INFLUENCE OF FRICTION ON THE CHAOTIC DYNAMICS IN COUPLED OSCILLATORS.

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Abstract - A route to chaos in the system with dry friction is analyzed. In spite of the complexity of the system, a similar transition to that discovered in the two-well potential anharmonic oscillator is described and illustrated. Dry friction weakens the chaotic dynamics and induces the occurrence of stick and slip transitions during the chaotic wandering of the trajectory in four dimensional phase space.

1. INTRODUCTION

The aim of this paper is to show the "qualitative universal" transition to chaos in a certain subclass of sinusoidally-driven nonlinear oscillators, i. e., systems with a two-well potential. The question of interest is whether or not the scenarios leading to chaotic orbits discovered in simple uncoupled oscillators are likely also hold for much more complicated systems, such as coupled nonlinear sinusoidally driven oscillators. Simulation experiments show that the potential has two wells, and that chaotic dynamics will obtain, in which each of the oscillators jumps between two wells in an unpredictable way.

2. THE SYSTEM

We consider a system of two coupled mechanical oscillators, both of which are externally driven. The governing equations are

$$m_1\ddot{x}_1+(C_3-C_1)\dot{x}_1-C_3\dot{x}_2+C_2x_1^2\dot{x}_1+(k_1+k_3)x_1$$
$$-k_3x_2+k_2x_1^3 \;+\mu m_1 g\,sgn(\dot{x}_1)= \;q_1\cos(\omega_1 t+\phi),$$
$$m_2\ddot{x}_2+(C_3-C_4)\dot{x}_2-C_3\dot{x}_1+C_5x_2^2\dot{x}_1+(k_3+k_4)x_2$$
$$-k_3x_1+k_5x_2^3 \;=\; q_2\cos(\omega_2 t), \qquad (1)$$

where m_1 and m_2 are the masses of the oscillators, C_1-C_5 and k_1-k_5 are damping and stiffness coefficients, respectively q_1 and q_2 are the amplitudes of the exciting forces with corresponding frequencies ω_1 and ω_2, and ϕ denotes a phase shift between the exciting forces.

In nondimensional form we have

$$\xi_1''+(\alpha_3-\alpha_1)\xi_1'-\alpha_3(KM^{-1})^{0.5}\xi_2'+\gamma_1\xi_1^2\xi_1'+(\kappa_1+\kappa_3)\xi_1$$
$$-\kappa_3(KM^{-1})^{0.5}\xi_2+\xi_1^3+R\,sgn(\xi_1') \;=\; B_1\cos(\tau+\phi),$$
$$\xi_2''+M(\alpha_3-\alpha_4)\xi_2'-M\alpha_3(MK^{-1})^{0.5}\xi_1'+\gamma_2K\xi_2^2\xi_2'+M(\kappa_3+\kappa_4)\xi_2$$
$$-M\kappa_3(MK^{-1})^{0.5}\xi_1+\xi_2^3 \;=\; M^{1.5}K^{-0.5}B_2\cos(\nu\tau), \quad (2)$$

where

$$\tau=\omega_1 t,\ \xi_1=(\omega_1 m_1^{0.5})^{-1}k_2^{0.5}x_1,\ \xi_2=(\omega_1 m_2^{0.5})^{-1}k_5^{0.5}x_2,$$
$$M=m_1m_2^{-1},\quad K=k_2k_5^{-1},\quad \nu=\omega_2\omega_1^{-1},$$
$$B_1=q_1\omega_1^{-3}m_1^{-1.5}k_2^{0.5},\ B_2=q_2\omega_1^{-3}m_1^{-1.5}k_2^{0.5},\ \alpha_1=C_1(m_1\omega_1)^{-1},$$
$$\alpha_3=C_3(m_1\omega_1)^{-1},\ \alpha_4=C_4(m_1\omega_1)^{-1},\ \kappa_1=k_1m_1^{-1}\omega_1^{-2},$$
$$\gamma_1=\omega_1C_2k_2^{-1},\quad \gamma_2=\omega_1C_4k_2^{-1},\quad \kappa_3=k_3m_1^{-1}\omega_1^{-2},$$
$$\kappa_4=k_4m_1^{-1}\omega_1^{-2},\quad R=\mu g\omega_1^{-3}k_2^{0.5}m_1^{-0.5}. \qquad (3)$$

Using the transformations (3), the nineteen parameters of equations (1) are reduced to fourteen parameters in (2).

Such a general system has been investigated earlier by the authors [1-3] using a systematical numerical approach. Transitions between quasiperiodic, strange chaotic and strange non-chaotic attractors have been reported as well as some special chaotic dynamics has been discussed and illustrated. Here an attention is focused on the influence of friction of the chaotic dynamics on the mentioned above system.

3. NUMERICAL ANALYSIS

We define: $F_{st}=\mu m_1 g$,

$$F=(k_1+k_3)x_1-k_3x_2+k_2x_1^3-c_3\dot{x}_2-q_1\cos\omega_1 t. \qquad (4)$$

When $\dot{x}_1=0$ and $|F|<|F_{st}|$, the first oscillator is in a stick state. During the transition from a slip-state to a stick-state, an acceleration jump occurs. During an exit from a stick-state to a slip-state the acceleration is continuous, but a jump in the third derivative of the displacement appears. The velocity in both cases is always continuous. During "the transition over a stick area", but without sticking, the acceleration jump also occurs. When a regular transition, however, from a slip-state to a stick takes place with $|F|<|(F_{st})_{max}|$, the one eigenvalue of the Jacobi matrix is equal to zero. When $|F|=|(F_{st})_{max}|$ and the acceleration jump does not appear, the Jacobi matrix is not singular. The following equations govern the dynamics in the stick-state:

$$\xi_1=0,$$
$$\xi_2''+M(\alpha_3-\alpha_4)\xi_2'+\gamma_2K\xi_2^2\xi_2'+M(\kappa_3+\kappa_4)\xi_2$$
$$-M^{1.5}\kappa_3K^{-0.5}\xi_1+\xi_2^3 \;=\; M^{1.5}K^{-0.5}B_2\cos\nu\tau. \qquad (5)$$

with the following transition-condition

$$R > |(\kappa_1+\kappa_3)\xi_1 - \kappa_3(K/M)^{0.5}\xi_2 + \xi_1^3$$
$$-\alpha_3(K/M)^{0.5}\xi_2' - B_1\cos\tau|. \qquad (6)$$

We consider the behavior of the system (2) for the following fixed parameters: $\nu = M = K = 1.0$, $\kappa_1 = \kappa_4 = -0.816326$, $\phi = 0.0$, $\gamma_1 = \gamma_2 = 0.3$, $B_1 = 0.05$, $B_2 = 0.2$, and for two values of friction R.

Example 1 (R=0.05).

We take $\alpha_1 = \alpha_4 = 0.01$ and $\alpha_3 = \kappa_3 = 0.3$. For these parameters and without friction (R=0), the system exhibits intermittent chaos. Friction dampens the chaotic dynamics of the orbits and for R=0.05 we find a quasiperiodic attractor. In the neighborhood of these parameters (for $\alpha_1 = \alpha_4 = 0.05$ and $\alpha_3 = \kappa_3 = 0.3$), a periodic attractor is found. An increase in α_1 ($\alpha_1 = \alpha_4$) results in an increase in the magnitude of the self-excited oscillations. The periodic orbit grows and finally leads to intersections of the stable and unstable manifolds and a trajectory starts to wander in an unpredictable way between two potential wells. This situation is illustrated for $\alpha_1 = \alpha_4 = 0.2$ in Fig.1.

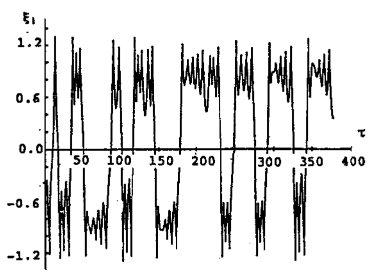

Fig.1. Time history of a strange chaotic attractor (R=0.05).

Example 2 (R=0.1).

In the second example we analyze the influence of the coupling between two oscillators. The numerical calculations have been carried out for the same parameters as in Example 1 and additionally for R=0.1 and $\alpha_1 = \alpha_4 = 0.2$. When two oscillators are strongly coupled ($\alpha_3 = 2.0$, $\kappa_3 = 0.3$), a periodic orbit is found. This orbit lies to the right of the origin. However, to the left of the origin there is also another small periodic orbit. These two orbits lie in two isolated potential wells. Decreasing α_3 causes the trajectory to move from the potential well and start to wander between the two potential wells (Fig.2). The escape, however, from one of the wells to the other is rather rare. The possibility of it occurring increases with a further decrease in α_3. For example, for $\alpha_3 = 0.3$, one of the projections of the Poincaré map shows a very complicated dynamics (Fig.3).

In order to understand how two oscillators move in a chaotic manner, two time histories (for relatively long time intervals of the same chaotic attractors) are presented in Fig.4. In this figure one can also observe stick states. These states correspond the a very short horizontal parts of $\xi_1(\tau)$.

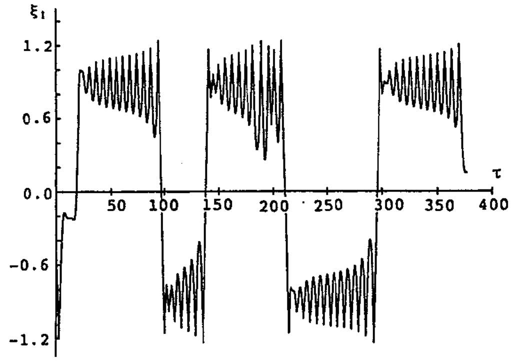

Fig.2. Time history for $\alpha_3 = 0.8$ (R=0.1).

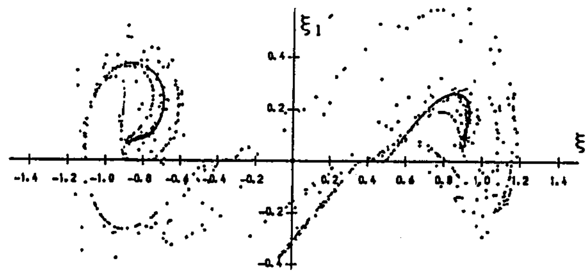

Fig.3. A strange chaotic attractor for $\alpha_3 = 0.3$

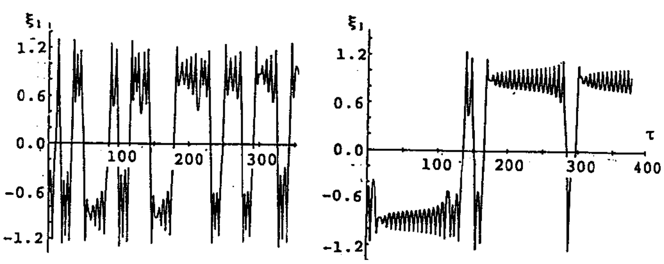

Fig.4. Two different time histories from the same chaotic attractor.

4. CONCLUSIONS

In the six-dimensional nonlinear mechanical system with friction that was investigated, quasiperiodic and chaotic attractors are detected. We have discussed and illustrated that in this case, the route to chaos is the same as in the simple two-well potential, sinusoidally-driven oscillator. An investigation of the influence of dry friction on the chaotic behavior of two coupled oscillators shows that increasing the friction weakens the chaotic dynamics of the orbits. During the chaotic motion of the first oscillator, stick-slip transitions are observed.

REFERENCES

1. J. Awrejcewicz, W.-D. Reinhardt, Some Comments About Quasi-Periodic Attractors, Journal of Sound and Vibration 139(2), 1990, 347.

2. J. Awrejcewicz, W.-D. Reinhardt, Quasiperiodicity, Strange Non-Chaotic and Chaotic attractors in the Forced System with Two Degrees of Freedom, Journal of Applied Mathematics and Physics ZAMP, 41, 1990, 713.

3. J. Awrejcewicz, W.-D. Reinhardt, Observation of Chaos in the Nonautonomous System with Two Degrees of Freedom, Journal of Applied Mathematics and Mechanics (in press).