CHAOTIC DYNAMICS OF A ROTOR WITH THE RECTANGULAR CROSS–SECTION

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ABSTRACT

Chaotic dynamic of the three degrees–of–freedom system is investigated in this paper. A rotor 2 with the rectangular cross–section is fixed in the rigid bearings of a rigid frame 1. The frame, which can move only horizontally, is nonlinearly supperted (see Fig.1). There are two harmonic excitations in the system. The first is caused by the unbalanced mass which is concentrated in the centre of the rotor. The second is an external excitation of the frame 1.

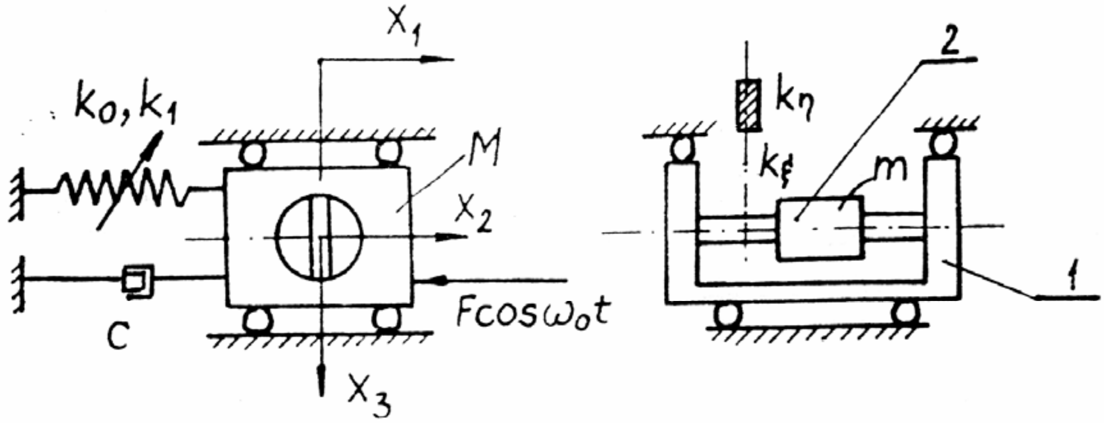

Fig. 1. The system analysed

The similar, but linear system, has been considered earlier by the author [1]. The use of the analytical perturbation method with two independent perturbation parameters has allowed investigating the effect of some particular parameters of the system for the shape of the limits of the unstable zones.

Based on the calculation model of the system under investigation presented in Fig. 2, and taking into account that a, ξ_r and η_r are small in relation to the inertia radius, the following governing equations are obtained:

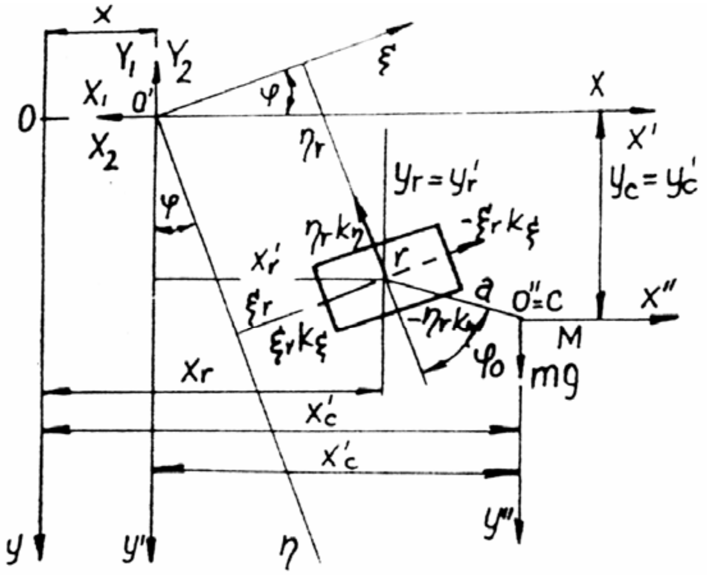

Fig. 2. The calculation model

$$
\begin{aligned}
&M\ddot{x}_1 + c\dot{x}_1 + k_0 x_1 + k_1 x_1^3 + x_1\left(\frac{k_\xi + k_\eta}{2} + \frac{k_\xi - k_\eta}{2}\cos 2\omega t\right) \\
&\quad - x_2\left(\frac{k_\xi + k_\eta}{2} + \frac{k_\xi - k_\eta}{2}\cos 2\omega t\right) + \\
&\quad + x_3\frac{k_\xi - k_\eta}{2}\sin 2\omega t = F\cos\omega_0 t \ , \\
&m\ddot{x}_2 - \left(\frac{k_\xi + k_\eta}{2} + \frac{k_\xi - k_\eta}{2}\cos 2\omega t\right)x_1 + \left(\frac{k_\xi + k_\eta}{2} + \frac{k_\xi - k_\eta}{2}\cos 2\omega t\right)x_2 \\
&\quad - x_3\frac{k_\xi - k_\eta}{2}\sin 2\omega t = ma\omega^2\sin(\omega t + \varphi_0) \ ; \\
&m\ddot{x}_3 + \frac{k_\xi - k_\eta}{2}x_3\sin 2\omega t - \frac{k_\xi - k_\eta}{2}x_2\sin 2\omega t + \\
&\quad + x_3\left(\frac{k_\xi + k_\eta}{2} - \frac{k_\xi - k_\eta}{2}\cos 2\omega t\right) = ma\omega^2\sin(\omega t + \varphi_0) + g \ .
\end{aligned}
\tag{1}
$$

Because of the small value of the moment of inertia $I_{2'} = mi_s^2$, the torsional vibrations are neglected and $\varphi = \omega t$. The following nomenclature is assumed additionaly: ξ_r, η_r are the coordinates of the point of puncture by the shaft in the coordinate system $0'\xi\eta$; $0'\xi\eta$ is the coordinate system, the axes of which are parallel to the main central inertia axes of the shaft cross–section; k_ξ, k_η are rigidities of the shaft in the direction of the axes ξ and η; a, φ_0 are the parameters characterising the placement of the mass centre of the disk c in relation to the point of puncture by the shaft; x_r, y_r are the coordinates of the point of puncture by the shaft in the Oxy system; X_1, Y_1 and X_2, Y_2 are the support reactions to the right and to the left tip of the shaft, respectively.

Thanks to the relations given below

$$\tau = (k_0 M^{-1})^{1/2} t ; \quad y_1 = k_1 k_0^{-1^{1/2}} x_1 ; \quad x_2 = 2mgy_2 / (k_\xi + k_\eta);$$
$$x_3 = 2mgy_3 / (k_\xi + k_\eta); \quad z = (k_\xi + k_\eta) / 2k_0 ; \quad b = (k_\xi - k_\eta) / 2k_0 ;$$
$$d = c / k_0 M^{-1/2}; \quad e = mg(k_1 k_0)^{-3^{1/2}} ; \quad v = \omega(M / k_0)^{1/2}; \quad \mu = m / M;$$
$$q = F(k_1 k_0)^{-3^{1/2}} ; \quad v_0 = \omega_0 (M / k_0)^{1/2}; \quad q_1 = \omega^2 a g^{-1} \cos \varphi_0 ;$$
$$q_2 = \omega^2 a g^{-1} \sin\varphi_0 , \tag{2}$$

the following nondimensional set of equations is obtained:

$$\ddot{y}_1 + d\dot{y}_1 + (1+z)y_1 + y_1^3 + by_1 \cos 2v\tau - ey_2 - \varepsilon e y_2 \cos 2v\tau$$
$$+ \varepsilon e y_3 \sin 2v\tau = q \cos v_0 \tau ; \tag{3}$$
$$\frac{\mu}{2}\ddot{y}_2 - \frac{2}{e} y_1 (1 + \varepsilon \cos 2v\tau) + (1 + \varepsilon \cos 2v\tau) y_2 - \varepsilon y_3 \sin 2v\tau = q_1 \sin v\tau + q_2 \cos v\tau ;$$
$$\frac{\mu}{2}\ddot{y}_3 + \frac{2}{e} \varepsilon y_1 \sin 2v\tau - \varepsilon y_2 \sin 2v\tau + y_3 (1 - \varepsilon \cos 2v\tau) - 1 = q_1 \cos v\tau + q_2 \sin v\tau ,$$

where $\varepsilon = \frac{b}{z} = \frac{k_\xi - k_\eta}{k_\xi - k_\eta}$, and $\cdot = d / d\tau$.

One can expect in the system governed by equations (3) very complicated dynamics when compared with the sinusoidally driven nonlinear oscillators. The equation set investigated has ten nondimensional parameters and the knowledge in full about the possible dynamics of the system analysed is practically impossible. This report concentrates only on the influence of some particular parameters for the evolution of chaotic orbits. Unfortunately, hitherto in order to trace the behaviour of chaotic orbits we have not been able to use a numerical technique based on solving a boundary value problem (shooting) as in the case of periodic orbits [2]. For this reason, in the investigations presented heve a numerical method based on solving an initial value problem is used. The equations (3) are numerically integrated with a Gear method.

First influence of a parameter q_1 on the behaviour of a strange chaotic attractor is analysed (the other parameters are fixed i.e. $z = 1.0$, $b = 0.0$, $d = 0.4$, $e = 10$, $\mu = 0.1$, $v = v_0 = 1.0$, $q = 0.4$, $q_2 = 0.0$). Note, that $b = 0.$ denotes also $\varepsilon = 0.$ ($k_\xi = k_\eta$) so in the system a parametric excitation does not appear and also a rotor oscillates in y_3 direction independently (only the variables y_1 and y_2 are coupled). For $q_1 = 0.1$ a periodic orbit has been found which with the increase in this parameter ($q = 0.2$) bifurcates into a narrow band chaotic attractor. A further increase in q_1 causes further bifurcations of this chaotic attractor. For example for $q_1 = 0.4$ there is another chaotic attractor, for $q_1 = 0.6$ a periodic orbit, for $q_1 = 0.8$ another periodic orbit, and then starting with $q_1 = 1.0$ a strange chaotic attractor appears again. In the interval of $q_1 \in [1.,10.0]$ this attractor remains chaotic. In the similar way we have investigated the influence of q_1 on the behaviour of the system (for the case where comparing with the previous one, three parameters are changed, i.e. $z = 0.1$, $e = 0.01$, $\mu = 10$). For this set of parameters we have found a chaotic motion of the mass M with small amplitudes and high frequencies where–as the mass m oscillates with very large amplitudes and low frequencies. The behaviour described above has not changed qualitatively in the interval of $q_1 \in [0.0,\ 10.0]$.

The second example shows the influence of the parametric excitation for $z = 0.1$, $q_1 = 10$, $e = 0.01$ (the other parameters are the same as in the first example). For $b = 0.001$ the frame 1 oscillates chaotically, whereas the mass m in a quasiperiodic manner. With an increase in b the situation has not changed qualitatively until the value $b = 0.1$ is reached. With a further increase in b (to the value of 0.8) the amplitudes of oscillation grow drastically and weak chaotic behaviour of the mass m is observed instead of the earlier quasiperiodic one. The system behaves in a qualitatively similar way also for $v = \pi$.

To summarize, the dynamics of the nonlinear subsystem 1 coupled lineary with the rotor with two degrees of freedom is numerically investigated. In the first case considered, when the rotor had a circle cross–section, bifurcation from regular to irregular motion, which accompanies the increase in q_1, is reported. We have also shown that two bodies of the system can behave in a different chaotic manner: the body 1 oscillates with small amplitudes and high frequencies, whereas the body 2 with very large amplitudes and low frequencies. In the second example the influence of the parametric excitation ε(in this case the rotor has a rectangular cross–section)for the chaotic attractor is reported.

REFERENCES

[1] J. Awrejcewicz, Vibration System: Rotor with self–Excited Support, Proc. Intern. Conf. Rotordyn. Tokyo, Sep. 14–17, 1986, pp. 517–522.

[2] J. Awrejcewicz, Three Examples of Different Routes to Chaos in Simple Sinusoidally Driven Oscillators, ZAMM (to appear).