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MULTIBODY VIBRO-IMPACT DYNAMICS

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ABSTRACT

In this paper a method of controlling and improvement of stability of periodic orbits of vibro-impact systems is proposed. This method is based on the feedback loop control with a time delay. The paper is focused on three main items.

First, an analytical method is proposed to estimate delay loop coefficients for improvement of stability of the vibro-impact motion for one degree-of-freedom systems. Then, control of periodic motion of the one-degree-of freedom vibro-impact oscillator is analysed numerically. Applications of the method to the multibody vibro-impact dynamical systems are given.

INTRODUCTION

It is worthless to say that mechanical vibro-impact systems have been employed in both theoretical and applied mechanics for a long time. The vibro-impact dynamics can be observed in many real engineering systems, such as hammer-like devices, balls and race dynamics in a ball bearing assembly, wheel-rail impact dynamics, etc. (Gryboś 1969; Peterka 1974).

Nowadays again this field of research has attracted strong interest, but in the frame of theories of modern dynamical systems. Recent industrial examples (tubes fretting wear through vibro-impact behaviour in nuclear reactors or impacts between old and high buildings excited by earthquakes belong to additional but not satisfactory solved questions of discontinual dynamical systems.

There are two parallel branches of investigations in the frame of vibro-impact dynamics. The first one is based on a better approximation of lows for impact motion and restitution coefficients, and it is more involved in the physics of materials. The second branch includes control of steady-state vibro-impact motion with a possibility of stability changes (either to destabilise or to stabilise the vibro-impact attractor considered).

Recently many papers have appeared, which are devoted to control of nonlinear oscillators, including also control of chaotic orbits (Pyragas 1992; Shinbrot *et al.* 1993).

In general, these methods could be devided for feedback control with a time delay (Youcef-Toumi and Wu 1992), sliding mode control (Slotine and Li 1991), repetitive control (Hara *et al.* 1988), iterative learning control (Arimoto 1990), adaptive control (Slotine and Li 1991), and so on. The main purpose of these methods is to control complicated systems, even with imprecise knowledge of their mathematical models. However, the control of the attractor or repiler analysed is based on the numerical observations of the required results by an introduction of the "helping" control coefficients. Theoretical predictions are rather not given. Here we address one, not satisfactory solved yet problem of the vibro-impact dynamics control with delay feedback and we give analytical prediction of the proper choice of control parameters.

CONTROL OF VIBRO-IMPACT PERIODIC ORBITS

We analyse the following one-degree-of-freedom vibro-impact system with one clearance, presented in Fig. 1.

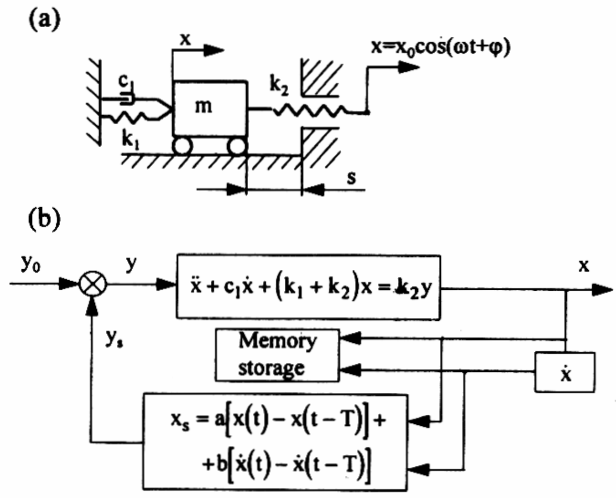

Figure 1. One-degree-of-freedom kinematically excited vibro-impact system with one clearance (a) and its control diagram (b) ("s" denotes clearance)

The equation of dynamics is as follows

$$\ddot{x} + c\dot{x} + \alpha^2 x = P_0 \cos\omega t + A[x(t) - x(t-T)] + \\ +B[\dot{x}(t) - \dot{x}(t-T)], \quad (1)$$

where:

$$P_0 = \frac{x_0 k_2}{m}, \quad c = \frac{c_1}{m}, \quad \alpha^2 = \frac{k_1 + k_2}{m},$$

$$A = \frac{k_2 a}{m}, \quad B = \frac{k_2 b}{m},$$

and T=2π/ω is the period of the considered periodic orbit being stabilised.

A key point of such a control is that a periodic solution possesses the same period as the excitation, i.e. $x_0=x_0(t-T)$, and x_0 is a particular solution of both the controlled and uncontrolled system (Krodkiewski and Faragher 1995). A delay loop is switched off where perturbations are not present. In the case of perturbations the controller causes the perturbations to vanish more quickly than in the case without control. The problem of analytical estimation of the influence of control coefficients for periodic orbit stability cannot be solved in a standard way. Here we propose the following approach. Because in fact the differencies x(t)-x(t-T) and $\dot{x}(t) - \dot{x}(t-T)$ are small, we express them by introducing the small parameter ε, which allows one then to apply the KBM method formally and next to take ε=1 (Awrejcewicz 1994).

We assume damping of the same order as ε, and from (1) we obtain

$$\ddot{x} + \alpha^2 x = P_0 \cos\omega t + \varepsilon A[x(t) - x(t-T)] + \\ +\varepsilon B\left[\left(1 - \frac{c}{B}\right)\dot{x}(t) - \dot{x}(t-T)\right]. \quad (2)$$

Introducing

$$x = z + \frac{P_0}{\alpha^2 - \omega^2}\cos\omega t, \quad (3)$$

we get

$$\ddot{z} + \alpha^2 z = \varepsilon f(a, \eta, \psi),$$

where:

$$\varepsilon f(a,\eta,\psi) = \varepsilon A\left\{\left(z + \frac{P_0}{\alpha^2 - \omega^2}\cos\omega t\right) - z(t-T) + \\ -\frac{P_0}{\alpha^2 - \omega^2}\cos\omega(t-T)\right\} + \\ +\varepsilon B\left\{\left(1 - \frac{c}{B}\right)\left(\dot{z} + \frac{P_0}{\alpha^2 - \omega^2}\cos\omega t\right) + \\ -\dot{z}(t-T) - \frac{P_0}{\alpha^2 - \omega^2}\cos\omega(t-T)\right\}, \quad (4)$$

$$\eta = \omega t, \quad \psi = \alpha t.$$

Using the KBM method we have truncated the ε series up to the order O(ε) and we have obtained

$$\frac{da}{dt} = \frac{1}{2}(B - c)a + \frac{Aa}{2\alpha}\sin\alpha T - \frac{Ba}{2\alpha}\cos\alpha T, \quad (5)$$

$$\frac{d\varphi}{dt} = \alpha - \frac{A}{2\alpha} + \frac{A}{2\alpha}\cos\alpha T + \frac{1}{2}B\sin\alpha T.$$

For A=B=0 we get the uncontrolled solution, which testifies the validity of our approach.

Therefore, we analyse the following equivalent solution

$$x = \frac{P_0}{\alpha^2 - \omega^2}\cos(\omega t + \varphi) + e^{RT}\left(C\cos\alpha_0 t + D\sin\alpha_0 t\right). \quad (6)$$

After the integration of Eqs. (5) we get

$$a(t) = C_0 e^{Rt},$$

$$\psi(t) = \alpha_0 t + \Theta_0, \quad (7)$$

$$R = \frac{1}{2}\left\{B\left[\left(1 - \frac{c}{B}\right) - \frac{1}{\alpha}\cos\alpha T\right] + \frac{A}{\alpha}\sin\alpha T\right\},$$

$$\alpha_0 = \alpha + \frac{A}{2\alpha}\left(\cos\alpha T - 1\right) + \frac{1}{2}B\sin\alpha T$$

and according to (7) and (6) one obtains

$$C = C_0\cos\Theta_0, \quad D = -C_0\sin\Theta_0, \quad C_0 = \sqrt{C^2 + D^2}.$$

STABILITY CONTROL

From Eq. (6) it is seen that when R<0 the assumed solution is stabilised more quickly in comparison to the case of R=0. However, a problem of stability investigation of the vibro-impact state is much more subtle. Before the impact number l, the mass possesses the velocity x_{l-}. This causes the following perturbation solution to occur

$$x + \delta x_l = e^{RT_l}\left[\left(C + \delta C_l\right)\cos\alpha_0\tau_l + \left(D + \delta D_l\right)\sin\alpha_0\tau_l\right] + \\ +a\cos\left(\omega\tau_l + \varphi + \delta\varphi_l\right). \quad (8)$$

A new time τ is measured from the l-th impact $\tau_l=\tau+\delta\tau_l$. For example, the next impact occurs for $\tau_{l+1} = \frac{2\pi}{\omega} + \delta T_l$, where δT_l denotes the period T=2π/ω perturbation.

After some calculations we get

$$\delta x_l = e^{R\tau}\left[-C\alpha_0\delta\tau_l\sin\alpha_0\tau + \delta C_l\cos\alpha_0\tau + \\ +D\alpha_0\delta\tau_l\cos\alpha_0\tau + \delta D_l\sin\alpha_0\tau + R\delta\tau_l C\cos\alpha_0\tau + \\ +R\delta\tau_l D\sin\alpha_0\tau\right] + a\delta\varphi_l\sin\left(\omega\tau + \varphi\right) + \\ -a\omega\delta\tau_l\sin\left(\omega\tau + \varphi\right),$$

$$\delta\dot{x}_l = e^{R\tau}\left[R\alpha_0\delta\tau_l\left(D\cos\alpha_0\tau - C\sin\alpha_0\tau\right) + R\delta C_l\cos\alpha_0\tau + \\ +R\delta D_l\sin\alpha_0\tau + R^2\delta\tau_l C\cos\alpha_0\tau + R^2\delta\tau_l D\sin\alpha_0\tau + \\ +\alpha_0\delta\tau_l\left(C\cos\alpha_0\tau + D\sin\alpha_0\tau\right) - \delta C_l\alpha_0\sin\alpha_0\tau + \\ +\delta D_l\alpha_0\cos\alpha_0\tau - R\delta\tau_l C\alpha_0\sin\alpha_0\tau + R\delta\tau_l D\alpha_0\cos\alpha_0\tau\right] \\ -a\omega\delta\varphi_l\cos\left(\omega\tau + \varphi\right) - a\omega^2\delta\tau_l\cos\left(\omega\tau + \varphi\right).$$

The following boundary conditions are introduced:

$$l:\quad \tau = 0,\quad \delta\tau_l = 0,\quad \delta x_l = 0,\quad \delta\dot{x}_l = \delta\dot{x}_{l+},$$

$$l+1:\quad \tau = \frac{2\pi k}{\omega} + \delta\tau_l,\quad \delta\tau_l = \delta T_l,\quad \delta x_l = 0, \quad (10)$$

$$\delta\dot{x}_l = \delta\dot{x}_{(l+1)-}$$

After some calculations we have obtained the following equations

$$\delta C_l - a\delta\varphi_l \sin\varphi = 0, \quad (11)$$

$$e^{2\beta R}\left[\left(\delta\varphi_{l+1} - \delta\varphi_l\right)\frac{RC - \alpha_0 Ctg\alpha_0\beta}{\omega} + \delta C_l \cos 2\beta\alpha_0 + \right.$$

$$\left. + \delta D_l \sin 2\beta\alpha_0\right] - \delta C_{l+1} = 0$$

$$R_r e^{2\beta R}\left[\left(\delta\varphi_{l+1} - \delta\varphi_l\right)\frac{(R^2 - \alpha_0^2)C - 2R\alpha_0 Ctg\alpha_0\beta}{\omega} + \right.$$

$$\left. + R\delta C_l \cos 2\beta\alpha_0 + R\delta D_l \sin 2\beta\alpha_0 - \delta C_l \alpha_0 \sin 2\beta\alpha_0\right]$$

$$+ R\delta C_{l+1} + \delta D_{l+1}\alpha_0 - (R_r + 1)a\omega\delta\varphi_{l+1}\cos\varphi = 0,$$

and $R_r \leq 1$ as usual denotes the restitution coefficient.

Assuming that

$$\delta\varphi_l = \delta\varphi_0 + \sum_{i=1}^{l}\omega\delta T_i \quad (12)$$

and introducing

$$\delta C_l = a_1\gamma^l,\quad \delta D_l = a_2\gamma^l,\quad \delta\varphi_l = a_3\gamma^l \quad (13)$$

we get the following characteristic equation

$$b_2\gamma^2 + b_1\gamma + b_0 = 0, \quad (14)$$

where

$$b_2 = \left[-e^{2\beta R}\frac{RC - \alpha_0 Ctg\alpha_0\beta}{\omega} + a\sin\varphi\right]\alpha_0, \quad (15)$$

$$b_1 = e^{2\beta R}\sin 2\beta\alpha_0\left[R_r e^{2\beta R}\frac{(R^2 - \alpha_0^2)C - 2R\alpha_0 Ctg\alpha_0\beta}{\omega} + \right.$$

$$\left. -(R_r + 1)a\omega\cos\varphi\right] - e^{2\beta R}\frac{RC - \alpha_0 Ctg\alpha_0\beta}{\omega}R_r e^{2\beta R} \times$$

$$\times(R\sin 2\beta\alpha_0 + \alpha_0\cos 2\beta\alpha_0) + e^{2\beta R}\frac{RC - \alpha_0 tg\alpha_0\beta}{\omega} +$$

$$-a\alpha_0 e^{2\beta R}\sin\varphi\cos 2\beta\alpha_0 + a\sin\varphi\left[R_r e^{2\beta R}(R\sin 2\beta\alpha_0 + \right.$$

$$\left. + \alpha_0\cos 2\beta\alpha_0)\right] - e^{-2\beta R}R\sin 2\beta\alpha_0,$$

$$b_0 = R_r \sin 2\beta\alpha_0\frac{(R^2 - \alpha_0^2)C - 2R\alpha_0 Ctg\alpha_0\beta}{\omega} +$$

$$+ C\frac{R - \alpha_0 tg\alpha_0\beta}{\omega}R_r e^{4\beta R}(R\sin 2\beta\alpha_0 + \alpha_0\cos 2\beta\alpha_0) +$$

$$-a\sin\varphi\cos 2\beta\alpha_0\left[R_r e^{4\beta R}(R\sin 2\beta\alpha_0 + \alpha_0\cos 2\beta\alpha_0)\right] +$$

$$-\sin 2\beta\alpha_0\left[R_r(R\cos 2\beta\alpha_0 - \alpha_0\sin 2\beta\alpha_0)\right].$$

Note that C(s) should be earlier obtained using a similar approach but without the perturbations.

Therefore, the problem of stability is reduced to consideration of the second order charakteristic equation (13). If the roots of Eq. (14) are $|\gamma_{1,2}| < 1$, then according to the assumed solutions (13) δC_l, δD_l and $\delta\varphi_l$ approach zero for $l \to +\infty$, and the solutions will be asymptotically stable. We can easily estimate the stability regions, which are defined by the following inequalities

$$\left|\frac{b_2}{b_0}\right| < 1 \quad \text{and} \quad \left|\frac{b_1}{b_0 + b_2}\right| < 1. \quad (16)$$

Taking into account Eq. (15) it is easy now to find parameters of the system (or a delay loop) which fulfil inequalities (16). Additionally, because of some mechanical reasons, we have $x(t) \leq s$.

SIMULATION RESULTS

During numerical simulations we have used the MATLAB-Simulink package and the MATLAB-model for equation (1) with the boundary conditions (Fig. 2).

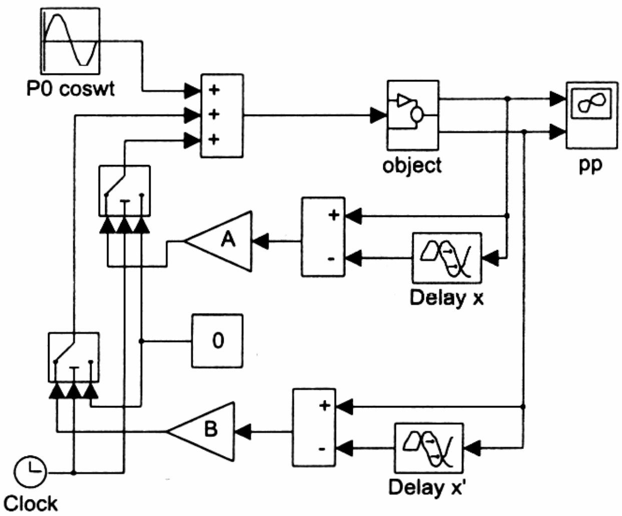

Figure 2. MATLAB-Simulink model of the vibro-impact system presented in Fig. 1

In Fig. 2 additional control of the delay loop is used. If the magnitude of the signal in this loop is smaller than $|\mu|$, then we know that the stabilised periodic orbit is achieved. We have taken the following parameters:

$$m = 1[kg],\quad k = k_1 = 1\left[\frac{N}{m}\right],\quad c_1 = 0{,}0\left[\frac{Ns}{m}\right],$$

$$x_0 = 1[m],\quad T = 8[s],\quad R_r = 0.65,\quad s = 0{,}4[m],$$

$$a = 0{,}12\left[\frac{N}{m}\right],\quad b = -0{,}05\left[\frac{Ns}{m}\right].$$

For these parameters according to (7) we get R = -0,0598, which shows that the delay loop control coefficients A and B allow us to obtain quicker damping of free oscillations in the solution (6) than without control. Additionally, for the given parameters we have found from Eq. (14) that $|\gamma_{1,2}|$ are lying closer to the origin for the system with the control coefficients than without control.

For the given parameters numerical simulations confirm analytical predictions. In Fig. 3 phase portraits of the analysed system are given.

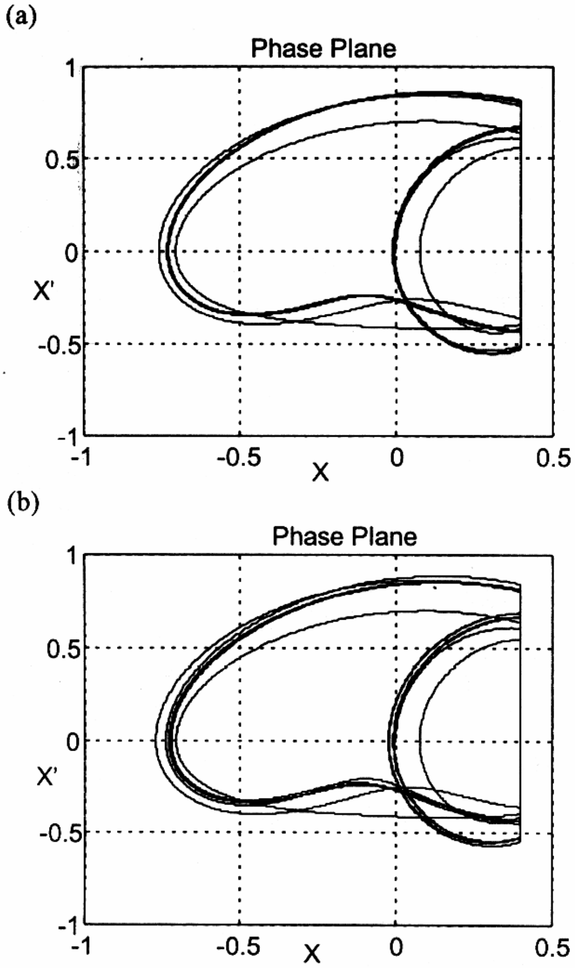

Figure 3. Phase portraits of the perturbation decay with control (a) and without control(b)

It can be seen that with control the transients vanish much more quickly than in the case without control. In the case presented above the periodic orbit is achieved after about 50 sec. for the system analysed without the delay loop and after 34 sec. for the system analysed with the delay loop ($|\mu| \leq 10^{-3}$), respectively.

CONCLUDING REMARKS

In this paper we have presented an analytical approach to estimate the delay control coefficients for efficient stabilisation or destabilisation of the periodic orbit under consideration. Although the efficiency of the method is presented for k=1 (periodic orbit with the same period as the excitation period) but our considerations are also valid for subharmonics (for arbitrarily taken k > 1). The validity of our analytical approach has been testified by numerical simulations. The obtained results give a good prognosis for analytical control of two-degree-of-freedom vibro-impact systems.

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