# Free Vibration Analysis of an Auxetic Honeycomb Sandwich Shallow Shell with an Arbitrary Planform Resting on Pasternak Elastic Foundation

Lidiya Kurpa[1](✉), Tetyana Shmatko[2], and Jan Awrejcewicz[3]

[1] Department of Applied Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine
kurpalidia@gmail.com

[2] Department of Higher Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine
ktv_ua@yahoo.com

[3] Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, Lodz, Poland
jan.awrejcewicz@p.lodz.pl

**Abstract.** This paper proposes an approach for analyzing the free vibrations of sandwich shallow shells with an auxetic core exhibiting a negative Poisson's ratio. The face sheets are made of functionally graded materials (FGMs). The method allows to consider the panels with arbitrary planform geometries. The shell is assumed to be resting on a Pasternak elastic foundation. Mathematical formulation is based on the first order shear deformation theory (FSDT). The core's unit cell is modeled with a hexagonal configuration, and established analytical relations are used to determine its material properties. Power law distribution is applied to characterize the effective properties of the FGM face sheets. To address shells with arbitrary planforms, the R-functions theory is integrated with the variational Ritz method. The accuracy and efficiency of the proposed methodology are demonstrated through comparison with existing results as well as new results obtained for auxetic shells of complex geometries.

**Keywords:** arbitrary shape · R-functions theory · sandwich shallow shells with auxetic core · FGM · first order shear deformation theory · elastic foundation

## 1 Introduction

Sandwich structures are widely recognized in engineering due to their ability to combine lightweight characteristics with high stiffness and strength. The performance of sandwich structures is highly influenced by the choice of materials for both face sheets and the core. Face sheets can include functionally graded materials (FGMs), which are attractive for different applications. In recent years auxetic honeycomb core materials have


H. Altenbach et al. (Eds.): CAMPE 2025, LNME, pp. 262–272, 2026.
https://doi.org/10.1007/978-3-032-16381-3_25

gained significant attention due to their negative Poisson's ratio, which leads to effective energy absorption, impact resistance, and enhanced strength capacity compared to conventional core architectures. This class of materials is deformed in a counterintuitive manner, becoming thicker perpendicular to an applied stretch, making them ideal for advanced core designs in highly productive sandwich structures. Note that such structures can serve as protective armor, reducing overall weight while improving acoustic insulation, thermal resistance, and impact mitigation against collisions or shock waves from explosions [7]. Extensive studies have been explored the modeling and performance of sandwich panels within the frameworks of classical plate theory (CPT), first order shear deformation theory (FSDT), and higher-order shear deformation theories (HSDT). A recent comprehensive review [1] has highlighted the progress in research on sandwich structures with auxetic cores. That work offers an in-depth discussion of design strategies, classifications of auxetic geometries, and material choices for both the core and the face sheets. A wide range of studies has been focused on problems such as static bending, buckling loads, dynamic response plates, and both linear and nonlinear vibrations [2–14]. Free vibration analyses of auxetic plates were presented in [2–4], while works [5–8] examined auxetic sandwich plates resting on elastic foundations. Further investigations [9–11] were addressed the vibration behavior of FGM shells with auxetic cores. The questions of the design of doubly curved sandwich panels with honeycomb cores were considered in [12, 13].

Although significant progress has been made in the study of sandwich structures with FGM face sheets and auxetic cores, relatively little attention has been devoted to shallow sandwich shells with complex planforms. To address this gap, the present work introduces a semi-analytical approach based on the variational Ritz method in combination with the R-functions theory [14, 15].

## 2 Mathematical Model for Vibration Analysis of Sandwich Panels with Auxetic Core on Elastic Foundation Using FSDT

The study focuses on sandwich shallow shells (panels) with an auxetic honeycomb core and FGM face sheets supported by an elastic foundation (Fig. 1). The three-layer configuration includes a core of thickness $h_c = h_2 - h_1$ and two FGM face sheets placed on the top (T) and bottom (B), each with thickness $h_f$. The total shell thickness $h$ is defined by $h = 2h_f + h_c$. Planform can take complex shapes with different boundary conditions. The effective properties of the FGM sheets, made of ceramic and metal, vary along the thickness following a power law function.

$$E^{(i)}(z) = (E_c - E_m)V^{(i)} + E_m,\ \rho^{(i)}(z) = (\rho_c - \rho_m)V^{(i)} + \rho_m,\ i = T, B, \tag{1}$$

where

$$V^{(T)}(z) = \left(\frac{z - h_2}{h/2 - h_2}\right)^p,\ V^{(B)}(z) = \left(\frac{h_1 - z}{h_1 + h/2}\right)^p. \tag{2}$$

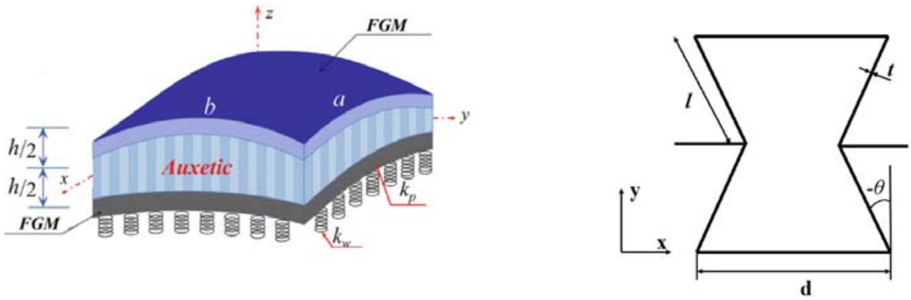


**Fig. 1.** Sandwich FGM shallow shell with honeycomb core

In formulas (1) and (2) $z$ represents the distance from the current point to the midsurface of the shell, while $z = h_1$ and $z = h_2$ correspond to the bottom and top surfaces of the core layer, respectively. The index $p$ $(0 \le p < \infty)$ denotes the ceramic volume fraction exponent, which is related to the metal volume fraction $V_m$ by expression $V_c + V_m = 1$. The subscripts $c$ and $m$ stand for "ceramic" and "metal," respectively, $E_m, E_c$ and $\rho_m, \rho_c$ denote the Young's modulus and mass density of the constituent materials.

Suppose that unit cell of an auxetic honeycomb core has form of the hexagonal shown in Fig. 1. Geometrical parameters $d, l$ define length of the horizontal and inclined sides respectively, parameter $t$ defines thickness of hexagonal rib and parameter $\theta$ corresponds to inclined angle of the rib. Then core's material properties for negative $\theta$ are adopted from [11]:

$$E_1^{(c)} = E\eta_3^3 \frac{\gamma}{cos\theta\left[cos^2\theta + \left(sin^2\theta + \eta_1\right)\eta_3^2\right]},\ \gamma = \eta_1 + sin\theta,\ \eta_1 = \frac{d}{l},\ \eta_3 = \frac{t}{l},$$
$$E_2^{(c)} = \frac{E\eta_3^3}{\gamma cos\theta\left(tan^2\theta + \eta_3^2\right)},\ G_{12}^{(c)} = \frac{E\eta_3^3}{\eta_1 cos\theta(1 + 2\eta_1)},\ G_{23}^{(c)} = \frac{G\eta_3 cos\vartheta}{\gamma},$$
$$G_{13}^{(c)} = \frac{G\eta_3}{2cos\theta}\left(\frac{\gamma}{1 + 2\eta_1} + \frac{\eta_1 + 2sin^2\theta}{2\gamma}\right),\ \rho^{(c)} = \rho\frac{\eta_3(\eta_1 + 2)}{2\gamma cos\theta}, \tag{3}$$
$$\nu_{12}^{(c)} = \frac{\gamma sin\theta\left(1 - \eta_3^2\right)}{cos^2\theta + \left(sin^2\theta + \eta_1\right)\eta_3^2},\ \nu_{21}^{(c)} = \frac{sin\theta\left(1 - \eta_3^2\right)}{\gamma\left(tan^2\theta + \eta_3^2\right)},$$

where $E, G$ are modulus of elasticity and shear, $\rho$ is a mass density of original core material. By FSDT the strain components $\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}\}^T$ for moderately large deformations can be expressed as $\{\varepsilon\} = \left\{\varepsilon^{(0)}\right\} + z\{\chi\}$, where

$$\left\{\varepsilon^{(0)}\right\} = \left\{u_{0,x} + w_0/R_x,\quad v_{0,y} + w_0/R_y,\quad u_{0,y} + v_{0,x}\quad\right\}^T,$$

$$\{\chi\} = \left\{\psi_{x,x},\quad \psi_{y,y}\quad \psi_{x,y} + \psi_{y,x}\right\}^T,\ \varepsilon_{13} = \psi_x + w_{0,x} - \frac{u_0}{R_x},\ \varepsilon_{23} = \psi_y + w_{0,y} - \frac{v_0}{R_y}, \tag{4}$$

here $(u_0, v_0, w_0)$ denote displacements of a point at the mid-plane to $x$, $y$, $z$ directions, $\psi_x, \psi_y$ are rotation angles of the transverse normal in $yz$ and $xz$ planes respectively, $R_x, R_y$ are radii of principal curvatures in $x$ and $y$ directions.

Note that we consider shallow shells of small curvature, the equations of which are described by quadratic functions that do not contain the product $xy$. In this case, the curvatures are constant. Thus, it is approximately assumed that the intrinsic geometry of the middle surface does not differ from the Euclidean geometry of the plane. The principal directions of curvature do not change and coincide with the directions of the $Ox$ and $Oy$-axes. The expressions for the in-plane force resultants $N = (N_{11}, N_{22}, N_{12})^T$, bending moment resultants $M = (M_{11}, M_{22}, M_{12})^T$ and transverse shear force resultants $Q = \left(Q_x, Q_y\right)$ can be written in matrix form as:

$$\{N\} = [A]\{\varepsilon\} + [B]\{\chi\}, \quad \{M\} = [B]\{\varepsilon\} + [D]\{\chi\},\ \{Q\} = \begin{bmatrix} D_{44} 0 \\ 0 D_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix}, \tag{5}$$

$$[A] = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix},\ [B] = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},\ [D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}.$$

Note that elements $A_{ij},\ B_{ij},\ D_{ij}, (i, j = 1,2, 6)$ of the matrices $[A]$, $[B]$ and $[D]$ in relations (5) are calculated by formulas:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-\frac{h}{2}}^{h_1} Q_{ij}^{(B)}\left(1, z, z^2\right)dz + \int_{h_1}^{h_2} Q_{ij}^{(C)}\left(1, z, z^2\right)dz + \int_{h_2}^{\frac{h}{2}} Q_{ij}^{(T)}\left(1, z, z^2\right)dz. \tag{6}$$

Elements $D_{ij}\ \ (i, j = 4,5)$ are defined as.

$$D_{ij} = K_s^2\left(\int_{-\frac{h}{2}}^{h_1} Q_{ij}^{(B)}dz + \int_{h_1}^{h_2} Q_{ij}^{(C)}dz + \int_{h_2}^{\frac{h}{2}} Q_{ij}^{(T)}dz\right),$$

where $K_s^2$ is a shear correction factor (it is assumed to be $K_s^2 = 5/6$ in the work). Values $Q_{ij}^{(B,T)}$ and $Q_{ij}^{(C)}$ $(i, j = 11, 22, 12, 66, 44, 55)$ are defined by the following expressions:

$$Q_{11}^{(i)} = Q_{22}^{(i)} = \frac{E^{(i)}(z)}{1-\nu^2},\ Q_{12}^{(i)} = \nu Q_{11}^{(i)},\ Q_{66}^{(i)} = Q_{44}^{(i)} = Q_{55}^{(i)} = \frac{E^{(i)}(z)}{2(1+\nu)},\ i = T, B, \tag{7}$$

$$Q_{11}^{(C)} = \frac{E_1^{(C)}}{1-\nu_{12}^{(C)}\nu_{21}^{(C)}},\ Q_{22}^{(C)} = \frac{E_2^{(C)}}{1-\nu_{12}^{(C)}\nu_{21}^{(C)}},\ Q_{12}^{(C)} = \frac{\nu_{12}^{(C)}E_2^{(C)}}{1-\nu_{12}^{(C)}\nu_{21}^{(C)}} Q_{66}^{(C)} = G_{12}^{(C)},$$

$$Q_{44}^{(C)} = G_{23}^{(C)},\ Q_{55}^{(C)} = G_{13}^{(C)}.$$

Here $E^{(i)}(z), \nu$, are effective Young's modulus, Poisson's ratio of the corresponding FGM layers $(i = T, B)$. For core (C) values $E_1^{(C)}, E_2^{(C)}, \nu_{12}^{(C)}, \nu_{21}^{(C)}, G_{12}^{(C)}, G_{23}^{(C)}, G_{13}^{(C)}$ are calculated by formulas (3).

The governing equations of the sandwich shallow shells with auxetic core are obtained by applying Hamilton's principle [11]:

$$N_{11,x} + N_{12,y} + \frac{Q_x}{R_x} = I_0\ddot{u}_0 + I_1\ddot{\psi}_x,$$

$$N_{12,x} + N_{22,y} + \frac{Q_y}{R_y} = I_0 \ddot{v}_0 + I_1 \ddot{\psi}_y,$$

$$Q_{x,x} + Q_{y,y} - \frac{N_{11}}{R_x} - \frac{N_{22}}{R_y} - k_w w_0 + k_p \vec{\nabla}^2 w_0 = I_0 \ddot{w}_0, \quad (8)$$

$$M_{11,x} + M_{12,y} - Q_x = I_1 \ddot{u}_0 + I_2 \ddot{\psi}_x,$$

$$M_{12,x} + M_{22,y} - Q_y = I_1 \ddot{v}_0 + I_2 \ddot{\psi}_y,$$

where cofactors $I_0, I_1, I_2$ in system (8) are defined as:

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{h_1} \rho^{(B)} \left(1, z, z^2\right) dz + \int_{h_1}^{h_2} \rho^{(C)} \left(1, z, z^2\right) dz + \int_{h_2}^{\frac{h}{2}} \rho^{(T)} \left(1, z, z^2\right) dz.$$

Here $\rho^{(r)}$ $(r = B, C, T)$ is a mass density of the $r$-th layer for bottom (B), top (T), and for auxetic core (C), $k_w$, $k_P$ are the parameters of elastic foundations.

## 3 Solution Method

To analyze the linear vibrations of FGM shallow shells with arbitrary planforms and various boundary conditions, the variational Ritz method is used. Assuming that the shell vibrations are harmonic, the problem can be formulated in variational form, which reduces to finding the stationary point of the following functional:

$$J = U\left(u, v, w, \psi_x, \psi_y\right) + V_e(w) - \lambda^2 T\left(u, v, w, \psi_x, \psi_y\right), \quad (9)$$

where $\lambda$ is a natural frequency, $U$, $T$ and $V_e$ are strain, kinetic and potential energies, which are defined by the expressions:

$$U = \int_\Omega \left(N_{11}\varepsilon_{11}^{(0)} + N_{22}\varepsilon_{22}^{(0)} + N_{12}\varepsilon_{12}^{(0)} + M_{11}\chi_{11} + M_{22}\chi_{22} + M_{12}\chi_{12} + Q_x\varepsilon_{13} + Q_y\varepsilon_{23}\right) d\Omega,$$

$$V_e = \frac{1}{2} \int_\Omega \left(k_w w_0^2 + k_p \left(\overrightarrow{\nabla w_0}\right)^2\right) d\Omega,$$

$$T = \int_\Omega \left(I_0 \left(u_0^2 + v_0^2 + w_0^2\right) + I_1 \left(\psi_x u_0 + \psi_y v_0\right) + I_2 \left(\psi_x^2 + \psi_y^2\right)\right) d\Omega.$$

Despite the effectiveness of the Ritz method as a tool for solving vibration problems of plates and shells, a key challenge arises when dealing with complex geometries and varied boundary conditions connected with the construction of admissible functions. This difficulty can be addressed using the R-functions theory [14, 15], which allows sequences of admissible functions to be systematically constructed for arbitrary geometries and boundary conditions. Importantly, these functions exactly satisfy the necessary prescribed boundary conditions. Consequently, the solution can be expressed in analytical form, representing a significant advantage of the R-functions method (RFM) over conventional numerical approaches.

Suppose that admissible functions $\{u_i\}, \{v_i\}, \{w_i\}, \{\psi_{xi}\}, \left\{\psi_{yi}\right\}$ have been constructed. Then according to the Ritz method, unknown functions $u(x, y), v(x, y), w(x, y), \psi_x(x, y), \psi_y(x, y)$ are presented as follows:

$$u = \sum_{i=1}^{N_1} a_i u_i,\ v = \sum\nolimits_{i=N_1+1}^{N_2} a_i v_i,\ w = \sum_{i=N_2+1}^{N_3} a_i w_i,$$

$$\psi_x = \sum\nolimits_{i=N_3+1}^{N_4} a_i \psi_{xi},\ \psi_y = \sum\nolimits_{i=N_4+1}^{N_5} a_i \psi_{yi}. \quad (10)$$

Coefficients of expansion $\{a_i\}$ $i = 1,2, \ldots, N_5$ in (10) are solution of the Ritz system:

$$\frac{\partial J}{\partial a_i} = 0,\ i = 1, 2, \ \ldots, N_5.$$

## 4 Results and Discussion

### 4.1 Shells of Rectangular Shape of Plan

**Problem 1.** Sandwich plates as well as cylindrical, spherical, and hyperbolic paraboloidal panels with auxetic core are examined. Each structure has a square planform with sides of length $a = b = 2$m and thickness $h = 0.1$m [11]. The shell edges are assumed to be simply supported and movable. The structural configuration includes two FGM face sheets composed of $Al/Al_2O_3$ and an auxetic honeycomb core layer made of $Al$. The material properties of the FGM mixture constituents are:

$$\text{Al} : E = 69GPa,\ \rho = 2700kg/m,\ \nu = 0.33;$$

$$Al_2O_3 : E = 380GPa,\ \rho = 3800kg/m,\ \nu = 0.33.$$

Table 1 shows fundamental frequencies computed by the approach proposed (RFM) and using Navier's form for analytical solution from work [11]. Comparison is carried out for shallow shells with the following parameters: $h = 0{,}1m$, $a = b = 20h$, $\eta_1 = \frac{d}{l} = 2$, $\eta_3 = \frac{t}{l} = 0.013857$, $\frac{h_c}{h_f} = 2$, angle $\theta = -55^0$ and different values $p = 0.5, 1, 5, 10$.

**Problem 2.** The second comparison was done for an auxetic honeycomb core sandwich square plate with isotropic face sheets *(Al)* and the same material of the core layer, but in this case the plate is resting on elastic foundation ($k_w = 0.1kN/m^3$, $k_P = 0.05kN/m$), inclined angle $\theta$ and ratio $\eta_1 = \frac{d}{l}$ vary. Comparison of results (Table 2) is presented for two works: [5], where authors used FEM and FSDT, and [8], where researchers employed HSDT theory and solved the problem analytically using the Navier solution.

**Table 1.** Comparison of fundamental frequencies (Hz) for auxetic sandwich shallow shells with square planform

| $R_x(m)$ | $R_y(m)$ | Method | $p = 0.5$ | $p = 1$ | $p = 5$ | $p = 10$ | Pure metal |
|---|---|---|---|---|---|---|---|
| $\infty$ | $\infty$ | [11] | 288.580 | 273.369 | 217,785 | 194.966 | 155.052 |
| | | RFM | 288.554 | 273.345 | 217.744 | 194.919 | 154.933 |
| $\infty$ | 10 | [11] | 296.045 | 280.085 | 222.851 | 199.673 | 159.411 |
| | | RFM | 296.013 | 280.056 | 222.803 | 199.618 | 159.348 |
| 10 | 10 | [11] | 319.520 | 301.356 | 239.022 | 214.613 | 173.206 |
| | | RFM | 319.476 | 301.321 | 238.967 | 214.550 | 173.047 |
| –10 | 10 | [11] | 286.209 | 271.122 | 215.992 | 193.363 | 153.781 |
| | | RFM | 286.183 | 271.095 | 215.950 | 193.314 | 153.663 |

**Table 2.** Comparison of the fundamental frequency (Hz) of the square simply supported FGM auxetic plate on elastic foundation

| $\eta_1$ | Method | $\theta = -10^0$ | $\theta = -35^0$ | $\theta = -55^0$ | $\theta = -80^0$ |
|---|---|---|---|---|---|
| 0.5 | RFM | 276.72 | 318.29 | 292.71 | 306.69 |
| | [8] | 276.542 | 317.795 | 292.415 | 306.313 |
| | [5] | 277.622 | 319.212 | 293.616 | 307.568 |
| 2 | RFM | 280.601 | 279.74 | 278.000 | 267.00 |
| | [8] | 280.387 | 279.537 | 277.803 | 266.875 |
| | [5] | 281.512 | 280.656 | 278.910 | 267.903 |
| 4 | RFM | 281.05 | 280.66 | 279.845 | 274.24 |
| | [8] | 280.836 | 280.454 | 279.645 | 274.095 |
| | [5] | 281.946 | 281.572 | 280.750 | 275.114 |

### 4.2 Shells of a Complex Planform

To illustrate the capabilities of the method proposed, a shallow shell with the complex planform shown in Fig. 2 is analyzed. As in the previous cases, the face sheets are assumed to be made of an FGM mixture $Al/Al_2O_3$, while the core consists of an auxetic aluminum $(Al)$ honeycomb structure. It should be noted that the manufacturing process of curved cores for double-curved sandwich panels is a highly complex task. This problem is discussed in [12], and in [13] with respect to flexible honeycomb cores. In the present work, we assume that the cell geometry is designed in such a way that the mechanical properties of the core are preserved.

The shell is assumed to be clamped along its entire boundary. Accordingly, the boundary conditions are specified as:

$$w(x, y) = 0, u(x, y) = 0, v(x, y) = 0, \psi_x(x, y) = 0, \psi_y(x, y) = 0, \forall(x, y).$$

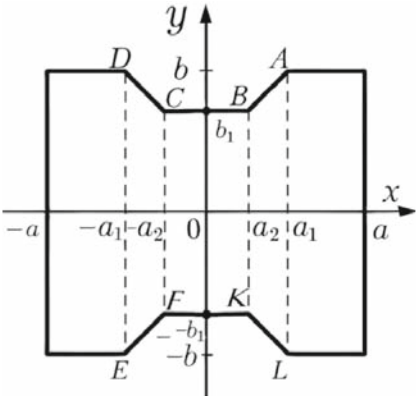


**Fig. 2.** Planform of the shallow shell

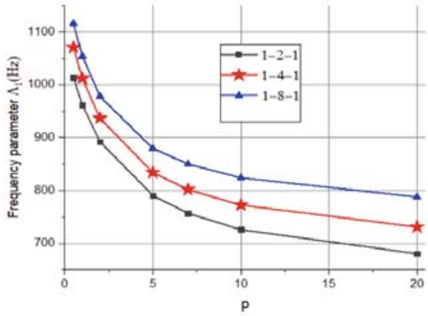


**Fig. 3.** Effect of gradient index on frequency

The solution structure [14, 15] for a fully clamped shell can be expressed in the form:

$$w = \omega\Phi_1, u = \omega\Phi_2, v = \omega\Phi_3, \psi_x = \omega\Phi_4, \psi_y = \omega\Phi_5, \quad (11)$$

where indefinite components $\Phi_i,\ i = \overline{1,5}$ are expanded in the form of an appropriate complete system such as polynomials, trigonometric functions, splines, etc. Function $\omega(x, y)$ constructed using the R-functions theory vanishes exactly on the shell boundary. In the present case this function is given by the following equation:

$$\omega(x, y) = (f_1 \wedge_0 f_2) \wedge_0 ((f_{AB} \vee_0 f_{CD}) \wedge_0 (f_{EF} \vee_0 f_{KL}) \ \vee_0 f_3),$$

symbols $\wedge_0,\ \vee_0$ denote the R-operations [14]. Functions $f_i,\ i = \overline{1,3}$ are defined as:

$$f_1 = \frac{(a^2 - x^2)}{2a} \geq 0, f_2 = \frac{(b^2 - y^2)}{2b} \geq 0, f_3 = \frac{(b_1^2 - y^2)}{2b_1} \geq 0.$$

Functions $f_{AB}$,$f_{CD}$,$f_{EF}$,$f_{KL}$ are defined by the expressions of the oriented [14] straight lines (AB, CD, EF, KL).

$$f_{AB} = ((b - b_1)(x - a_1) - (a_2 - a_1)(y - b))/\sqrt{(b - b_1)^2 + (a_2 - a_1)^2} \geq 0,$$

$$f_{CD} = ((b_1 - b)(x + a_2) + (a_2 - a_1)(y - b_1))/\sqrt{(b - b_1)^2 + (a_2 - a_1)^2} \geq 0,$$

$$f_{EF} = ((b - b_1)(x + a_1) + (a_2 - a_1)(y + b))/\sqrt{(b - b_1)^2 + (a_2 - a_1)^2} \geq 0,$$

$$f_{KL} = ((b - b_1)(x - a_2) + (a_1 - a_2)(y + b_1))/\sqrt{(b - b_1)^2 + (a_2 - a_1)^2} \geq 0.$$

Functions $\Phi_i,\ i = \overline{1,5}$ in (11) were expanded in a power series upon consideration of the problem symmetry [14, 15]. To approximate $\Phi_1$ twenty eight terms of the polynomial's series were retained, and the remaining components were approximated by fifteen terms. Figure 3 illustrates influence of fraction index $p$ and thickness of layers on fundamental natural frequency $\Lambda$ of the FGM auxetic sandwich spherical shells $\left(R_x = R_y = 10m\right)$ on elastic foundation ($k_w = 0.1kN/m^3$, $k_P = 0.05kN/m$). Values of geometric parameters were taken as: $h = 0{,}1m$; $2a = 2b = 20h$; $2a_1 = 1{,}4m$; $2a_2 = 0.6m$; $2b_1 = 1.2m$; $\theta = -55^0$, $\eta_3 = \frac{t}{l} = 0.013857$; $\eta_1 = \frac{d}{l} = 2$; $\frac{h_c}{h_f} = 2, 4, 8$. As shown in Fig. 3, an increase in the gradient index $p$ results in a higher metal content in the FGMs, which, in turn, reduces the overall stiffness of the shell and consequently leads to lower fundamental frequencies. However, increasing the thickness of the core layer contributes to a rise in these frequencies.

The effect of the inclined angle $\theta$ of the auxetic honeycomb unit cell core on the fundamental frequency of the shell is shown in Fig. 4. The results are presented for a spherical shell with the same geometric and mechanical parameters as for Problem 2. The volume fraction index $p$ and thickness ratio $\frac{h_c}{h_f}$ between the face sheets and the core are kept constant: $p = 2$; $\frac{h_c}{h_f} = 8(1 - 8 - 1)$.

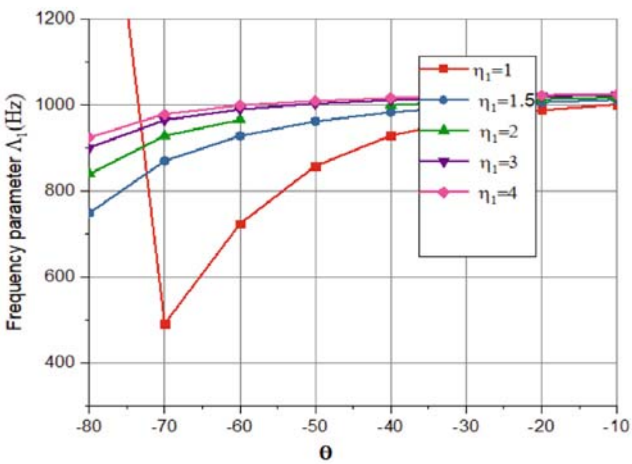


**Fig. 4.** Effect of the inclined angle on fundamental frequency of the clamped spherical shell

As shown in Fig. 4 for values of inclined angle from –80° to –50°, the fundamental frequency increases significantly for $\eta_1 = 1.5$, 2, 3, 4. But for $\eta_1 = 1$ frequency behavior is extraordinary. This phenomenon should be taken into account when using metamaterials in practical applications.

## 5 Conclusions

This study presents the first application of the R-functions theory combined with the variational Ritz method to analyze the free vibrations of sandwich shallow shells with auxetic cores and functionally graded face sheets. The validity and efficiency of the proposed approach have been confirmed through comparisons with previously published results. As a novel contribution, the work investigates the vibration response of a clamped spherical shallow square shell with trapezoidal cutouts on two opposite edges, resting on an elastic foundation.

**Acknowledgment.** The research was carried out within the NATO project "Composite Metamaterials for Aerospace Structures CoMetA", grant number G6176, under the framework of the Science for Peace and Security (SPS) Programme.

## References

1. Bohara, R. P., Linforth, S., Nguyen, T., Ghazlan, A., Ngo, T.: Anti-blast and -impact performances of auxetic structures: a review of structures, materials, methods, and fabrications. Eng. Struct. **276**. 115377 (2023). https://doi.org/10.1016/j/engstruct.2022.115377
2. Singh, S.J., Harsha, S.P.: Free vibration analysis of sandwich plate with honeycomb core and FGM face sheets. In: Saran, V.H., Misra, R.K. (eds.) Advances in Systems Engineering. LNME, pp. 905–917. Springer, Singapore (2021). https://doi.org/10.1007/978-981-15-8025-3_85
3. Sadiq, E., Sadeq, H.B., Munsin, J.J.: Optimum vibration characteristics for honeycomb sandwich panel used in aircraft structure. J. Eng. Sci. Technol. **16**(2), 1463–1479 (2021)
4. Quoc, T.H., Tu, T.M., Tham, V.V: Free vibration and dynamic response of sandwich composite plates with auxetic honeycomb core. J. Sci. Technol. Civ. Eng. HUCE (NUCE) **15**(4), 1–14 (2021). https://doi.org/10.31814/stce.huce(nuce)2021-15(4)-01
5. Tran, T.T., Pham, Q.H., Thoi, T.N., Tran, V.T.: Dynamic analysis of sandwich auxetic honeycomb plates subjected to moving oscillator load on elastic foundation. Adv. Mater. Sci. Eng. (2020). https://doi.org/10.1155/2020/6309130
6. Thang, N.T., Van Long, N., Tu, T.M., Nam, N.H., Anh, M.C.: Navier solution for static and free vibration analysis of sandwich plate with auxetic honeycomb core resting on Pasternak elastic foundation. J. Sci. Technol. Civ. Eng. HUCE (NUCE) **16**(3), 18–28 (2022). https://doi.org/10.31814/stce.huce(nuce)2022-16(3)-02
7. Ghazwani, M.H., Alnujaie, A., Vinh, P.V.: A general viscoelastic foundation model for vibration analysis of functionally graded sandwich plate with auxetic core. Defence Technol. **46**, 40–58 (2024). https://doi.org/10.1016/j.dt.2024.12.008
8. Binh, D.C., Huan, D.T., Hai, N.T., Le V.Q., Chau, L.T-M., Anh, H.X.: Free vibration analysis of sandwich plates with auxetic core and porous FGM faces resting on winkler/pasternak/kerr elastic foundations. Vietnam J. Agric. Sci. **8**(2), 2493–2508 (2025). https://doi.org/10.31817/vjas.2025.8.2.06
9. Shariyat, M., Alipour, M.M.: Analytical bending and stress analysis of variable thickness FGM auxetic conical/cylindrical shells with general tractions. Latin Am. J. Solids Struct. **14**, 805–843 (2017). https://doi.org/10.1590/1679–78253413
10. Cong, P.H., Long, P.T., Nguyen Van Nhat, N.V., Duc, N.D.: Geometrically nonlinear dynamic response of eccentrically stiffened circular cylindrical shells with negative Poisson's ratio in auxetic honeycombs core layer. Int. J. Mech. Sci. **152**, 443–453 (2019). https://doi.org/10.1016/j.ijmecsci.2018.12.052
11. Pham, H.-A., Tran, H.-Q., Tran, M.-T., Nguyen, V.-L., Huong, Q.-T.: Free vibration analysis and optimization of doubly-curved stiffened sandwich shells with functionally graded skins and auxetic honeycomb core layer. Thin-Walled Struct. **179**, 109571 (2022). https://doi.org/10.1016/j.tws.2022.109571

12. Evans, K.E.: The design of doubly curved sandwich panels with honeycomb cores. Compos. Struct. **17**(2), 95–111 (1991)
13. Avramov, K., Uspensky, B.: Nonlinear vibrations of doubly curved composite sandwich shells with FDM additively manufactured flexible honeycomb core. Acta Mech. **234**, 1183–1210 (2023). https://doi.org/10.1007/s00707-022-03426-w
14. Rvachev, V.L.: The R-functions Theory and Its Some Application. Naukova Dumka, Kiev (in Russian) (1982)
15. Kurpa, L., Pellicano, F., Shmatko, T., Zippo, A.: Free vibration analysis of porous functionally graded material plates with variable thickness on an elastic foundation using the R-functions method. Math. Comput. Appl. **29**(1), 10 (2024). https://doi.org/10.3390/mca29010010