Chapter 13
Topological Optimization of Multilayer Structural Elements of MEMS/NEMS Resonators with an Adhesive Layer Subjected to Mechanical Loads

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Abstract The paper considers the problem of topological optimization of multilayer structural elements of MEMS/NEMS resonators with an adhesive layer under the action of mechanical loads. The purpose of this work is to obtain a design solution that is least susceptible to destruction due to an increase in the rigidity of the elements to be joined and, as a consequence, providing smoothing of stress peaks in the adhesive layer. To demonstrate the operation of the topological optimization algorithm for this class of problems, several examples are given that show significant improvements in the set target indicators. The problems were solved by the finite element method with the application of the sliding asymptotes method.

Key words: Topological optimization, Shear stresses, Adhesive, Method of sliding asymptotes

13.1 Introduction

Adhesive bonding technology, alone or in combination with mechanical fastening, can significantly improve the mechanical performance of a structure, both in terms

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of rigidity and in terms of strength and fatigue (Hart-Smith, 1982; Kelly, 2006; da Silva et al, 2018). Adhesive joints have advantages over alternative bonding methods (bolted or riveted), as they provide stress distribution over a wider area of the joints, minimal thermal effect (as opposed to welding), high rigidity and high strength-to-weight ratio. In addition, adhesives have better corrosion resistance as well as good damping performance. In contrast to the uneven distribution of loads when joining fasteners, the transfer of load between glued or soldered components is continuous throughout the entire layer. This allows simpler and lighter connections to be used. In other words, the adhesive bond provides the ability to reduce the weight of the structure while providing mechanical strength. Based on this, such connections are increasingly used in the design of mechanical systems (Adams et al, 1997; Dixon, 2005; Watson, 2005).

Quite a lot of studies have been devoted to the problem of increasing the strength of adhesive joints. Most of them were based on parametric optimization, where it was assumed that the design variables were changed in a selected range using specified intervals Groth and Nordlund (1991); Hildebrand (1994); Rispler et al (2000); Taib et al (2006). For example, in da Silva et al (2011), such factors influencing the strength of the joint, as the adhesive properties of the material, the thickness of the intermediate layer, the contact area, and residual stresses, were determined.

In recent years, a number of works have been devoted to the problem of improving and researching structures with an adhesive layer, including research based on analytical formulations (Spaggiari and Dragoni, 2014; da Silva and Lopes, 2009), numerical modeling (Pires et al, 2003; Nimje and Panigrahi, 2014) or a combination of these two methods (das Neves et al, 2009; das Neves et al, 2009; Carbas et al, 2014). In all the studies considered, modifications were made to the shape of the elements to be joined or the shape and location of the adhesive layer.

Awrejcewicz et al (2020) developed a technique based on a combination of topological optimization methods (moving asymptotes method) and a finite element method for obtaining an optimal structure to reduce the stress level in a soldered joint. Krysko et al (2019) constructed a mathematical model and a technique for solving a wide class of problems of topological optimization of the adhesive layer under the action of both mechanical and thermal loads to obtain an optimal microstructure and gradient properties in order to reduce the stress level in it. It is shown that it is possible to achieve almost uniform shear stresses in the solder, arising due to the difference in the coefficients of linear thermal expansion. Krysko et al (2018) investigated the nonlinear dynamics of inhomogeneous beams with an optimal distribution of material over thickness and length. Comparison of static and dynamic results of optimal and homogeneous beams for different values of the scale parameter of material length and temperature was carried out. The influence of the scale parameter of the length of the material on the chaotic behavior of the beam was investigated. Scenarios of transition to chaos were constructed for various values of temperature, both for a homogeneous beam and for a beam with an optimal microstructure.

Zhu et al (2017) presented a systematic approach to the design of membrane structures for a piezoresistive pressure sensor using topology optimization. The design problem was interpreted as the problem of optimizing a three-dimensional topology with calculated dependent loads, in which the dependence was considered due to the transferred loads. The topological optimization problem was solved using the popular SIMP (Solid Isotropic Material with Penalization) method.

A topological optimization method for a local resonator was presented in Jung et al (2020) to adapt flexural band gaps in plate structures. Topological optimization was performed with simulated annealing (SA) and using the finite element method. Numerical examples demonstrated the effectiveness of the presented method of creating a band gap at frequencies below 500 Hz. The above studies show that, under the action of mechanical loads, the destruction of structures occurs mainly due to peak stresses in the adhesive layer.

This paper poses the problem of topological optimization of the shape of the connected elements under the action of mechanical loads in order to obtain a design solution that is least susceptible to destruction. The solution was achieved by increasing the rigidity of the elements to be joined, which ensures the smoothing of stress peaks in the adhesive layer.

13.2 Statement of the Topological Optimization Problem

At present, the most widely used approaches to solving problems of topological optimization of structures are methods of explicit parameterization, which work on a fixed domain of finite elements; however, instead of a set of elastic properties of the microstructure, each finite element contains only one design variable. This variable is often understood as the density of the element material, ρ_e. To determine the defining characteristics of the material, one of the most well-known methods was chosen - the SIMP method. Power-law interpolation is used; in the case of setting the problem on a region containing a void and one phase of the material, it has the following form (Bendsøe and Sigmund, 2004)

$$E_e(\rho_e) = \rho_e^p; \quad 0 \leq \rho_{\min} \leq \rho_e \leq 1, \tag{13.1}$$

where ρ stands for amount of penalty. Design variable ρ is bounded from below by a small positive constant $\rho_{\min}$, which is introduced in order to prevent the degeneracy of the finite element matrix. Note that for the values $\rho_{\min} \leq \rho \leq 1$ and positive ρ, modulus $E_e(\rho_e)$ are limited to small value at density $\rho_e = \rho_{\min}$ and the value of Young modulus of the phase of the base material E_0, for $\rho_e = 1$.

Here, the optimization problem relies on achieving a structure with maximum rigidity by modifying the structure of the elements connected by the adhesive layer while maintaining a given amount of modeling material. The redistribution of the material should ensure a decrease in stresses both in the elements to be joined and in the adhesive layer, which is the most susceptible to destruction.

Here, the algorithm minimizes strain energy W_s by increasing density in areas of higher sensitivity

$$\min\left(\frac{1}{W_{s0}}\int\limits_{\Omega} W_s(x)\,\mathrm{d}\Omega\right), \tag{13.2}$$

where: Ω - area of the structure under consideration, W_{s0} - normalizing factor. At the same time, restrictions on the amount of material used for modeling must be met in the area of solving the optimization problem

$$0 \leq \int\limits_{\Omega} \rho_i(x)\,\mathrm{d}\Omega_{\mathrm{opt}} \leq \gamma_i A, \tag{13.3}$$

where: A - optimized area Ω_{opt}, γ - material volumetric ratio. To eliminate the checkerboard effect in the optimal structure, a penalty function is introduced in the form

$$\frac{h_0 h_{\max}}{A}\int\limits_{\Omega} |\nabla\rho(x)|^2 \mathrm{d}\Omega, \tag{13.4}$$

where: h_0 - initial grid size, which controls the size of the elements in the split, $h_{\max}$ - the current size of the element at the given level. The penalty function is dimensionless and has a value of the order of unity for the worst possible solution. Dimensionless target function (13.2) and penalty function (13.4) must be consistent, for example, in the form of a linear combination (13.2) and (13.4) with a given parameter q, i.e. we have

$$f = \frac{1-q}{W_{s0}}\int\limits_{\Omega} W(x)\,\mathrm{d}\Omega + q\int\limits_{\Omega} |\nabla\rho(x)|^2 \mathrm{d}\Omega. \tag{13.5}$$

Below we will consider several examples for the problems of topological optimization of multilayer structures with an adhesive layer under the action of mechanical loads. The problems are solved by the finite element method, and linear triangular elements are used. The optimization algorithm is based on the sliding asymptote method.

13.3 Case Study 1

Consider a three-layer elastic structure, the dimensions and boundary conditions for which are shown in Fig. 13.1. Area Ω_1 filled with aluminium 2024-T3 with Young modulus equal to E_1 = 73.1 GPa, and Ω_2 stands for the area for solving the problem of topological optimization, in which it is necessary to find the optimal microstructure of the distribution of a given amount of aluminium 2024-T3, while Ω_3 is the area of evenly distributed adhesive FM73-M solder with E_2 = 2260 MPa. Mechanical load acting on the right $F = 100\frac{\mathrm{kN}}{\mathrm{m}^2}$, the left border is fixed. The material data are taken from Mubashar et al (2011).

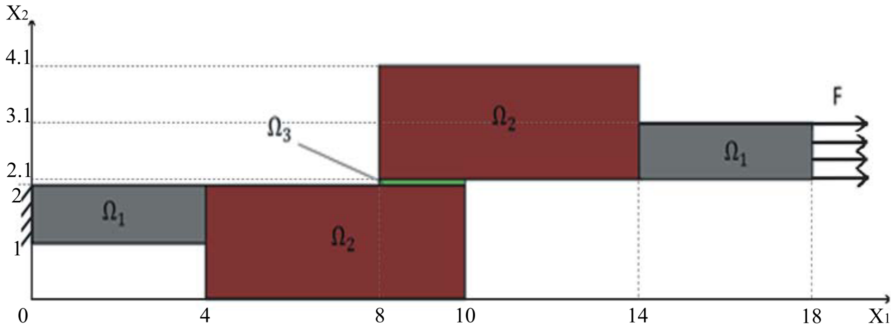


Fig. 13.1 Design and boundary conditions.

When overlapping, there are tears at the ends of the adhesive line. These inhomogeneities lead to bending moments due to eccentric loading as well as uneven distribution of moments around the adhesive layer. These moments create breaking stresses in the adhesive layer. Geometric rupture also creates high shear stresses in the adhesive. There are ways to reduce this eccentric load in lap joints. For example, it has been shown to be effective for this to taper the edges of the layers to be joined. A decrease in the maximum shear and peel stresses can be achieved by increasing the length of the joint, the thickness of the solder, and the thickness of the layers to be brazed. In this example, all geometrical and physical parameters of the solder remain constant, and a decrease in the maximum stress values at the ends of the solder is achieved due to the topological optimization of the microstructure of the layers to be joined.

Figure 13.2 shows three-layer constructions commonly found in practical applications (Fig. 13.2 (A, B)) and the optimal design obtained as a result of solving the problem of topological optimization (Fig. 13.2 (C)). The construction in Fig. 13.2 (B) features beveled corners of the elements to be joined, which is a classic engineering technique for reducing shear stresses. Note that the amount of duralumin and silver solder material in structures (Fig. 13.2 (A, B, C)) is the same, while solving the optimization problem, the coefficient γ should be taken equal to 0.5.

The Table 13.1 shows the numerical results: maximum values of von Mises stresses in the solder layer, maximum values of shear stresses in the solder layer, and total strain energy throughout the structure. For main stresses $\sigma_1, \sigma_2, \sigma_3$ the von Mises stress formula is defined as follows:

$$\sigma_{\mathrm{vM}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \tag{13.6}$$

It can be seen from the table that when using the classical design option to reduce shear stresses (b), there is a slight improvement in this parameter. However, in terms of von Mises stresses σ_{vM} and deformation energy W_s this design is slightly inferior to the original one. For a topologically optimal design in solder, the maximum values

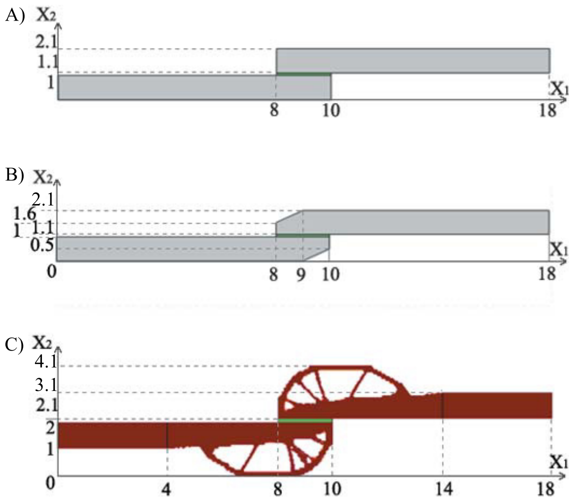


Fig. 13.2 Three-layer constructions (A, B, C) with an adhesive layer.

Table 13.1 Values of maximum stresses and strain energy.

Construction	Maximum value of σ_{vM} (Pa) in the solder	Maximum shear stress (Pa) in the solder	W_s (Nm) by construction
Straight (original construction) (a)	196620	97941	9118,3
Bevel (Classic design for reducing shear stresses) (b)	217390	93368	9383,9
Topologically optimal design (c)	100356	56308	456,0

are as σ_{vM} and the maximum values of shear stresses decrease by more than 2 times. The strain energy target for the entire structure has improved more than 20 times.

The graphs in Figs. 13.3, 13.4 show distribution σ_{vM} and shear stresses, respectively, in the adhesive layer for the cases of structures A, B, C. Here and further

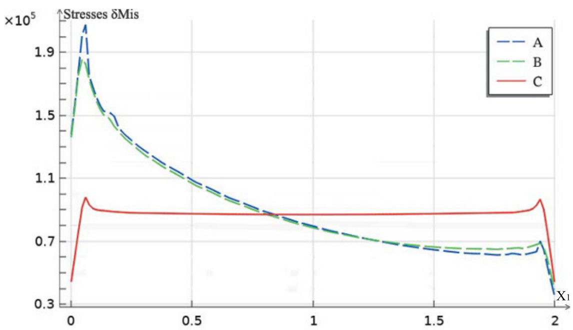
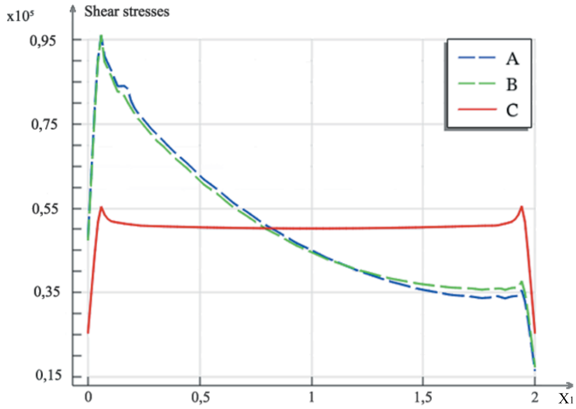


Fig. 13.3 Stresses σ_{vM} (Pa) for structures A, B, C.

Fig. 13.4 Shear stresses (Pa) for structures A, B, C.



in the work, the graphs are given for the coordinate axes passing through the center of the adhesive layer (as shown in Fig. 13.5). These plots confirm the previous conclusions, and also demonstrate the uniformity of stress distribution in the adhesive layer for a topologically optimal design.

13.4 Case Study 2

Consider the construction shown in Fig. 13.6 having two uniform overlapping adhesive joints. The physical properties of the materials are similar to the previous case.

Problems were solved for four different cases of load action:

- Symmetrical action of loads on the top and bottom of the structure $F = 100\frac{\text{kN}}{\text{m}^2}$;
- Symmetrical action of loads on the top and bottom of the structure $F = 150\frac{\text{kN}}{\text{m}^2}$;

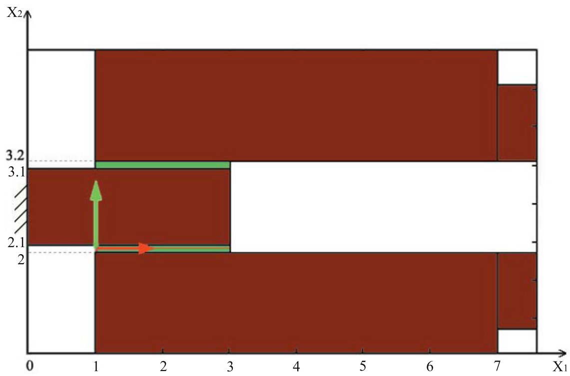


Fig. 13.5 Location of coordinate axes for graphs.

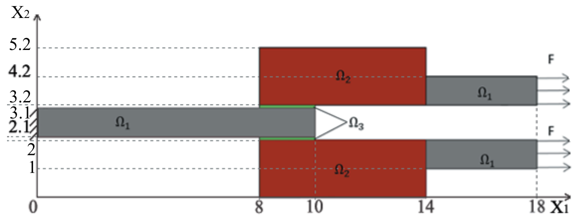


Fig. 13.6 Construction and boundary conditions.

- Load on the top of the structure $F = 100\frac{\text{kN}}{\text{m}^2}$, to the bottom $F = 150\frac{\text{kN}}{\text{m}^2}$;
- Load on the top of the structure $F = 100\frac{\text{kN}}{\text{m}^2}$, to the bottom $F = 200\frac{\text{kN}}{\text{m}^2}$.

Table 13.2 shows the numerical results for each of these load cases. Figure 13.7 shows the optimal topology of structures under symmetric loading (cases 1, 2). The optimal topologies of structures for different load intensities are almost identical, however, with a stronger impact (case 2), the result has a finer structure, which can be explained by the higher sensitivity of the objective function.

The graphs in Figs. 13.8 and 13.9 show the distribution σ_{vM} and shear stress for load case 2 at the center of the upper adhesive layer. In the case of a symmetric action of loads, the distribution graphs are symmetrical, but have different signs.

For example, in Fig. 13.10 the optimal topology of the structure with an asymmetric action of loads, for the case of loading 3, is reported.

Table 13.2 Numerical results for different loading cases.

Load	Construction	Adhesive layer	Maximum σ_{vM} (Pa) in the solder	Maximum shear stress (Pa) in the solder	Integral von Mises stress (Pa) over the solder area	W_s (Nm) designs
First case	elementary	-	134870	62601	17892	4019,7
	optimized	-	98299	56604	17423	961,6
Second case	elementary	-	202310	93902	26838	9044,3
	optimized	-	147390	84884	26137	2164,4
Third case	elementary	top	120030	53499	17931	8298,9
		bottom	237550	109640	26837	
	optimized	top	98824	56968	17455	1552,6
		bottom	147190	84827	26093	
Fourth case	elementary	top	132430	58057	18016	17117,0
		bottom	340430	156390	35795	
	optimized	top	99920	56914	52145	4722,2
		bottom	296110	17046	17553	

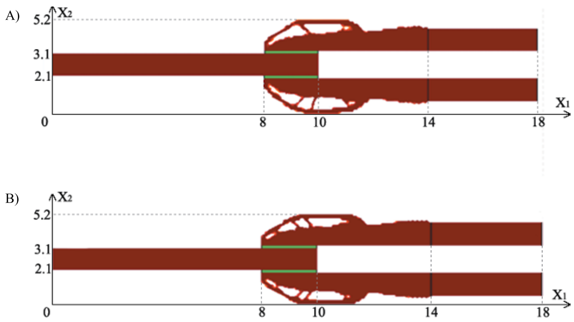


Fig. 13.7 Optimal topologies for two load cases.

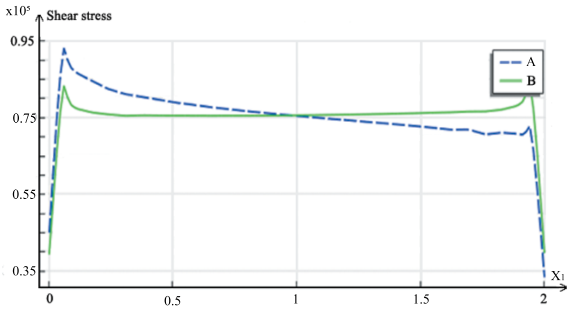


Fig. 13.8 Shear stress (Pa) along the central axis of the solder region for loading case 2 (A - not optimal, B - optimal).

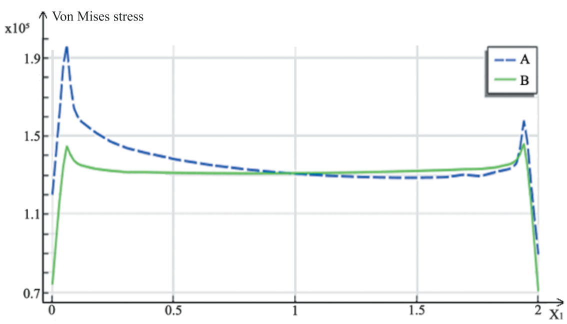


Fig. 13.9 Von Mises stress (Pa) along the central axis of the solder region for loading case 2 (A - not optimal, B - optimal)

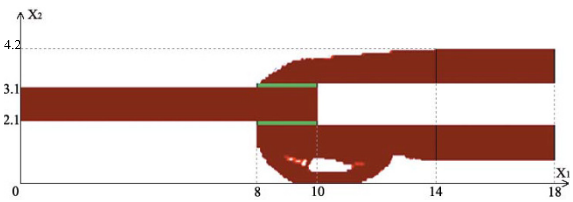


Fig. 13.10 Optimal topology with asymmetric loads.

In Figs. 13.11 and 13.12 shear stresses along the central axis of the region of the solder for the case of loading 3 along the upper and lower layers of the adhesive are shown.

For optimal structures, the stresses in all cases are distributed almost evenly, in contrast to the original structures. The optimization process makes it possible to reduce the stress drops for both symmetrical action of loads and for different intensities of loads in different parts of the structure.

13.5 Concluding Remarks

In this paper, the topological optimization algorithm was used to optimize sandwich structures with an adhesive layer under the action of mechanical loads in order to reduce peak stresses. The results show that the obtained optimal structures significantly reduce the peak shear and von Mises stresses in the solder layer in comparison with other common engineering solutions for this class of problems.

As a result of solving the problem of topological optimization, a design solution was obtained that is least susceptible to destruction due to an increase in the rigidity of the elements being connected and, as a consequence, provides smoothing of stress

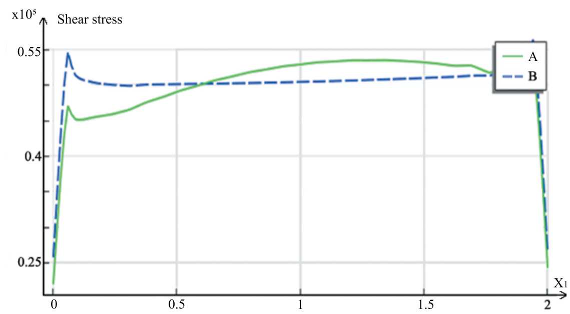


Fig. 13.11 Shear stress (Pa) along the central axis of the solder region for the case of loading 3 (A - not optimal, B - optimal) along the upper layer.

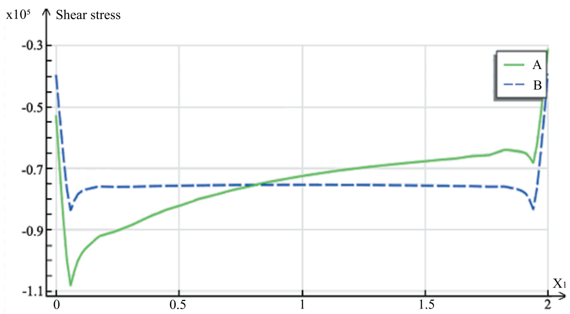


Fig. 13.12 Shear stress (Pa) along the central axis of the solder region for loading case 3 (A - not optimal, B - optimal) along the bottom layer.

peaks in the adhesive layer. This was achieved by modifying the optimization area with the same amount of material.

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