Magnetic and Electromagnetic Springs Forces: Determination and Usage in Damping Vibrations

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1 Introduction

Springs are commonly used in the technical industry, thanks to their ability to store energy or, in certain cases, to damp some movement. Technological progress allows creating devices that act as a spring (e.g., rubber, hydro-pneumatic, magnetic, or electromagnetic springs). Electromagnetic spring has an advantage because its parameters can be controllable by changing the current in the coil. The idea of using an electromagnet to achieve varying stiffness has already found usage in biomechanics. Liang et al. [1] designed a finger exoskeleton with a variable stiffness which can be used in hand rehabilitation. Liu et al. [2] developed a controllable electromagnetic vibration isolator. Authors have introduced negative stiffness which opposes the excited movement in some working range. As a result, the amplitude of this vibration has been decreased. Since electromagnets can operate with varying forces, they can be successfully used for shock absorption [3, 4]. We can treat two or more magnets moving coaxially as a nonlinear magnetic spring. By electromagnetic spring we mean here a magnet moving coaxially in the vicinity of the powered coil. Usually, the identification of spring parameters, such as stiffness or damping coefficient, is biased with an error resulting from dry friction. Experimental stand consisting of aerostatic bearings allows us to minimize this error. There are numerous publications that try to model the strength of a magnet or powered coil depending on the distance from it [5–7]. In this work, the interaction between a pair of magnets and between a magnet and a coil has been described and experimentally validated. Sometimes in an application, one needs a spring with a particular characteristic, for example, linear or progressive. Finding

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a proper combination of springs to achieve the desired characteristic might be troublesome, especially for complex stiffness shapes. This work presents a method for tailoring any characteristic of electromagnetic spring. It should be emphasized that by changing characteristics of certain components of the system during its motion in an appropriate way, we can dampen or additionally force the motion. That property allows one to create active damping, which, unlike passive damping, can adapt to changing conditions in which the device is operated. In this article, we present the current controller with a feedback loop, which allows us to dampen any movement that appears as a result of some external excitation. This article extends our previous work [6] by introducing a new formula to approximate electromagnetic force. Furthermore, a concept of tailoring electromagnetic stiffness characteristic is introduced. This paper is constructed in the following way: Sect. 2 – description of the experimental setup, Sect. 3 – investigations of spring stiffness characteristic, Sect. 4 – shaping the electromagnetic stiffness characteristic, Sect. 5 – active damping.

2 Experimental Rig

Our experimental rig allows us to test a wide spectrum of properties of stiffness elements (springs). Thanks to the use of nonferromagnetic elements, we can study not only mechanical springs but also magnetic and electromagnetic springs. The setup consists of a guide 6 suspended in two closed aerostatic supports (see Fig. 1) which allows for full elimination of dry friction between the guide and the supports. Universal spring holders are placed at the ends of the guide 6 and on the brackets 7. The distance between the handles and the guide's face can be adjusted by axial movement of the brackets 7 in the t-slots of the base 9. It is possible to measure the displacement of the guide with an accuracy of 1 micrometer. Position of the guide is measured with Hall sensor 3 and gathered by data acquisition card. The value of force F is changed by setting the angle α, relative to the direction of gravity, with a self-locking worm gear 12. Inclinometer 2 ensures accurate angle reading.

3 Determining Spring Stiffness Characteristics

We study two different types of springs which will be subject to theoretical and experimental analysis: magnetic and electromagnetic (see Fig. 2). Their forces will be denoted as F_m and F_e, where F_m is the force of magnetic spring and F_e indicates the force of electromagnetic spring. Static stiffness characteristic was collected using a dynamometer.

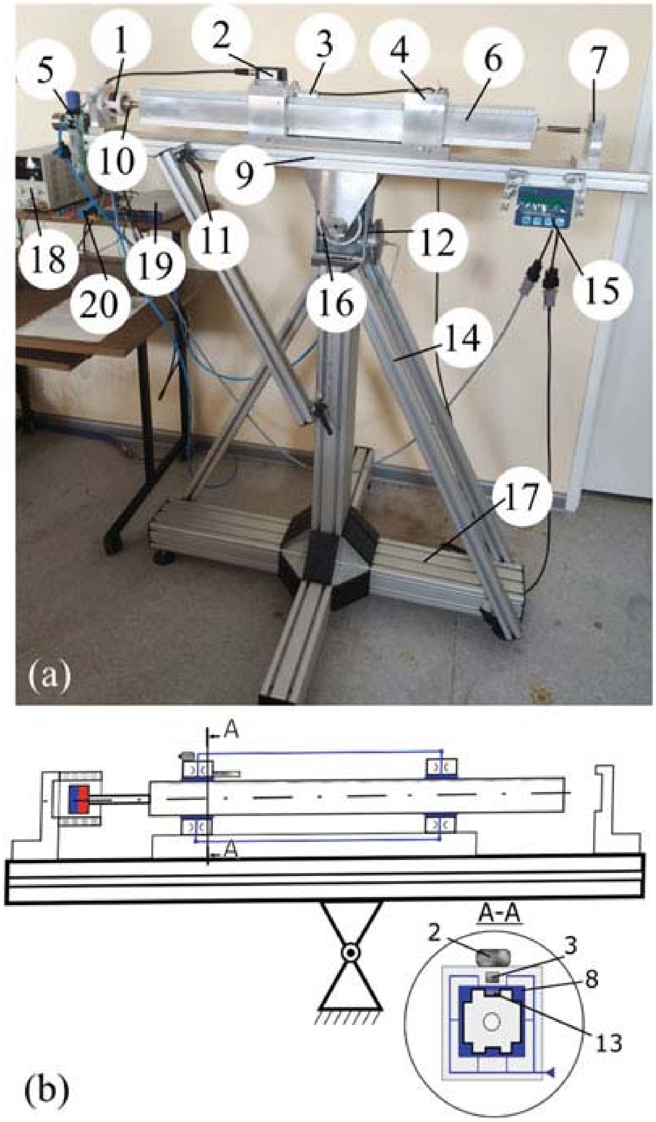

Fig. 1 Photo (**a**) and diagram (**b**) of the experimental rig with aerostatic supports: 1 – coil, 2 – inclinometer, 3 – position sensor, 4 – aerostatic supports, 5 – air preparation system, 6 – guide, 7 – brackets, 8 – compressed air, 9 – base, 10 – magnet, 11 – angle adjustment locks, 12 – worm gear, 13 – magnetic ruler, 14 – supports, 15 – displacement display, 16 – angular scale, 17- rig frame, 18 – power supply, 19 – data acquisition card, 20 – H-bridge

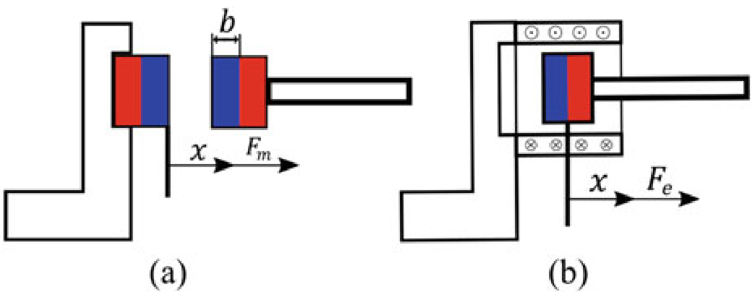

Fig. 2 Test objects: (**a**) magnetic spring (magnet - magnet), (**b**) electromagnetic spring (powered coil - magnet)

3.1 Magnetic Spring

A magnetic spring is defined as two magnets arranged coaxially with identical poles relative to each other as it is shown in Fig. 2a. One magnet was attached to the fixed

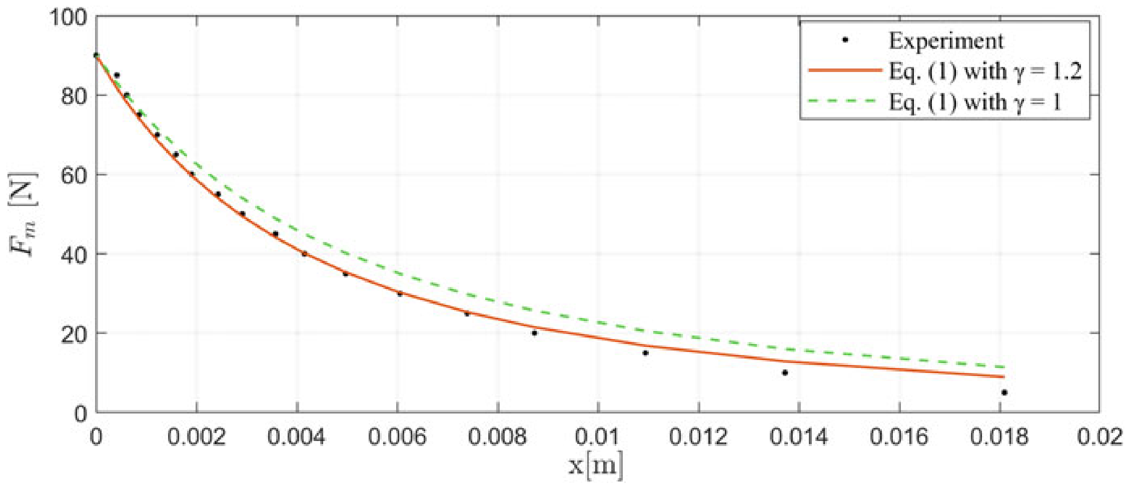

Fig. 3 Graph representing repulsive force vs. displacement of magnetic spring. For $x = 0$, the surfaces of magnets adjoin. Dots represent experimental values; red (green) curve represents Eq. (1) with (without) regard to environmental conditions

mounting, while the other to the guide. Repulsive force of two magnets moving coaxially on the guide without friction is nonlinear and can be determined by the formula:

$$F_m = A(\gamma x + 2b)^{-2}, \tag{1}$$

where A is a magnet constant, b is the distance between the center of the magnet and the magnet's front surface, while γ is a constant of distance influences for given conditions. Constant A can be determined from the formula:

$$A = F_0(2b)^2, \tag{2}$$

where F_0 is the value of force that occurs between connected identical poles of magnets. The green curve shown in Fig. 3 displays formula (1) without taking into consideration environmental conditions represented by parameter γ, while the red curve represents formula (1) with regard to them.

3.2 Electromagnetic Spring

An electromagnetic spring is defined as a pair of a magnet and powered coil arranged coaxially, so that magnet can move freely inside and outside the coil (see Fig. 2b). Depending on the polarization of the current, the magnet is repelled or attracted to the center of the coil. Measurement was carried out in the same way as magnetic spring. The coil used in the experiment has 30 mm in length, 26 mm on the inner diameter, and 45 mm in outer diameter. Static tests have shown that the stiffness of the electromagnetic spring is linear inside the coil and strongly nonlinear

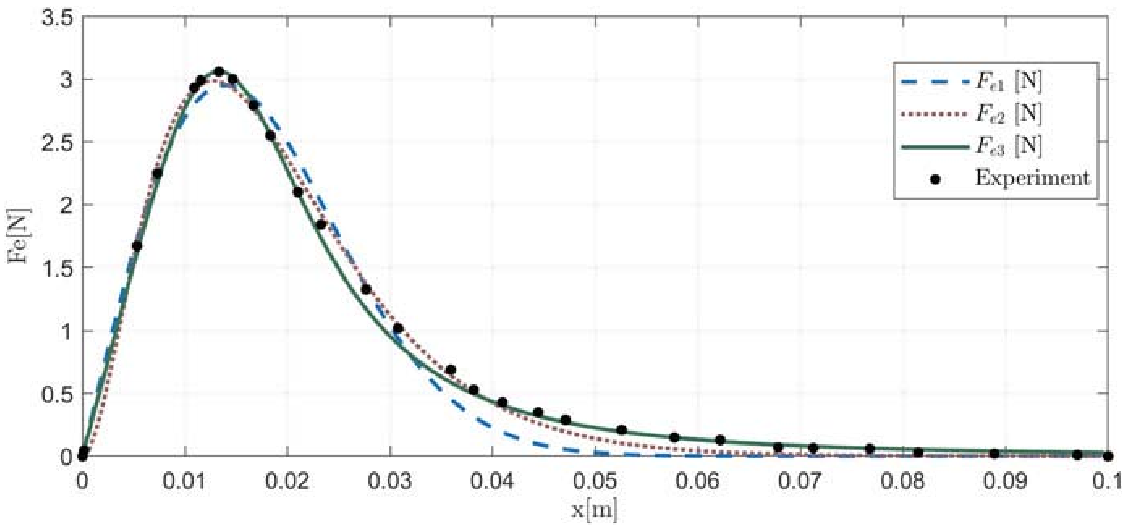

Fig. 4 Fitting of Eqs. (3), (4), and (5) to the experimental data for $I = 2A$

Table 1 Values of parameters used in Eqs. (3), (4), and (5)

a_1 $[Nm]$	$a_2[m^2]$	$b_1\left[\frac{N}{m^{b_2}}\right]$	b_2	$b_3\left[\frac{1}{m}\right]$	$c_1\left[\frac{N}{A}\right]$	$c_2[m]$
0.6782	0.000388	106100	1.953	−153.6	2.6857	0.0175

outside the coil (see Fig. 4). Such a line can be approximated with many formulas. To distinguish and compare them, they are named F_{e1}, F_{e2}, and F_{e3}. In each of three mathematical descriptions of the electromagnetic force, the force reaches a maximum value when the center of magnet meets the frontal surface of the coil. Force is equal to 0 when center of the magnet is located in the center of the coil ($x = 0$). Figure 4 shows a correspondence between the experimental dots and an analytical curve in all three variants. The parameters in formulas of electromagnetic spring are purely mathematical. They have no physical meaning, unlike parameters in formula (1) and (2). All values have been adjusted by Matlab curve fitting tool, using nonlinear least squares method. Obtained values are given in Table 1.

Term F_{e1} is described by the expression [8]

$$F_{e1} = \frac{2a_1}{a_2} \cdot x \cdot e^{-\frac{x^2}{a_2}}, \tag{3}$$

where a_1 $[Nm]$ and $a_2[m^2]$ are constant coefficients for the given current amplitude.

Term F_{e2} is described by the expression [6]

$$F_{e2} = b_1 x^{b_2} e^{b_3 x}, \tag{4}$$

where $b_1\left[\frac{N}{m^{b_2}}\right]$, b_2, and $b_3\left[\frac{1}{m}\right]$ are coil-current-dependent parameters.

Term F_{e3} we propose to describe by the following expression

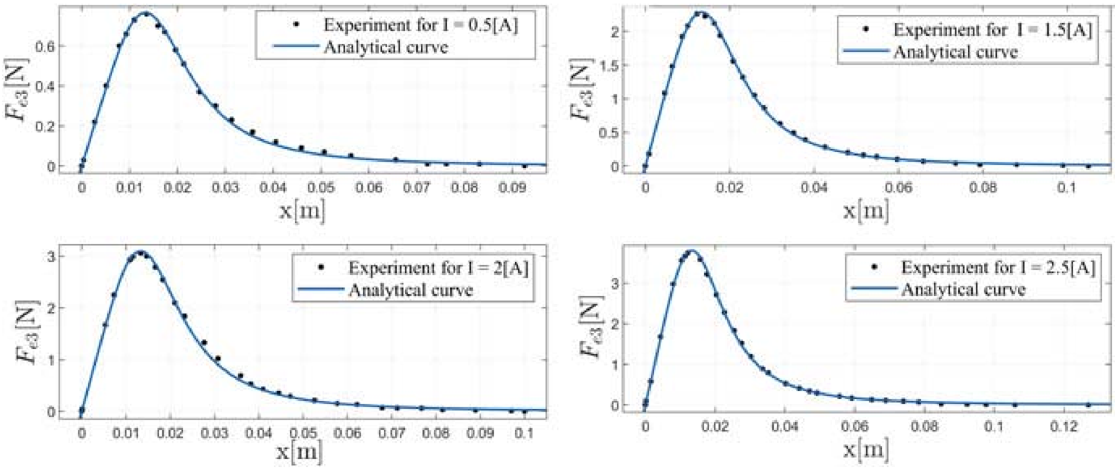

Fig. 5 Congruence of Eq. (5) for various values of current

$$F_{e3} = c_1 \cdot I \cdot \frac{\frac{x}{c_2}}{1 + \left(\frac{x}{c_2}\right)^4}, \tag{5}$$

where $c_1\left[\frac{N}{A}\right]$ and $c_2[m]$ are constant parameters for a given coil-magnet pair. In our case, $c_1 = 2.6857\ \frac{N}{A}$ and $c_2 = 0.0175\ m$. The electric current in the coil is represented by I.

Analysis of all three mathematical description methods shows that F_{e3} has the best fitting with experimental data. To measure the value of fitness, coefficient of measurement(R^2) was determined. For F_{e1}: $R^2 = 0.9853$, for F_{e2}: $R^2 = 0.9971$, and for F_{e3}: $R^2 = 0.9985$. Moreover, the dependency between c_1 and c_2 parameters and current is linear, which is a very useful phenomenon for tailoring characteristic of electromagnetic spring. Figure 5 shows fitting F_{e3} to values of current from 0.5A to 2.5A.

4 Tailoring Characteristics of the Electromagnet

Knowing that the force of an arbitrary spring is denoted as $F = kx$, we can use it along with Eq. (5) to tailor the characteristic of the electromagnetic spring. Let us denote k_e as an established linear stiffness that we want to achieve in electromagnetic spring. Transforming Eq. (5), we achieve following expression:

$$\begin{cases} F_e = k_e \cdot x \\ I = \frac{F_e c_2}{c_1 x}\left(1 + \left(\frac{x}{c_2}\right)^4\right) \end{cases}. \tag{6}$$

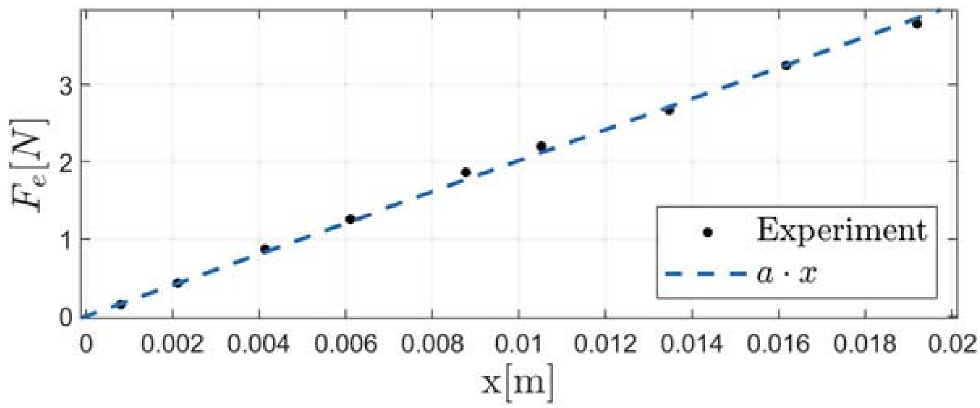

Fig. 6 Force vs. displacement chart of electromagnetic spring for the value of current calculated by Eq. (6), under assumption that $k_e = 200$ (dashed curve represents arbitrary linear function)

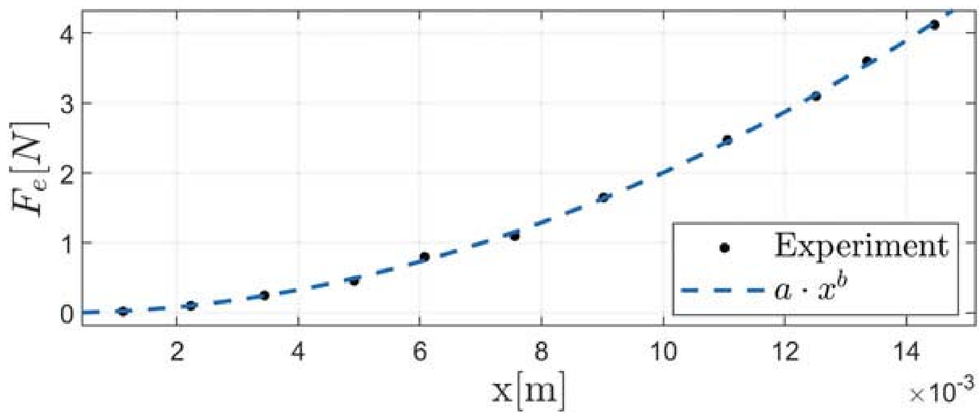

Fig. 7 Force vs. displacement chart of electromagnetic spring for the value of current calculated by Eq. (6), under assumption that $k_e = 20000x$ (dashed curve represents arbitrary progressive power function)

Figures 6, 7, and 8 show force versus displacement of an electromagnet with various linear and nonlinear set stiffness k_e. The idea is to make linear, progressive, and regressive characteristic.

This section has shown that by manipulating the current in a coil, we can use electromagnetic spring as a substitute for an arbitrary spring. Of course, above certain distance from the coil, needed current grows exponentially, so in order to use this solution efficiently, working space should be limited to double or triple of a coil length (depending on the specification of a coil).

5 Active Damping

Since force characteristic of electromagnetic spring can be tailored arbitrary, it can also be amplified or diminished in such a way, so that the resulting force will be opposite to the movement. To achieve such behaviour, a control system with a

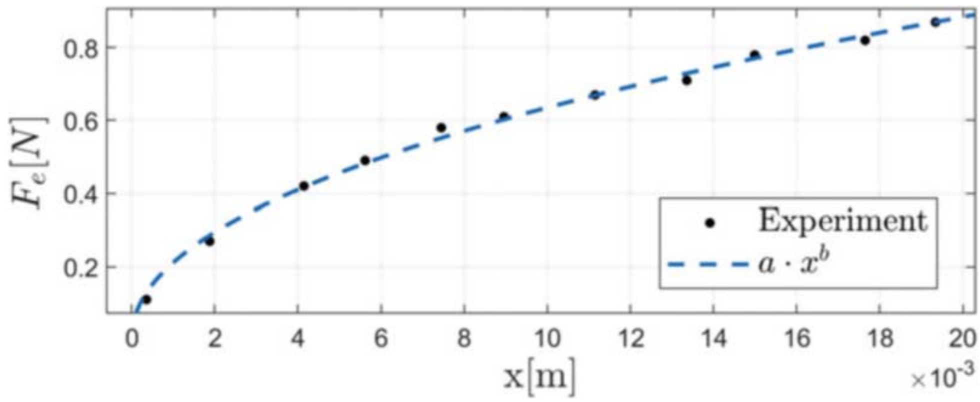

Fig. 8 Force vs. displacement chart of electromagnetic spring for the value of current calculated by Eq. (6), under assumption that $k_e = \frac{200000}{\sqrt{x}}$ (dashed curve represents arbitrary regressive power function)

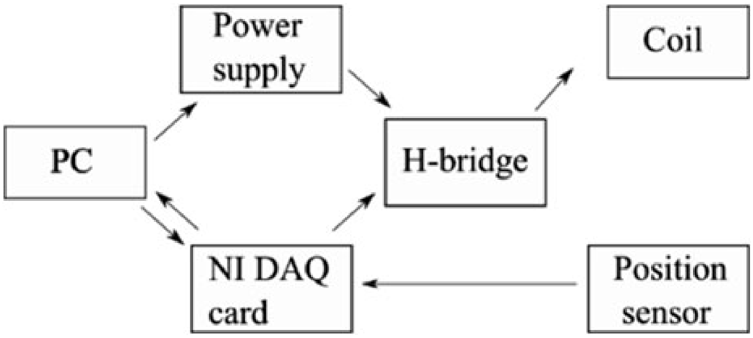

Fig. 9 Diagram showing communications between components of the control system

feedback loop has been developed. It allows data to be acquired and processed in real time. To acquire measurements, we use the NI USB-6341 device, sensors, and a computer connected to the card (see Fig. 9). In order for the current to flow in both directions, the H-bridge is used. The program shows parameters such as guide inclination, displacement, and current sent to the coil in runtime.

The algorithm, which controls the current, is represented by the following expression:

$$I(x, \dot{x}) = K_p x + K_d \dot{x} + K_i \int x dt, \quad (7)$$

where K_p, K_d, and K_i are coefficients of the proportional, derivative, and integral terms, respectively. All tune settings parameters have been adjusted experimentally by trails and error method. In each loop iteration, the displacement is numerically differentiated to obtain the velocity and it is proportionally converted to the coil current. The fourth order Runge-Kutta method was used to integrate displacement in time in order to get a second component of the algorithm which is first amplified by a tuning constant. Thanks to this, we achieve the effect in which the guide moving toward the electromagnet is repelled from it, while moving in the opposite

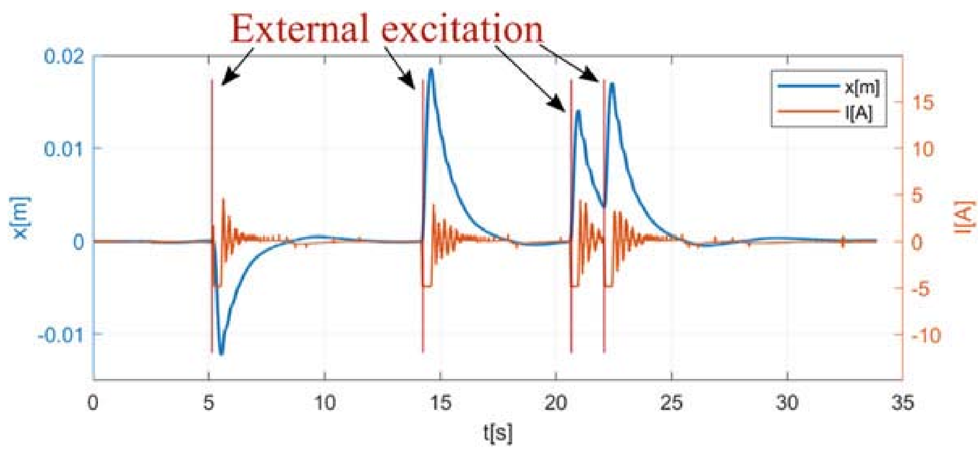

Fig. 10 Time history of the guide movement and value of the coil current. The blue curve stands for the displacement of the guide, whereas orange curve stands for coil current

direction, it is attracted. Figure 10 shows the damping of the guide movement using the presented algorithm. External excitation is an impulse force which is used for presentation purposes. Without active damping, the guide would move in the direction compatible with the direction of external excitation until it meets one of the brackets.

6 Concluding Remarks

The formula for the magnetic force between two magnets is proposed and experimentally validated. It might be very useful for engineers who need to calculate the force of a magnet in a given distance from each other, knowing only maximum force between them, and length of the magnets. The formula for tailoring characteristic of electromagnet has been presented and proven experimentally. It is shown that electromagnetic spring can also be a successful shock absorber. It may be useful for applications where there is a need for varying stiffness of the spring. For example, in suspension systems for active vibration damping or in systems where it is necessary to pass through resonance to achieve target rotational speeds.

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