\omega_{15} = 2\omega_8$, and $\omega_{21} = 2\omega_{14}$. Besides, the new frequencies appear, which are linear combination of ω_8 and ω_{14} : $\omega_5 = \omega_{14} - \omega_8$, $\omega_{19} = \omega_{14} + \omega_8$. In the interval of $7.9 < \omega \le 8$ oscillations are spanned on two-four independent frequencies. This behaviour corresponds to crisis [2]. The post crises state is related to $8 < \omega \le 9.2$ (Fig.3), where oscillations include three frequencies ω_3, ω_5 and ω_{26}. The following synchronization of oscillations occurs, because of ω_5 : $\omega_{14} = 3\omega_5$, $\omega_{19} = 5\omega_5$. Then the plate starts to oscillate on two frequencies ω_5 and ω_{27}, and the synchronization on one frequency ω_5 occurs: $\omega_8 = 2\omega_5$, $\omega_{14} = 3\omega_5$, $\omega_{15} = 4\omega_5$, $\omega_{19} = 5\omega_5$, $\omega_{21} = 6\omega_5$ $(\omega \in (9.2;\ 13))$. For $P_0 > 20$ the plate exhibits chaos (Fig. 4).

In Figures 2-4 the following characteristics related to the plate centre are introduced: $w(t)$, $w_{in}(t) = \int_0^{0.5}\int_0^{0.5} w(x,y,t)dxdy$, Poincaré pseudomaps, the volume relation $(w_{tt},\ w_t,\ w)$ and its projection onto the corresponding plane, Poincaré sections, FFT, power spectrum for $(x = y = 0.5)$ and for an integral characteristics w, as well as modal phase portraits $(w_{xx},\ w_x,\ w)$ for $(x = y = 0.25)$ and the time histories. All of the plate points behave in a similar way.

5. Conclusions

It should be pointed out that a separation of a motion to regular and chaotic parts has rather a conventional character. Each of observed chaotic behaviour is characterized by regular parts, timing rules and space structures. A problem of investigation of chaotic behaviour is related to detection and analysis of timing and spaces rules having either regular or chaotic behaviour.

6. References

1. Andronov A. A., Witt A. A., Chajkin S. E.: *Theory of oscillations*, Fizmatgiz, Moscow (1959).
2. Grebogi C., Yorke J. A.: *Crises, sudden changes in chaotic attractors and transient chaos*, Physica 7D, (1983), 181-200.

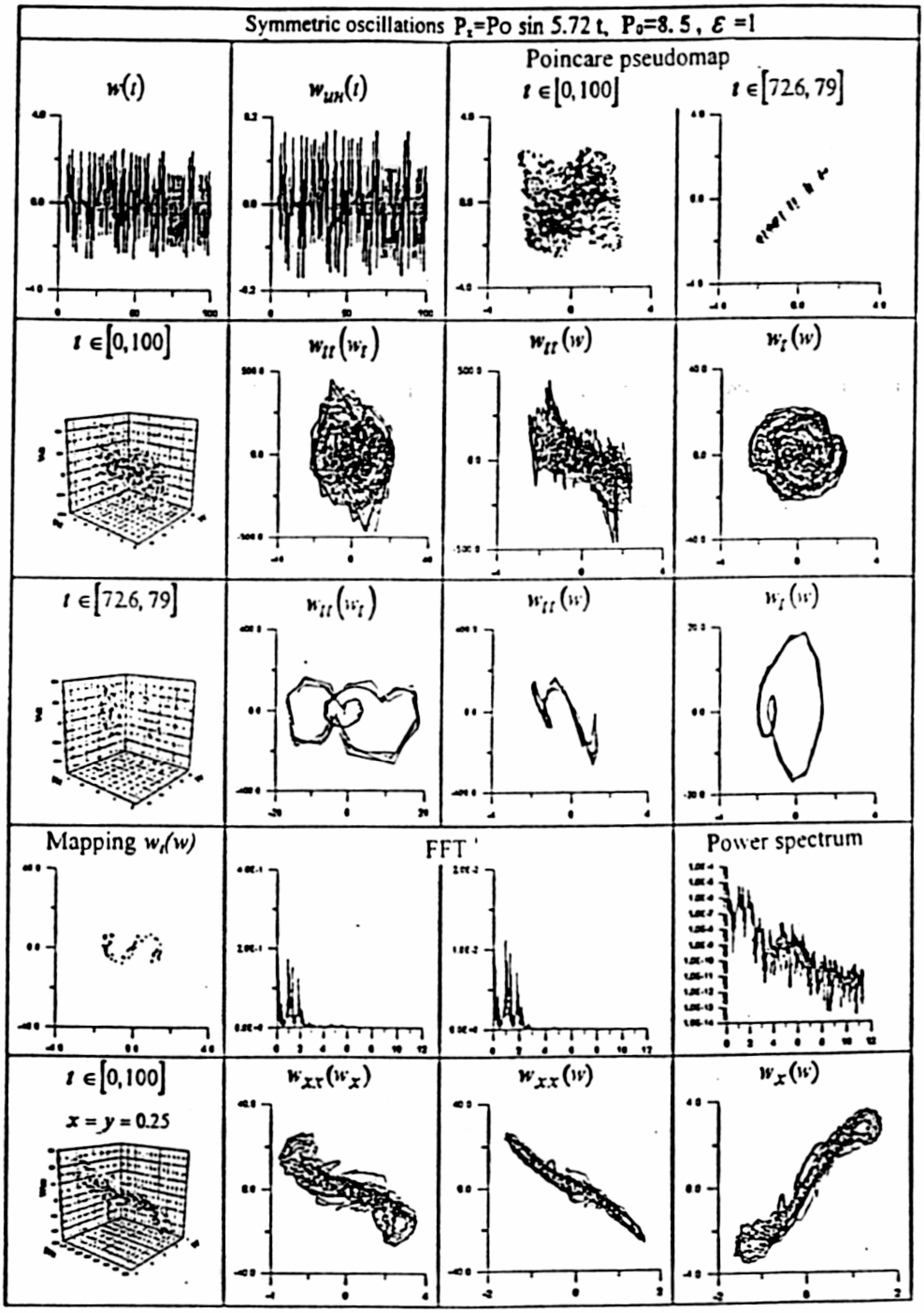

Fig.3. Same as in Figure 2

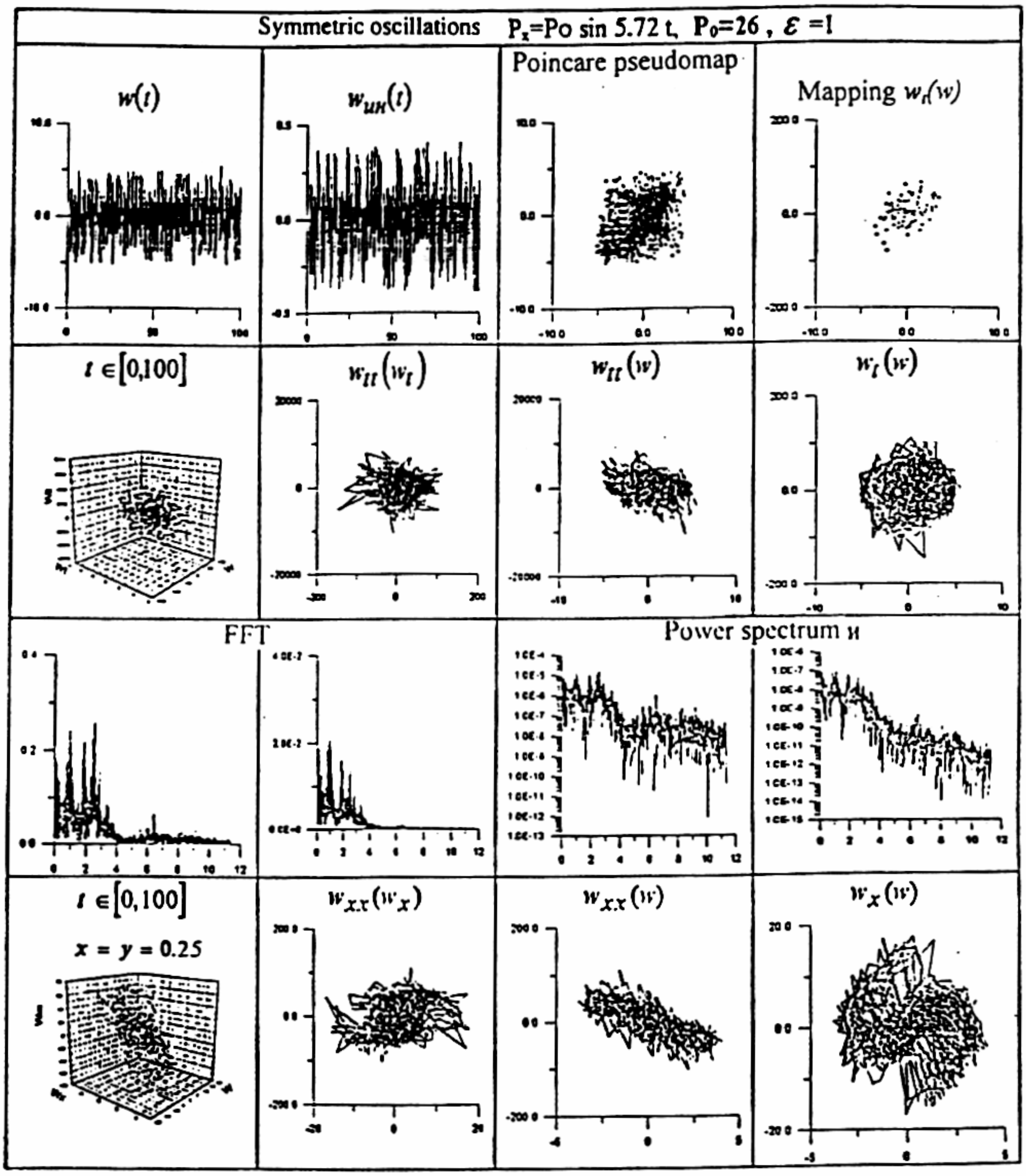

Fig. 4. Same as in Figure 2.

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