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**NON-SYMMETRIC OSCILLATIONS AND TRANSITION TO CHAOS IN  
FREELY SUPPORTED FLEXIBLE PLATE SINUSOIDALLY EXCITED.**

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*Abstract.* In this contribution timing and spatial chaos exhibited by a flexible plate sinusoidally excited is analysed.

**1. Introduction**

Flexible rectangular plates periodically excited can exhibit complex behaviour including regular and chaotic oscillations. In addition, many other dynamical features depending on the parameters such as jump phenomena, complex resonances, symmetric and unsymmetric oscillations leading to space - time dynamical configuration, standing or travelling waves, can appear.

In this contribution complex non-symmetric oscillations of a plate sinusoidally excited are analysed and this contribution extends the results presented in reference [1] printed in this Proceedings. A method of solution as well as the boundary and initial conditions are the same as in reference [1]. However, now a symmetry conditions are not introduced and a whole space due to spatial co-ordinates is taken into account ( $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ ). Contrary to the previous work here the algebraic equations have been solved on each iterational step using a relaxation method.

## 2. Analysis

The area of integration includes a square ( $\lambda = 1$ ) and, as in reference [1], the initial conditions:  $w|_{t=0} = 1 \cdot 10^{-3} \sin \pi x \sin \pi y$ ;  $\dot{w}|_{t=0} = 0$  and the boundary conditions are symmetric. However, an unsymmetric mode of stability loss is observed. In Table 1 frequencies  $\omega$  versus amplitude of excitation  $P_0$  ( $P_0 = 4; 7; 7,5; 8; 8,25; 8,5; 9; 10; 10,25; 10,5$ ) are presented. A scenario from a steady state to chaos is illustrated in Fig. 1. The various dynamical states of the system is reported in Figs 2-5. In addition, the plate surface deflections for  $P_0 = 7,5$  for the freely taken time moments are reported. The obtained results lead to conclusion that from an energetical point of view a non-symmetric mode corresponding to chaos transition is more suitable. In this case a magnitude of longitudinal excited force is two times smaller than for symmetric mode transition to chaos.

The various synchronizations in time are characterized by a periodical behaviour of discrete or continuous systems. An occurrence of spatial order is less investigated in comparison to timing synchronization. The up dated results are referred to investigations of Bernard cells in convective flows, Karman vortices, or spatial-timing changes of biological cells structure, and other.

An investigation of plate oscillations should be carried out from a point of view of spatial-temporal order or chaos, because the governing equation is a partial one and the being sought functions  $w(x, y, t)$  and  $F(x, y, t)$  depend on two spatial co-ordinates and time. The derivatives of

$w(x,y,t)$  and  $F(x,y,t)$  possess both geometrical and physical meaning. Because  $\frac{\partial w}{\partial x}$ ,  $\frac{\partial w}{\partial y}$  are the tangences of rotation angles of a normal with the corresponding co-ordinates.  $\dot{w}$  is a velocity;  $\frac{\partial^2 w}{\partial x^2}$ ,  $\frac{\partial^2 w}{\partial y^2}$ ,  $\frac{\partial^2 w}{\partial x \partial y}$  correspond to plate curvatures, which together with the being sought functions  $w$  and derivatives  $\frac{\partial w}{\partial x}$ ,  $\frac{\partial w}{\partial y}$  define a change of plate geometry  $\forall x, y \in [0,1; 0,1]$ .

## 3. Conclusions

In this work spatial-temporal chaos is investigated. A complex spatial behaviour is observed leading to an occurrence of spatial chaos which is indicated by the dependencies ( $w_{xx}$ ,  $w_y$ ,  $w$ ) and their projections onto the corresponding hyperplanes for an arbitrary point of the considered space.

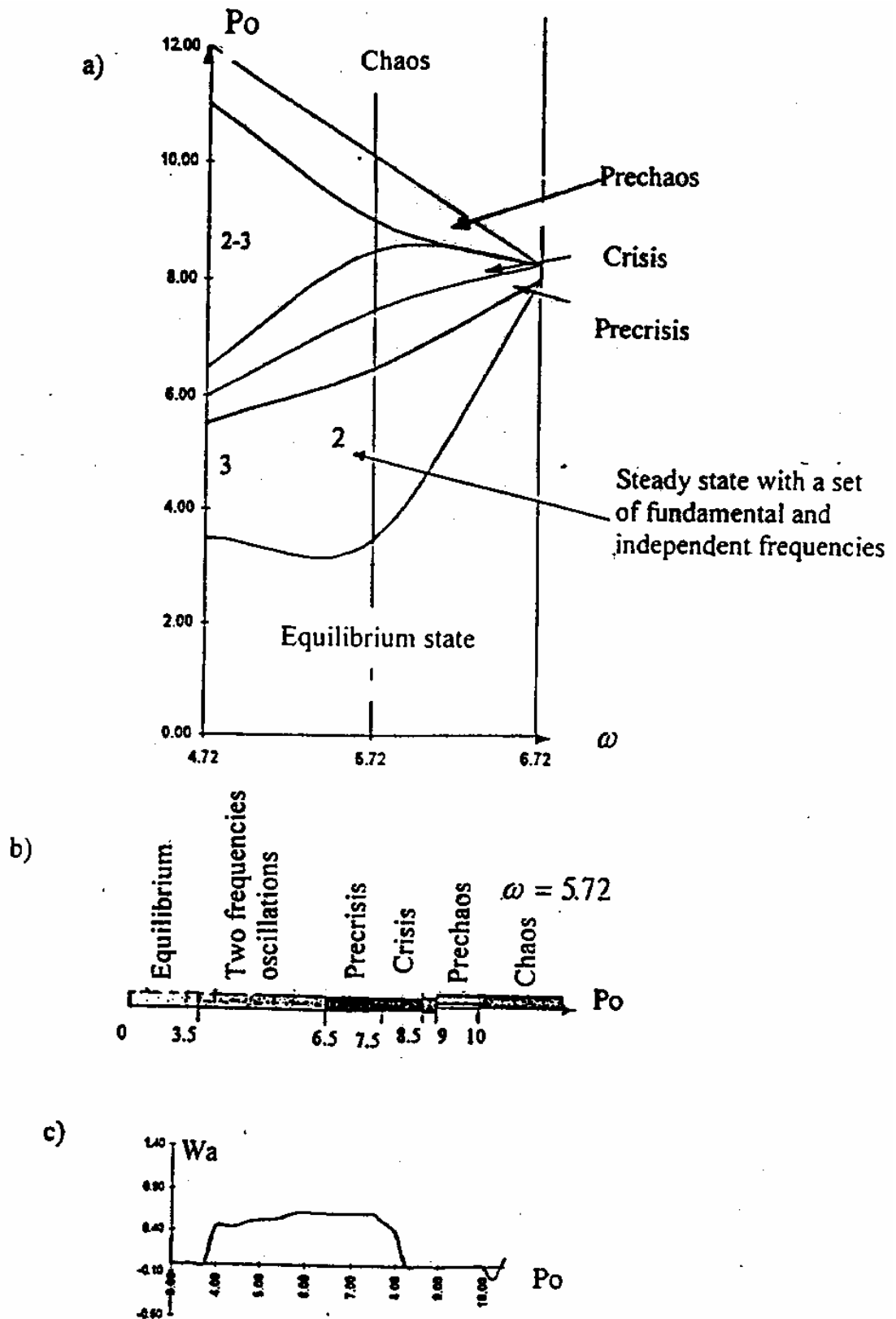


Fig. 1. Different steady states in two (a) and one (b) dimensional parameter space and average plate deflection versus  $P_0$  (c)

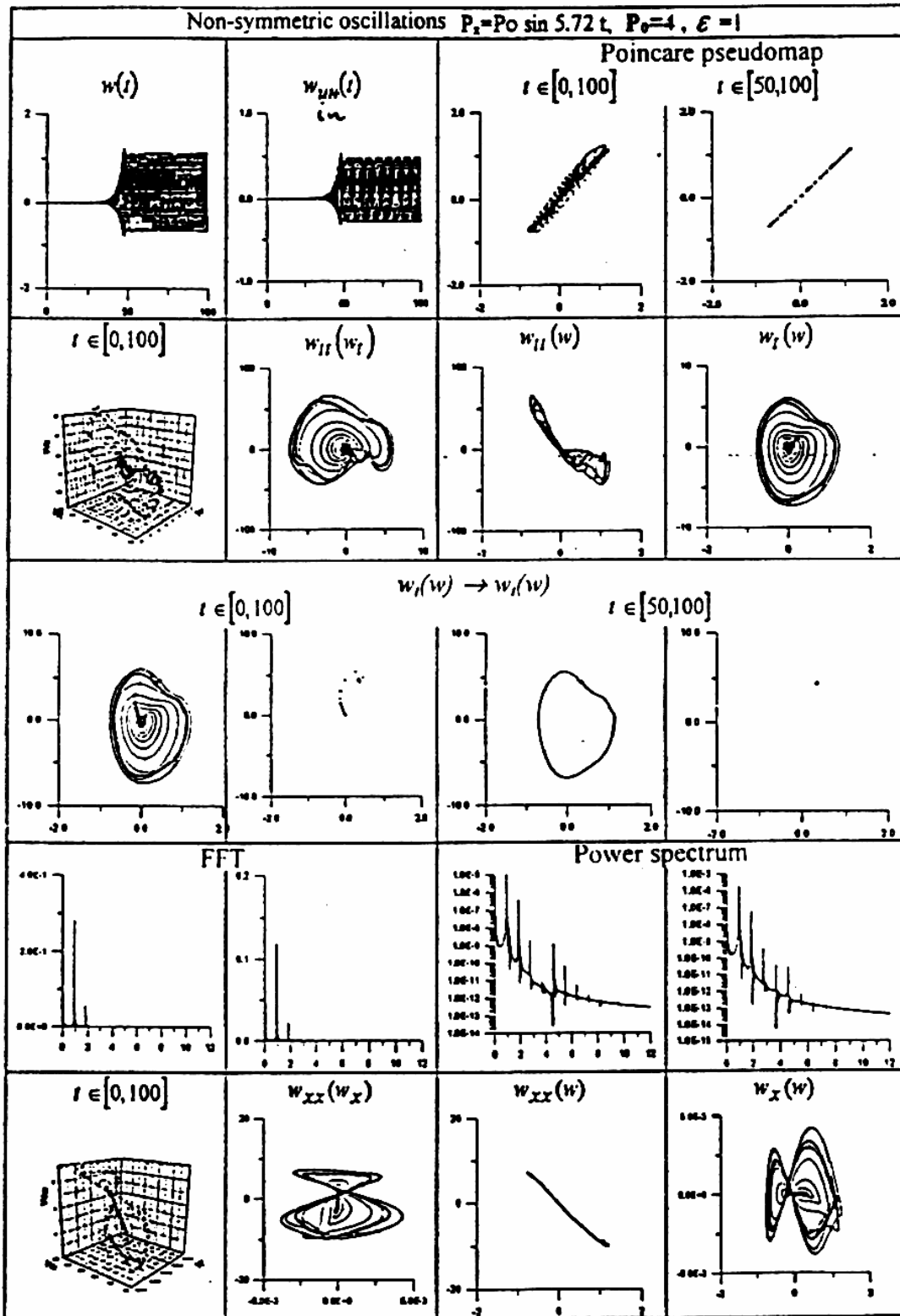


Fig. 2. Time histories, Poincaré maps and pseudomaps, phase and modal portraits, Fast Fourier Transform (FFT) and power spectrum of non-symmetric plate oscillations.

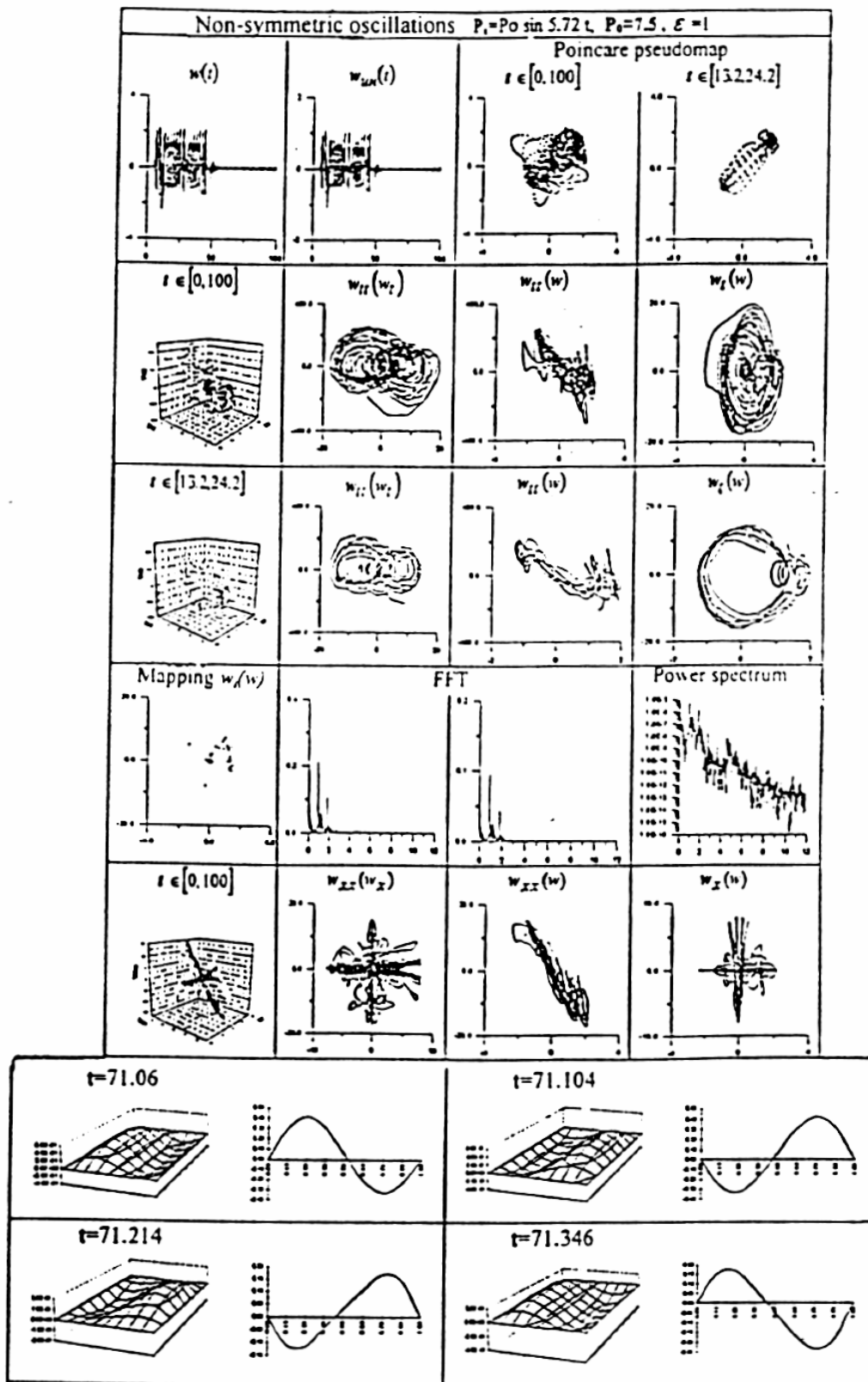


Fig.3. Same as in Figure 2 and the plate configurations for different time moments  $t$ .

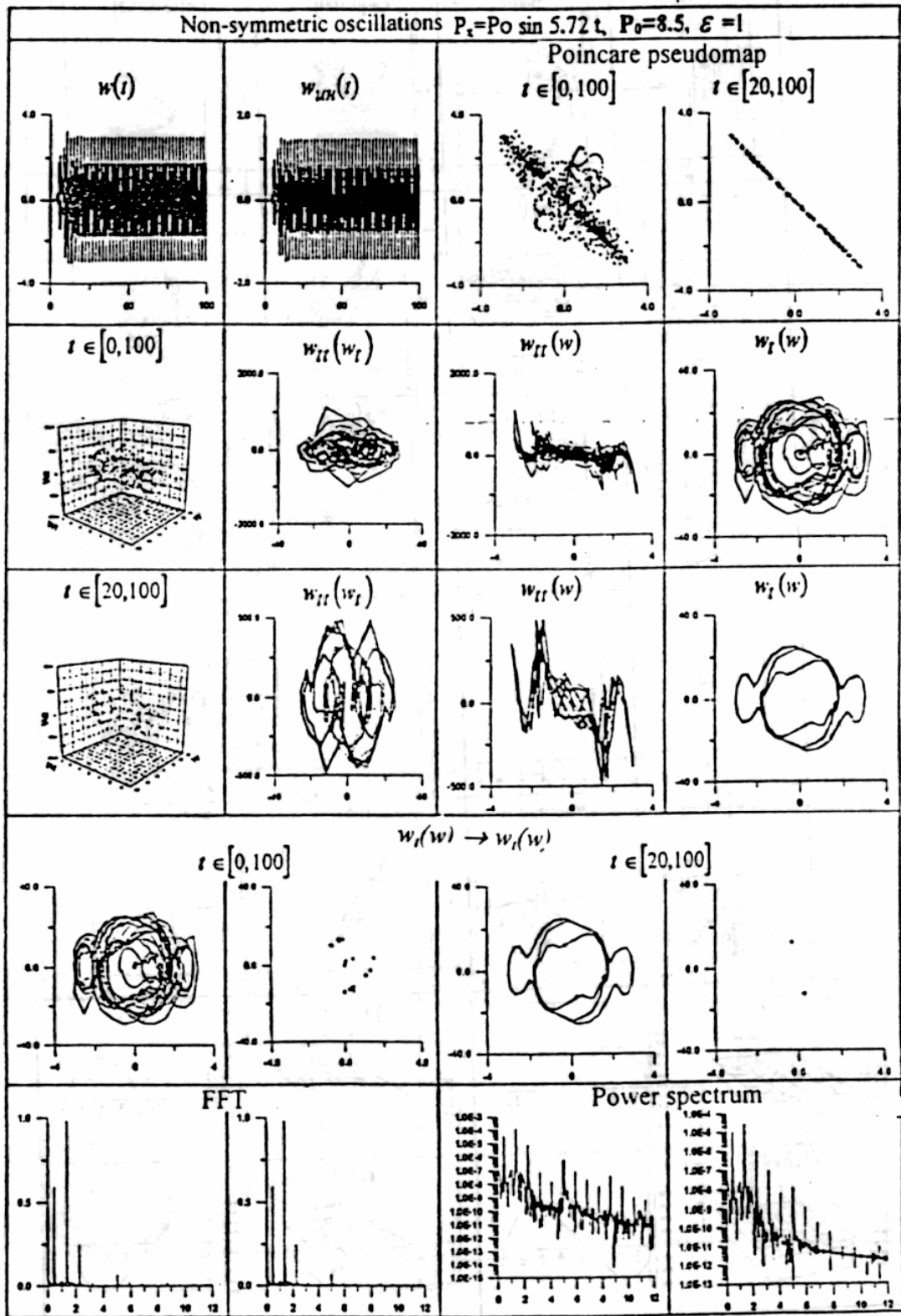


Fig. 4. Same as in Figure 2.

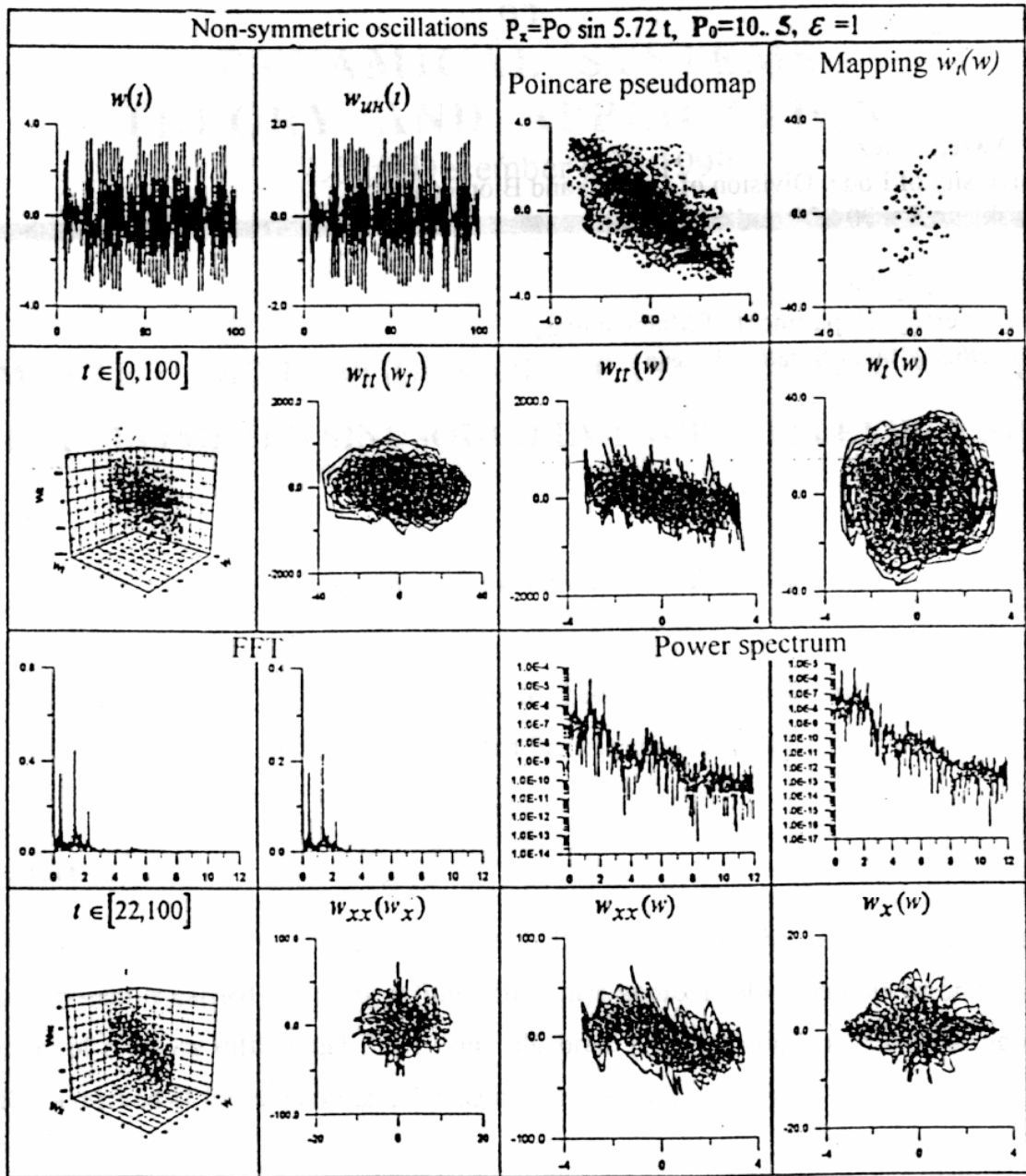


Fig. 5. Same as in Figure 2.

#### **4. References**

1. Awrejcewicz J., Krysko V. A., Krysko A. V.: Symmetric oscillations and transition to chaos in a freely supported sinusoidally excited flexible shell (this Proceedings).

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