Paper presented at the 1st International Conf. on Motion and Vibration Control, Yokohama, September 1992
Session : Application of Control Theory and Control Method IV

VIBRATION CONTROL OF NONLINEAR DISCRETE-CONTINUOUS SYSTEMS WITH DELAY

Jan Awrejcewicz
Research Center for Advanced Science and Technology, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153, Japan. On leave from: Technical University, Division of Dynamics and Control (K-13), B. Stefanowskiego 1/15, 90-924 Lodz, Poland
Tsuneo Someya
Musashi Institute of Technology, Department of Mechanical Engineering, 1-28-1 Tamazutsumi, Setagaya-ku, Tokyo 158, Japan

ABSTRACT

The paper presents an analytical method of determining one-frequency periodic oscillations in nonlinear autonomous discrete-continuous mechanical systems with time delay, on the basis of the asymptotic approach. The periodic solutions are sought in the form of some particular asymptotic series in relation to two independent bifurcation parameters - one is related to nonlinearity, and the other to delay. Some technical problems, which can only be solved using this approach, are demonstrated.

KEY WORDS: perturbation technique, bifurcation points.

INTRODUCTION

One of the important problems of mechanics and automatic control engineering is active control of the oscillations of the mechanical objects by means of control units, which can frequently be treated as inertial systems with concentrated parameters and time delay [1]. The objects subject to control can be nonlinear mechanical systems with concentrated (further referred to discrete mechanical systems) or distributed parameters. The latter, referred to as continuous systems, are dealt with in this paper.

In real control systems of this type, the control unit influences the object subject to control and the state of the controlled object is monitored only in certain isolated points. It is usually possible to find controlled objects, which are governed by partial differential nonlinear equations as well as control units, which can be modelled by ordinary nonlinear differential equations.

As has been mentioned above, the systems governed by nonlinear partial and ordinary equations have many technical applications and they are considered in this work. It is a continuation of earlier work, where the two-variable asymptotic expansion technique has been used to analyze periodic oscillations in nonlinear parametrically excited mechanical systems [2-4], bifurcated oscillations [5,6] as well as oscillations in discrete-continuous systems. The presented research develops the approach from [7], where similar systems were sought in the form of power series of two independent perturbation parameters. The recurrent set of linear differential equations obtained by means of comparing the expressions found at the same powers of two perturbation parameters were then solved using the harmonic balance method. Using this approach, however, enables one to analyze only the steady states of the considered mechanical systems. The technique developed here is more universal. By the use of such a method the steady and unsteady (transient) oscillation can be analyzed and, as it will be shown in a future paper, static-type catastrophes during oscillations can be detected.

The presented technique is a generalization of classical asymptotic methods, which are widely treated in the literature [8-17], to the analysis of discrete-continuous mechanical systems governed by partial and ordinary nonlinear equations with two independent parameters.

METHOD

Let us consider a discrete-continuous system governed by the following equations:

$$\frac{\partial^2 u(t,x)}{\partial t^2} = L_x^{(2m)}\{u(t,x)\} + \epsilon f_1\{x, u(t,x), y(t-\mu)\},$$

$$\frac{dy(t)}{dt} = \sum_{p=0}^{P} A_p y(t-\tau_p) + \epsilon F_1\{y(t-\mu), u(t-\mu, t-\xi)\} \tag{1}$$

subject to the following non-homogeneous boundary conditions

$$L_x^{(h,j)}\{u(t,x)\}|_{x\epsilon S} = \epsilon g_{hj}\{y(t-\mu)\}; \qquad h = 1,\dots,m. \tag{2}$$

The coordinate t denotes time and $t \in R$; x is the vector of the coordinates and $x \in (G \cup S)$, while S is the limiting set of G; u(t,x) is a certain scalar function determined in the set $R \times G$ and $L_x^{(h,j)}$ is a linear operator of order 2m on x; $L_x^{(2m)}$ is the linear differential operator of $j \le 2m-1$; y and F_1 are vectors of an m - dimensional space; A_p are constant matrices of $(m \times m)$ order; F_1, f_1 and g_{hj} are functions of $y(t-\mu)$, $u(t-\mu,\xi)$, $\xi \in (G \cup S)$, while τ_p and μ are time delays. Finally, we assume that ϵ and μ are small positive parameters.
Thanks to this mathematical formulation of the problem, the presented analytical approach can be further used for many different discrete - continuous mechanical systems governed by equations (1).
The problem including non-homogeneous boundary conditions (2) can be reduced [1,7] to one of homogeneous boundary conditions. Thus we analyze the following system:

$$\frac{\partial^2 v(t,x)}{\partial t^2} = L_x^{(2m)}\{v(t,x)\} + \epsilon f\{x, v(t,x), y(t-\mu)\},$$

$$\frac{dy(t)}{dt} = \sum_{p=0}^{P} A_p y(t-\tau_p) + \epsilon F\{y(t-\mu), v(t-\mu), \xi)\} \tag{3}$$

where $v(t,x)$ fulfils the homogeneous boundary conditions

$$L_x^{(h,j)}\{v(t,x)\}|_{x\epsilon S} = 0, \qquad h = 1,\dots,m. \tag{4}$$

From the first equation of system (3), and for $\epsilon = 0$, we obtain

$$L_x^{(2m)}\{X(x)\} + \sigma X(x) = 0,$$

$$L_x^{(h,j)}|_{x\epsilon S} = 0, \qquad h = 1,\dots,m, \tag{5}$$

while from the other we obtain the following characteristic equation:

$$D(\rho) = det\left\{\sum_{p=0}^{P} A_p e^{-\tau_p \rho} - E\rho\right\}. \tag{6}$$

In the considered dynamical system, oscillations will appear if $\sigma_s = \omega_{vs}^2$ and (or) if the characteristic equation (6) has imaginary eigenvalues $\rho_k = \pm i\omega_{yk}$. In this research we shall consider the case where $\sigma_1 = \omega_{v_1}^2 = \omega_1^2$ and the other eigenvalues of the first equation of the system (5) amount to $\sigma_s \ne \{(p/q)\omega_1\}^2$, where p and q are integers. Moreover, it is assumed that the characteristic equation (6) does not possess imaginary eigenvalues. We seek a one-frequency solution of the dynamic system (1) with the frequency approaching ω_1 for $\epsilon \to 0$ and $\mu \to 0$. To this aim the approach suggested by Krylov-Bogolubov-Mitropolski will be used. We look for a solution in the form

$$v(t,x) = a(t)X_1(x)\cos\psi t + \sum_{k=1}^{K}\sum_{l=0}^{L} \epsilon^k \mu^l V_{kl}\{x, a(t), \psi(t)\},$$

$$y(t) = \sum_{k=1}^{K}\sum_{l=0}^{L} y_{kl}\{a(t), \psi(t)\}, \tag{7}$$

where

$$\frac{da}{dt} = \sum_{k=1}^{K}\sum_{l=0}^{L}\epsilon^k\mu^l A_{kl}\{a(t)\},$$

$$\frac{d\psi}{dt} = \omega_1 + \sum_{k=1}^{K}\sum_{l=0}^{L}\epsilon^k\mu^l B_{kl}\{a(t)\}, \tag{8}$$

and $X_1(x)$ is the solution of the boundary problem (5). Proceeding calculations with a similar way to the standard KBM-method we have got the following sequence of the recurrent differential equations

$$\omega_{kl}^2\frac{\partial^2 V_{kl}(x,a,\psi)}{\partial\psi^2} = L_x^{(2m)}\{V_{10}\} + 2\omega_1 B_{kl}X_1 a\cos\psi +$$

$$2\omega_1 A_{kl}X_1\sin\psi + f_{\epsilon^k\mu^l}(x,a,\psi),$$

$$\omega_1\frac{\partial y_{kl}(a,\psi)}{\partial\psi} = \sum_{p=0}^{P}A_p y_{kl}(a,\psi-\tau_p\omega_1) + F_{\epsilon^k\mu^l}(a,\psi); \tag{9}$$

where, for example, for $k\le 2$ an l=1 we have:

$$f_\epsilon = f(x,v_0),$$

$$F_\epsilon = F(x,v_0),$$

$$f_{\epsilon^2} = \frac{\partial f}{\partial v}V_{10} + \sum_{l=1}^{m}\frac{\partial f}{\partial y_l}y_{(10)l} - 2\omega_1 B_{10}\frac{\partial^2 V_{10}}{\partial\dot\psi^2}$$

$$-2\omega_1 A_{10}\frac{\partial^2 V_{10}}{\partial a\partial\psi} - \left(A_{10}\frac{dA_{10}}{da} + B_{10}^2 a\right)X_1\cos\psi$$

$$-\left(2A_{10}B_{10} + a\frac{dB_{10}}{da}A_{10}\right)X_1\sin\psi,$$

$$F_{\epsilon^2} = \frac{\partial F}{\partial v}V_{10} + \sum_{l=1}^{m}\frac{\partial F}{\partial y_l}y_{(10)l} - \sum_{l=1}^{m}\frac{\partial y_{(10)}l}{\partial a}A_{10},$$

$$f_{\epsilon\mu} = 0,$$

$$F_{\epsilon\mu} = \frac{\partial F}{\partial\dot v_1}a\omega_1 X_1\sin\psi. \tag{10}$$

During the calculations y and v are expressed as power series,

$$y(t-\mu) = \sum_{n=0}^{N}\frac{1}{n!}\frac{d^n y(t)}{dt^n}(-\mu)^n,$$

$$v(t-\mu,\xi) = \sum_{n=0}^{N}\frac{1}{n!}\frac{d^n v(t,\xi)}{dt^n}(-\mu)^n, \tag{11}$$

and calculations are limited to the value of n=1. After expanding the function $f_{(\bullet)}$ into a Fourier series one obtains

$$f_{(\bullet)} = \sum_{n=1}^{\infty}\{b_{(\bullet)n}(a)\cos n\psi + c_{(\bullet)n}(a)\sin n\psi\}, \tag{12}$$

where

$$b_{(\bullet)n}(a) = \frac{1}{2\pi l}\int_0^l dx\int_0^{2\pi}f_{(\bullet)}(x,a,\psi)X_1(x)\cos n\psi\, d\psi,$$

$$c_{(\bullet)n}(a) = \frac{1}{2\pi l}\int_0^l dx\int_0^{2\pi}f_{(\bullet)}(x,a,\psi)X_1(x)\sin n\psi\, d\psi. \tag{13}$$

If we equate the coefficients of $X_1(x)\sin\psi$ and $X_1(x)\cos\psi$ to zero, we obtain A_{kl} and B_{kl}. According to (8) we get

$$\Phi(a)=\frac{da}{dt}=\epsilon A_{10}+\epsilon^2 A_{20}+\epsilon^3 A_{30}+\epsilon\mu A_{11}$$

$$+\epsilon^2\mu A_{21}+\epsilon\mu^2 A_{12}+0(\epsilon^k\mu^l;\ \ k+l=4),$$

$$\omega(a)=\frac{d\psi}{dt}=\omega_1+\epsilon B_{10}+\epsilon^2 B_{20}+\epsilon^3 B_{30}+\epsilon\mu B_{11}$$

$$+\epsilon^2\mu B_{21}+\epsilon\mu^2 B_{12}+0(\epsilon^k\mu^l;\ \ k+l=4), \tag{14}$$

at the initial conditions $a(t_0)=a_0$, $\psi(t_0)=\psi_0$.
From the first equation of (14) we obtain the dependence a(t), which upon introduction into the latter equation of (14) enables us to determine the dependence $\psi\{a(t)\}$. Thanks to this it is possible to analyze the slow transient processes leading to steady state. The latter are analyzed by assuming that da/dt=0, which leads to the algebraic equation

$$G(a,\epsilon,\mu)\ =\ A_{10}+\epsilon A_{20}+\epsilon^2 A_{30}+\mu A_{11}+\epsilon\mu A_{21}+\mu^2 A_{12}=0. \tag{15}$$

If the calculations are limited up to order ϵ, we get from (15)

$$A_{10}\ =\ 0, \tag{16}$$

which enables us to find: (a) one isolated solution; (b) few isolated solutions; (c) no solutions. However, sometimes the phase flow of the considered starting equations can be very sensitive to changes in the amplitude "a" and (or) the parameters ϵ and μ. For these reasons the full equation (15) should be taken into consideration. The solution of (16) can serve as a first approximation for the numerical solution of the full equation (15). Now we briefly indicate the variety of problems which can be solved using this approach, and that can not be solved by the use of a single perturbation method. A. Suppose that the parameter ϵ undergoes slight changes, which are impossible to avoid. We want to control such changes by treating μ as a control parameter. Inserting $a=a^0=const$ into (15) we can find $G(\epsilon,\mu,a^0)=G(\epsilon,\mu)=0$. Thus, in accordance with the changes of ϵ we can find the values of μ in order to maintain a constant amplitude. B. Suppose that we would like to have $a=a(\epsilon)$ and because the shape of $a(\epsilon)$ should be fixed a priori. The problem is then again reduced to the implicit algebraic functions of second order. Equation (15) is transformed into the form

$$A_{30}\epsilon^2+2A'_{21}\epsilon\mu+A_{12}\mu^2+2A'_{20}\epsilon+2A'_{11}\mu+A_{10}=0, \tag{17}$$

where

$$A'_{21}=\frac{1}{2}A_{21},\qquad A'_{20}=\frac{1}{2}A_{20},\qquad A'_{11}=\frac{1}{2}A_{11}. \tag{18}$$

Equation (17) presents implicit second-order algebraic functions if A_{30}, A'_{21} and A_{12} are not equal to zero at the same time. The form of the function is determined by the following expressions:

$$W=\det\begin{pmatrix}A_{30} & A'_{21} & A'_{20}\\ A'_{21} & A_{12} & A'_{11}\\ A'_{20} & A'_{11} & A_{10}\end{pmatrix},\quad V=\det\begin{pmatrix}A_{30} & A'_{21}\\ A'_{21} & A_{12}\end{pmatrix},\quad S=A_{30}+A_{12},$$

$$W_{22}=A_{30}A_{10}-(A'_{20})^2,\quad W_{11}=A_{12}A_{10}-(A'_{11})^2. \tag{19}$$

By means of shifting the origin of the coordinate system and turning the axis, it is possible to obtain the following functional forms (expressions W, V, S are the invariants of such shifts and turns):

1. $V>0$, $AW<0$. Curve (17) is the ellipse $\epsilon^2/A^2+\mu^2/B^2=1$.

2. $V>0$, $W=0$. Equation (17) can be transformed to $\epsilon^2/A^2+\mu^2/B^2=0$ and the solution is point (0,0).

3. $V>0$, $AW>0$. Curve (17) is an imaginary ellipse (no real curve exists).

3. $V>0$, $AW>0$. Curve (17) is an imaginary ellipse (no real curve exists).

4. $V<0$, $W\neq 0$. Equation (17) is the equilateral hyperbola $\epsilon^2/A^2-\mu^2/B^2=1$.

5. $V<0$, $W=0$. The solution of (17) is a pair of intersecting lines $\epsilon^2/A^2-\mu^2/B^2=0$.

6. $V=0$, $W\neq 0$. The curve governed by (17) is a parabola $\mu^2=2p\epsilon$.

7. $V=0$, $W=0$, $W_{11}<0$ or $W_{22}>0$. Equation (17) presents a pair of parallel lines $\mu^2-A^2=0$.

8. $V=0$, $W=0$, $W_{11}>0$ or $W_{22}>0$. The solution of (17) are imaginary parallel lines $\mu^2+A^2=0$ (no real curve exists).

9. $V=0$, $W=0$, $W_{11}=0$ or $W_{22}=0$. The solution of (17) is a double line $\mu^2=0$.

The coefficients of the equation (17) are functions of the amplitude a and their values are determined by the functions $f_{(\bullet)}$.
C. Different branching phenomena can be expected. We can find the hysteresis variety points defined by the following equations:

$$G(a,\epsilon,\mu) = 0, \; G_a(a,\epsilon,\mu) = 0, \; G_{aa}(a,\epsilon,\mu) = 0. \tag{20}$$

If it is possible to eliminate the amplitude "a" from one of the equations (20), then the other two enable us to for find the hysteresis points.
The bifurcation and isola variety points are defined by the following three equations:

$$G(a,\epsilon,\mu) = 0, \; G_a(a,\epsilon,\mu) = 0, \; G_\epsilon(a,\epsilon,\mu) = 0. \tag{21}$$

As mentioned above, equation (21) can possess several different solutions for "a". Thus m-multiply limit variety can be defined by the following equations:

$$G(a_1,\epsilon,\mu) = 0, \; \ldots, \; G(a_m,\epsilon,\mu) = 0,$$

$$G_a(a_1,\epsilon,\mu) = 0, \; \ldots, \; G_a(a_m,\epsilon,\mu) = 0. \tag{22}$$

Using μ as a parameter, we can control the branching phenomena mentioned above. D. We can find the (ϵ,μ) set of parameters for which no real solutions of (15) exist. Thus, a domain of the assumed solution (7) can be defined in the two-parameter space. E. Suppose that we want to change the amplitude of oscillations, but the frequency of oscillations should not undergo any changes (or it should be controlled only by the linear part of the equations). In order to fulfil such requirements we have

$$G(a,\epsilon,\mu) = A_{10}+\epsilon A_{20}+\epsilon^2 A_{30}+\mu A_{11}+\epsilon\mu A_{21}+\mu^2 A_{12}=0,$$

$$H(a,\epsilon,\mu) = B_{10}+\epsilon B_{20}+\epsilon^2 A_{30}+\mu B_{11}+\epsilon\mu B_{21}+\mu^2 B_{12}=0. \tag{23}$$

After eliminating "a" from one of equations (23) there remains one equation, which defines the implicit algebraic function of second order in ϵ and μ. One can freely choose one parameter and then calculate the value of the second one. Thus, by such an appropriate choice of the parameters ϵ and μ the amplitude of the one-frequency oscillations will change, however, the frequency ω_1 will always remain constant.

CONCLUDING REMARKS

This paper has presented a local analytical method for determining the periodic one-frequency oscillations in dynamical nonlinear discrete-continuous systems with delay. This method employed the classical KBM technique (Krylov - Bogolubov - Mitropolskii) and, in a new approach, the solution is sought in the form of certain power series in terms of two independent perturbation parameters ϵ and μ. The former is connected with nonlinearity and the latter with time delay. It is assumed that both parameters are small, and the amplitude of oscillations is small.
Thanks to this method the problem of analyzing the transient nonstationary states leading to the steady state has been reduced to the analysis of two

first order differential equations. The first is an equation with separable variables, and its solution after its introduction into the second enables us to determine how the frequency of the sought solution changes in time, and the influence of the parameters μ and ϵ, which appear explicitly in the solution.

A general discussion of the benefits of using the two-perturbation technique is provided. Such problems, important from the point of view of applications, are demonstrated. These problems can not be solved by the use of a classical single-perturbation technique.

REFERENCES

1. Rubanik, W. P., Vibrations in Complex Quasilinear Systems with Delay, University Press, Minsk (1985), in Russian.
2. Awrejcewicz, J., In: Proceedings of the International Conference on Rotordynamics, Tokyo (1986), pp. 517-522.
3. Awrejcewicz, J., Intern. J. of Non-Linear Mech. 23-1(1988), pp. 87-94.
4. Awrejcewicz, J., Korean Soc. of Mech. Eng. J. 4-2(1990), pp. 156-166.
5. Awrejcewicz, J., PAN American Congress of Applied Mechanics, Rio de Janeiro(1988), pp. 640-643.
6. Awrejcewicz, J., Bifurcation and Chaos in Simple Dynamical Systems, World Scientific, Singapore(1989).
7. Awrejcewicz, J., Int. J. of Solids and Struct. 27-7(1991), pp. 825-832.
8. Malkin, I. G., Some Problems in the Theory of Nonlinear Oscillations, Nauka, Moscow (1956), in Russian.
9. Giacaglia, G. E., Perturbation Methods in Non-Linear Systems, Springer Verlag, New York (1972).
10. Bogolubov, N. N., Mitropolski, Y. A., Asymptotic Methods in the Theory of Non-Linear Oscillations, Hindustan Publication Corporation, Delhi (1961).
11. Hale, J. K., Oscillations in Nonlinear Systems, McGraw Hill, New York (1963).
12. Nayfeh A. H., Mook, D. T., Nonlinear Oscillations, Wiley Interscience, New York (1981).
13. Nayfeh, A. H., Perturbation Methods, Wiley Interscience, New York (1979).
14. Nayfeh, A. H., Introduction to Perturbation Techniques, Wiley Interscience, New York (1981).
15. O'Malley, R. E., Introduction to Singular Perturbations, Academic Press, New York and London (1974).
16. Moiseev, N. N., Asymptotic Methods in Nonlinear Mechanics, Nauka, Moscow, (1981), in Russian.
17. Bajaj, A. K., Johnson, J. M., Intern. J. of Applied Mechanics 25-2/3(1990), pp. 211-226.