On the dynamics of blood through the circular tube along with magnetic properties

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Abstract: In this article, we will investigate a blood flow model with suspended magnetic particles. The fluid is influenced by an external magnetic field and an oscillating pressure gradient. Exact solutions for the velocity of fluid and velocity of magnetic particles will be obtained by means of integral transforms. Obtained results will be expressed in terms of post transient and transient parts. Moreover, to study the influence of the material parameters, numerical simulations and graphical illustrations will be used and useful consequences will be summarized.

1. Introduction

Basically, blood consists of multiple components, a mixture of various cells in plasma which behaves like an incompressible Newtonian fluid [1]. Moreover, plasma in a capillary flow behaves like Newtonian fluid [2]. It is more likely a bio-magnetic fluid, so its flow is effected by the magnetic field [3]. Furthermore, blood magnetic property is significantly influenced by the state of oxygenation [4]. The use of magnetic field for streamlining the flow of blood in the body could be utilized to control poor circulation of blood and the risk of heart attack to a person [3]. In [5] Haik et al. developed a bio fluid dynamics model closely resembling to the ferro-hydrodynamics. Further Varshney et al. [6] numerically investigated the effect of magnetic field on the blood flow in artery having multiple stenosis, Bourhan and Magableh [7] studied the effects of magnetic field on heat transfer and fluid flow characteristics of blood flow in multi-stenosis arteries. In 2015, Sharma et al. [4] have numerically investigated the fluid flow parameters of blood together with magnetic particles in a cylindrical tube.

Our aim is to investigate the dynamics of proposed blood flow model with magnetic particles through a cylindrical tube under the influence of magnetic field and oscillatory pressure gradient in the axial direction [4]. However, we look for the exact solutions for the dimensionless form of the fluid velocity and magnetic particles velocity and express the obtained results in terms of steady state and transient parts. Furthermore, influence of the external magnetic field, particles concentration parameter and particles mass parameters on the dynamics of fluid and particles is investigated via numerical simulations and graphical illustrations.

2. Description of the problem

The artificial blood (75% water and 25% Glycerol) along with magnetic particles (iron oxide) is assumed to be flowing in a cylindrical glass tube under the influence of axial pressure gradient. The magnetic particles are supposed to be uniformly distributed throughout the blood. The blood is flowing in the axial direction and a uniform transverse magnetic field is applied. We assume that:

a. No-slip condition at the wall of tube is applied that the blood and magnetic particles have zero velocities at the wall of the tube.

b. The magnetic Reynolds number is very small; hence the induced magnetic field effect is neglected [7].

2.1. Proposed geometry of the model

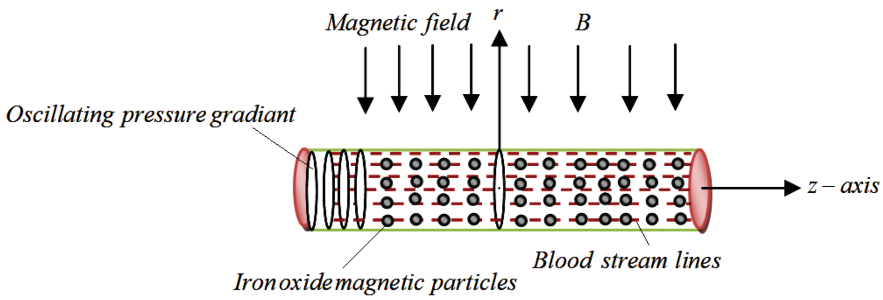

Figure 1. Proposed geometry of the model

Nomenclature

$\vec{J}$ Current density

σ Electrical conductivity

$\vec{E}$ Electric field intensity

$\vec{V}$ Velocity vector

$\vec{B}$ Magnetic flux intensity

μ_0 Magnetic permeability

ρ Density of the fluid

μ Dynamic viscosity of the fluid

ν Kinematic viscosity of the fluid

N Number of magnetic particles per unit volume

S Stokes constant

Ha Hartmann number

C Particles concentration

M Particles mass parameter

2.2. Mathematical model

Based on the fact that, when a magnetic field is applied on an electrically conducting fluid, an electromagnetic force is generated due to the interaction of current with magnetic field. In our model, iron oxide (magnetic particles) are suspended in blood (bio-magnetic fluid) which makes the blood more conducting and a strong electromagnetic force is experienced (due to the interaction of current with magnetic field). The strength of this electromotive force also depends on the speed of motion of the magnetic particles as well as magnetic flux intensity [8]. The governing equations of our problem involves both Navier-Stokes equations describing the fluid flow and Maxwell's relations for magnetic field interactions. Now, from

(Ohm's law) $$\vec{J} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right), \tag{1}$$

(Maxwell's equations) $$\nabla \cdot \vec{\mathrm{B}} = 0,\ \nabla \times \vec{\mathrm{B}} = \mu_0 \vec{J},\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{2}$$

(Electromagnetic force included

in the momentum equation) $$\vec{F}_m = \vec{J} \times \vec{\mathrm{B}} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right) \times \vec{B} = -\sigma B^2 u_b' \hat{e}_z, \tag{3}$$

where $u_b'(r,t)$ is the axial velocity of the blood.

Consider blood is flowing in an axi-symmetric cylindrical tube of radius "a" with axis of the cylinder along z' - axis, subject to the pressure gradient $\frac{\partial p'}{\partial z'}$ and transverse magnetic field of strength B. The governing momentum equation for fluid flow in cylindrical polar coordinates is given by [4], [5]

$$\frac{\partial u_b'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial z'} + \nu\left(\frac{\partial^2 u_b'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u_b'}{\partial r'}\right) + \frac{SN}{\rho}(u_p' - u_b') - \frac{\sigma B^2 u_b'}{\rho}, \tag{4}$$

where $u_p'(r',t')$ is the velocity of the particles. For small Reynolds number of the relative velocity the force between blood and magnetic particle is proportional to the relative velocity.

Moreover, motion of the magnetic particles is governed by

$$m_{av}\frac{\partial u_b'}{\partial t'} = S(u_b' - \mathrm{u}_p'), \tag{5}$$

where m_{av} is the average mass of the magnetic particles.

Like in [9], pressure gradient is considered as

$$-\frac{\partial p'}{\partial z'} = \psi_0 + \psi_1 \cos(\omega' t'), \tag{6}$$

where ψ_0 is the constant amplitude of the pressure gradient and ψ_1 is the amplitude of the pulsatile component giving rise to systolic and diastolic pressure. In experiments this kind of pressure gradient is maintained by peristaltic pump.

The initial and boundary conditions on the velocity field are given by [4]

$$u'_b(r',0) = u'_p(r',0) = 0 \ \text{ for } \ r' \in (0,a), \tag{7}$$

$$\left.\frac{\partial u'_b(r',t')}{\partial r'}\right|_{r'=0} = 0 \ \text{ for } \ t' > 0, \tag{8}$$

$$u'_b(a,t') = u'_p(a,t') = 0 \ \text{ for } \ t' > 0, \tag{9}$$

By introducing the following dimensionless variables and parameters [4] in to Eqs. (4) – (9)

$$r^* = \frac{r'}{a}, \quad u_b{}^* = \frac{a u'_b}{\nu}, \quad u_p{}^* = \frac{u'_p a}{\nu}, \quad t^* = \frac{\nu t'}{a^2}, \quad p^* = \frac{a^2 p'}{\rho \nu^2}, \tag{10}$$

and dropping star notation, we have the following dimensionless initial-boundary problem

$$\frac{\partial u_b(r,t)}{\partial t} = \psi_0 + \psi_1 \cos(\omega t) + \left(\frac{\partial^2 u_b(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u_b(r,t)}{\partial r} \right) + C\left[u_p(r,t) - u_b(r,t) \right] - Ha^2 u_b(r,t), \tag{11}$$

$$M \frac{\partial u_b(r,t)}{\partial t} = u_b(r,t) - u_p(r,t), \tag{12}$$

$$u_b(r,0) = u_p(r,0) = 0 \ , \ r \in [0,1], \tag{13}$$

$$\left.\frac{\partial u_b(r,t)}{\partial r}\right|_{r=0} = 0\,, \tag{14}$$

$$u_b(1,t) = u_p(1,t) = 0 \quad t > 0, \tag{15}$$

where $Ha = Ba\sqrt{\frac{\sigma}{\mu}}$, $C = \frac{SNa^2}{\mu}$ and $M = \frac{m\mu}{\rho a^2 S}$.

3. Solution of the problem

Applying Laplace transform to Eqs. (11), (12) and (15) and using initial conditions (13), we get

$$q\bar{u}_b(r,q) = \frac{\psi_0}{q} + \psi_1 \frac{q}{q^2 + \omega^2} + \left(\frac{\partial^2 \bar{u}_b(r,q)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}_b(r,q)}{\partial r} \right) + C\bar{u}_p(r,q) - (C + Ha^2)\bar{u}_b(r,q), \tag{16}$$

$$\bar{u}_p(r,q) = \frac{\bar{u}_b(r,q)}{Mq+1}, \tag{17}$$

$$\bar{u}_b(1,q)=0,\ \ \bar{u}_p(1,q)=0. \tag{18}$$

Using Eq. (17) into Eq. (16), we obtain

$$q\bar{u}_b(r,q)=\frac{\psi_0}{q}+\psi_1\frac{q}{q^2+\omega^2}+\left(\frac{\partial^2\bar{u}_b(r,q)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{u}_b(r,q)}{\partial r}\right)+C\frac{\bar{u}_b(r,q)}{Mq+1}-(C+Ha^2)\bar{u}_b(r,q). \tag{19}$$

Eq. (19) can be written in an equivalent form

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\bar{u}_b(r,q)=\frac{Mq^2+\left(1+M(C+Ha^2)q+Ha^2\right)}{Mq+1}\bar{u}_b(r,q)-\left(\frac{\psi_0}{q}+\psi_1\frac{q}{q^2+\omega^2}\right). \tag{20}$$

Applying finite Hankel transform [10] of order zero and using the boundary conditions, we get

$$\bar{u}_{bH}(r_n,q)=G(q)\frac{J_1(r_n)}{r_n}\cdot\frac{Mq+1}{Mq^2+(1+M(C+H_a^2+r_n^2)q+(H_a^2+r_n^2)}, \tag{21}$$

where $G(q)=\dfrac{\psi_o}{q}+\psi_1\dfrac{q}{q^2+\omega^2}$.

Now, Eq. (21) can be rewritten as

$$\begin{aligned}\bar{u}_{bH}(r_n,q)&=\frac{J_1(r_n)}{r_n}G(q)\frac{Mq+1}{Mq^2+\alpha_n q+\beta_n}\\&=\frac{J_1(r_n)}{r_n}\frac{1}{M}\left[\chi_{1n}\frac{1}{q+a_{1n}}+\chi_{2n}\frac{1}{q+a_{2n}}+\frac{\psi_o M}{q\beta_n}+\chi_{3n}\frac{q}{q^2+\omega^2}+\frac{\chi_{4n}}{\omega}\frac{\omega}{q^2+\omega^2}\right],\end{aligned} \tag{22}$$

where $\alpha_n=1+M(C+H_a^2+r_n^2)$, $\beta_n=H_a^2+r_n^2$, $a_{1n}=\dfrac{1}{2M}\left(\alpha_n-\sqrt{\alpha_n^2-4M\beta_n}\right)$,

$a_{2n}=\dfrac{1}{2M}\left(\alpha_n+\sqrt{\alpha_n^2-4M\beta_n}\right)$, $b_{1n}=a_{2n}M\left(\omega^2M-\beta_n\right)+\alpha_n\beta_n$, $b_{2n}=a_{1n}M\left(\omega^2M-\beta_n\right)+\alpha_n\beta_n$,

$b_{3n}=\dfrac{M}{\beta_n}\left[\psi_o\omega^2M\left(M\beta_n-\alpha_n\right)-M\beta_n^2\left(\psi_o+\psi_1\right)\right]$, $c_{1n}=M\left(\omega^2M-\beta_n\right)-\alpha_n\left(a_{2n}M-\alpha_n\right)$,

$c_{2n}=M\left(\omega^2M-\beta_n\right)-\alpha_n\left(a_{1n}M-\alpha_n\right)$, $c_{3n}=\dfrac{M^2}{\beta_n}\left[\beta_n\left(1-\alpha_n\right)\left(\psi_o+\psi_1\right)-\psi_o M\omega^2\right]$,

$\chi_{1n}=\dfrac{b_{3n}c_{2n}-b_{2n}c_{3n}}{b_{1n}c_{2n}-b_{2n}c_{1n}}$, $\chi_{2n}=\dfrac{b_{1n}c_{3n}-b_{3n}c_{1n}}{b_{1n}c_{2n}-b_{2n}c_{1n}}$, $\chi_{3n}=-\left(\dfrac{\psi_o M}{\beta_n}+\chi_{1n}+\chi_{2n}\right)$,

$\chi_{4n}=M\left(\psi_o+\psi_1\right)-\dfrac{a_{2n}M-\alpha_n}{M}\chi_{1n}-\dfrac{a_{1n}M-\alpha_n}{M}\chi_{2n}$.

Taking the inverse Laplace transform, we obtain

$$u_{bH}(r_n,t)=\frac{J_1(r_n)}{r_n}\left[\frac{\chi_{1n}}{M}e^{-a_{1n}t}+\frac{\chi_{2n}}{M}e^{-a_{2n}t}+\frac{\psi_o}{\beta_n}+\frac{\chi_{3n}}{M}\cos(\omega t)+\frac{\chi_{4n}}{M\omega}\sin(\omega t)\right]. \tag{23}$$

Taking the inverse Hankel transform [11], we obtain

$$u_b(r,t)=2\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\left[\frac{\psi_o}{\beta_n}+\frac{\chi_{3n}}{M}\cos(\omega t)+\frac{\chi_{4n}}{M\omega}\sin(\omega t)\right]+\frac{2}{M}\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\left[\chi_{1n}e^{-a_{1n}t}+\chi_{2n}e^{-a_{2n}t}\right] \quad (24)$$

or as the sum of steady-state part $u_{st}(r,t)$ and transient part $u_t(r,t)$ can be written

$$u_b(r,t)=u_{b\,st}(r,t)+u_{b\,t}(r,t), \quad (25)$$

where

$$u_{b\,st}(r,t)=2\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\left[\frac{\psi_o}{\beta_n}+\frac{\chi_{3n}}{M}\cos(\omega t)+\frac{\chi_{4n}}{M\omega}\sin(\omega t)\right] \quad (26)$$

and

$$u_{b\,t}(r,t)=\frac{2}{M}\sum_{n=1}^{\infty}\frac{J_0(rr_n)}{r_nJ_1(r_n)}\left[\chi_{1n}e^{-a_{1n}t}+\chi_{2n}e^{-a_{2n}t}\right]. \quad (27)$$

The particles velocity $u_p(r,t)$ can be obtained introducing Eq. (24) into (12) and using Eq. (18), we get

$$u_p(r,t)=\frac{1}{M}\int_0^t u_b(r,t)\exp\left(-\frac{t-s}{M}\right)ds. \quad (28)$$

or

$$\begin{aligned}u_p(r,t)=&2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{r_nJ_1(r_n)}\left[\frac{\psi_o}{H_a^2+r_n^2}+\frac{\psi_1}{(a_{1n}^2+\omega^2)(a_{2n}^2+\omega^2)}\times\right.\\&\times\Big(\big(a_{1n}a_{2n}+\omega^2(Mc_n-1)\big)\cos(\omega t)+\omega\big(a_{1n}+a_{2n}+M(\omega^2-a_{1n}a_{2n})\big)\sin(\omega t)\Big)+\\&\left.+\frac{1-Ma_{1n}}{a_{1n}-a_{2n}}\left(\frac{\psi_o}{Ma_{1n}}+\frac{a_{1n}\psi_1}{a_{1n}^2+\omega^2}\right)e^{-a_{1n}t}-\frac{1-Ma_{2n}}{a_{1n}-a_{2n}}\left(\frac{\psi_o}{Ma_{2n}}+\frac{a_{2n}\psi_1}{a_{2n}^2+\omega^2}\right)e^{-a_{2n}t}\right].\end{aligned} \quad (29)$$

Moreover, in more elegant form we can write

$$u_p(r,t)=u_{p\,st}(r,t)+u_{p\,t}(r,t), \quad (30)$$

where

$$\begin{aligned}u_{p\,st}(r,t)=&2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{r_nJ_1(r_n)}\left[\frac{\psi_o}{H_a^2+r_n^2}+\frac{\psi_1}{(a_{1n}^2+\omega^2)(a_{2n}^2+\omega^2)}\right.\\&\left.\Big(\big(a_{1n}a_{2n}+\omega^2(Mc_n-1)\big)\cos(\omega t)+\omega\big(a_{1n}+a_{2n}+M(\omega^2-a_{1n}a_{2n})\big)\sin(\omega t)\Big)\right],\end{aligned} \quad (31)$$

$$u_{p\,t}(r,t)=2\sum_{n=1}^{\infty}\frac{J_o(rr_n)}{r_nJ_1(r_n)}\left[\frac{1-Ma_{1n}}{a_{1n}-a_{2n}}\left(\frac{\psi_o}{Ma_{1n}}+\frac{a_{1n}\psi_1}{a_{1n}^2+\omega^2}\right)e^{-a_{1n}t}-\frac{1-Ma_{2n}}{a_{1n}-a_{2n}}\left(\frac{\psi_o}{Ma_{2n}}+\frac{a_{2n}\psi_1}{a_{2n}^2+\omega^2}\right)e^{-a_{2n}t}\right] \quad (32)$$

the steady-state and transient components of the particles velocity.

4. Numerical results and discussion

In this section our interest is to analyze the influence of the system parameters i.e. Hartmann number, particles concentration parameter and particles mass parameter on the velocity of the fluid as well as the flow velocity of the magnetic particles. In order to evaluate numerical values of the velocities, we need the positive roots of the Bessel function J_0. These roots are generated by a numerical subroutine using MATHCAD 15.

The profiles of velocities versus r are plotted as shown in the Fig. 2 in order to discuss the influence of the Hartmann number on the flow of fluid and particles at different values of dimensionless times t. Here we have considered $Ha \in \{1,2,3,4,5\}$ and $M = 0.5, C = 2$. As expected it is noticed that both fluid velocity $u_b(r,t)$ and particles velocity $u_p(r,t)$ decreases with the increasing values of Hartmann number. Because, Lorentz force (that appears when transverse magnetic field is applied to a moving electrically conducting fluid) resists the flow of fluid and magnetic particles. From the Fig. 2, it is reported that the effect of Hartmann number on $u_b(r,t)$ and $u_p(r,t)$ is quite significant about the axis of the cylinder and both the velocities decreases from maximum value to zero as $r \to 1$. In comparison the fluid flows faster than the particles flow. Moreover, as the time progresses both the velocities increases.

The diagrams of Fig. 3 are plotted in order to discuss the influence of the particle concentration parameter C on both the velocities $u_b(r,t)$ and $u_p(r,t)$. In this case we take $C \in \{2,4,6,8,12\}$ and we have used the other parameters $M = 1, Ha = 3$. A similar trend is observed like the case of Ha. Because, due to the increase of concentration, collision of the particles results in the displacement from their initial positions and leave the fluid stream lines. This deviation from their dynamic equilibrium state will induce relative velocity between the particles and the fluid, resulting in an additional energy dissipation and gives rise to an increase in effective viscosity. Moreover, from the profiles it is also noticed that $u_p(r,t) > u_b(r,t)$ and the influence of C on $u_b(r,t)$ is significant but less significant on $u_p(r,t)$ and diminishes with time. Here also influence of C is significant near the axis of cylinder and both the velocities approaches to zero as $r \to 1$.

From the profiles of velocities versus r for different values of particles mass parameter M and at different values of dimensionless times, as noticed from Fig. 4 that velocity of particle $u_p(r,t)$

increases with the increasing values of parameter M, but the parameter M has no significant influence on the fluid velocity and this influence diminishes with time. Here, we take $M \in \{0.6, 0.7, 0.8, 0.9, 1.2\}$ and $C = 4$, $Ha = 3$.

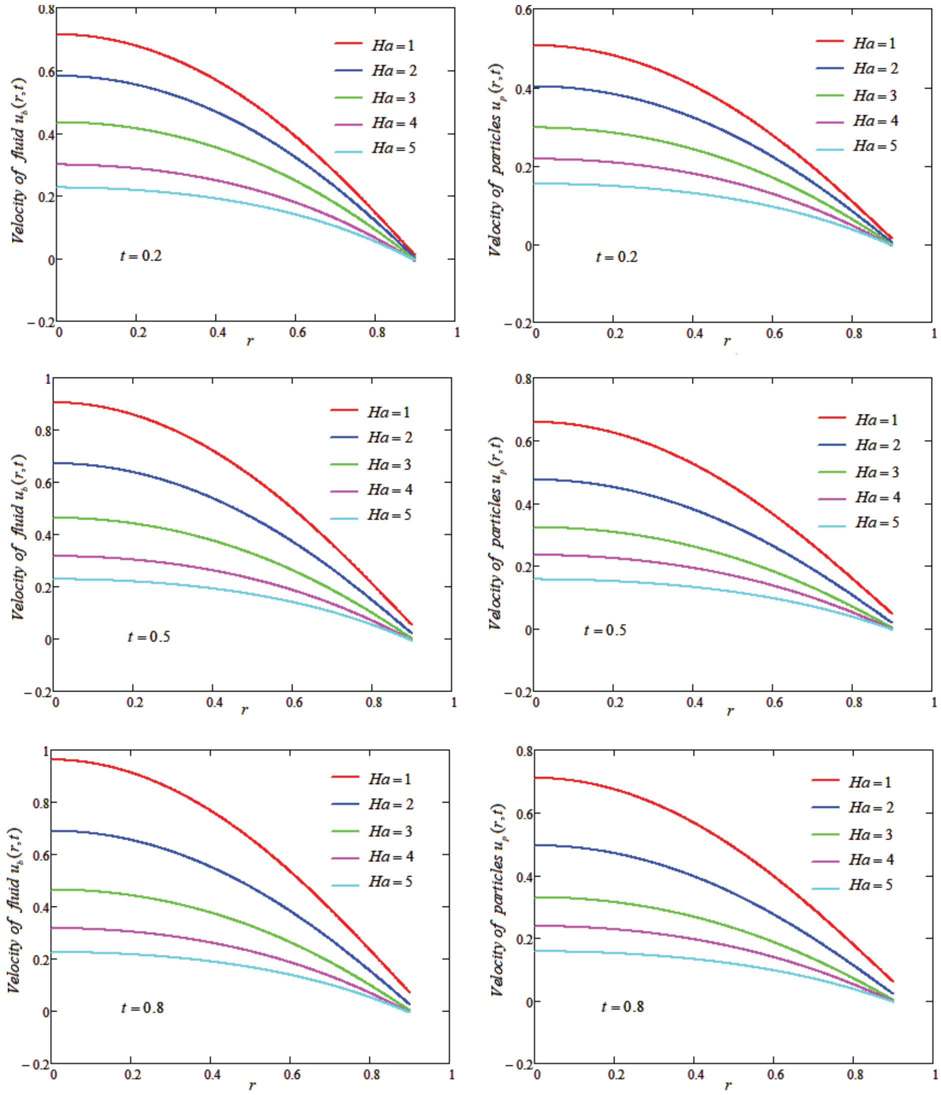

Figure 2. Profiles of velocities for different values of Hartmann number and times.

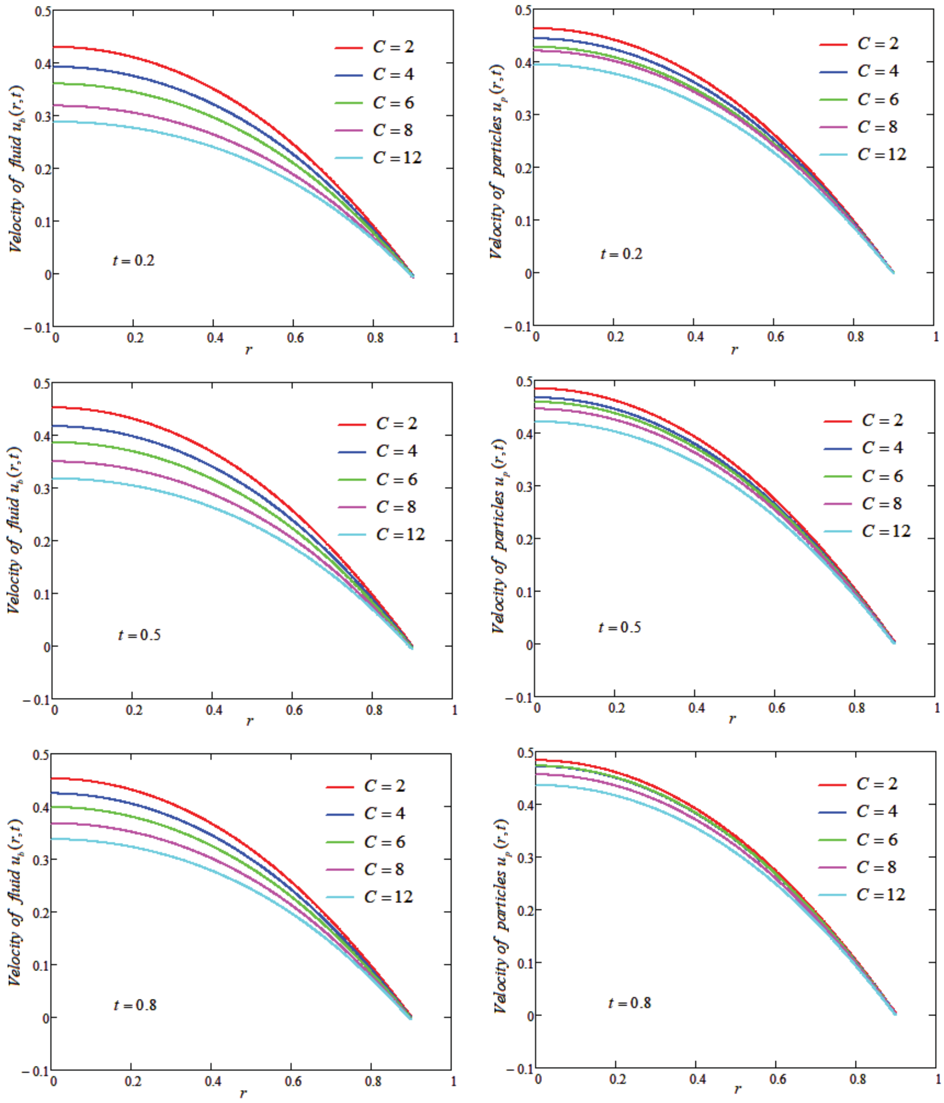

Figure 3. Profiles of velocities for different values of particles concentration parameter and times.

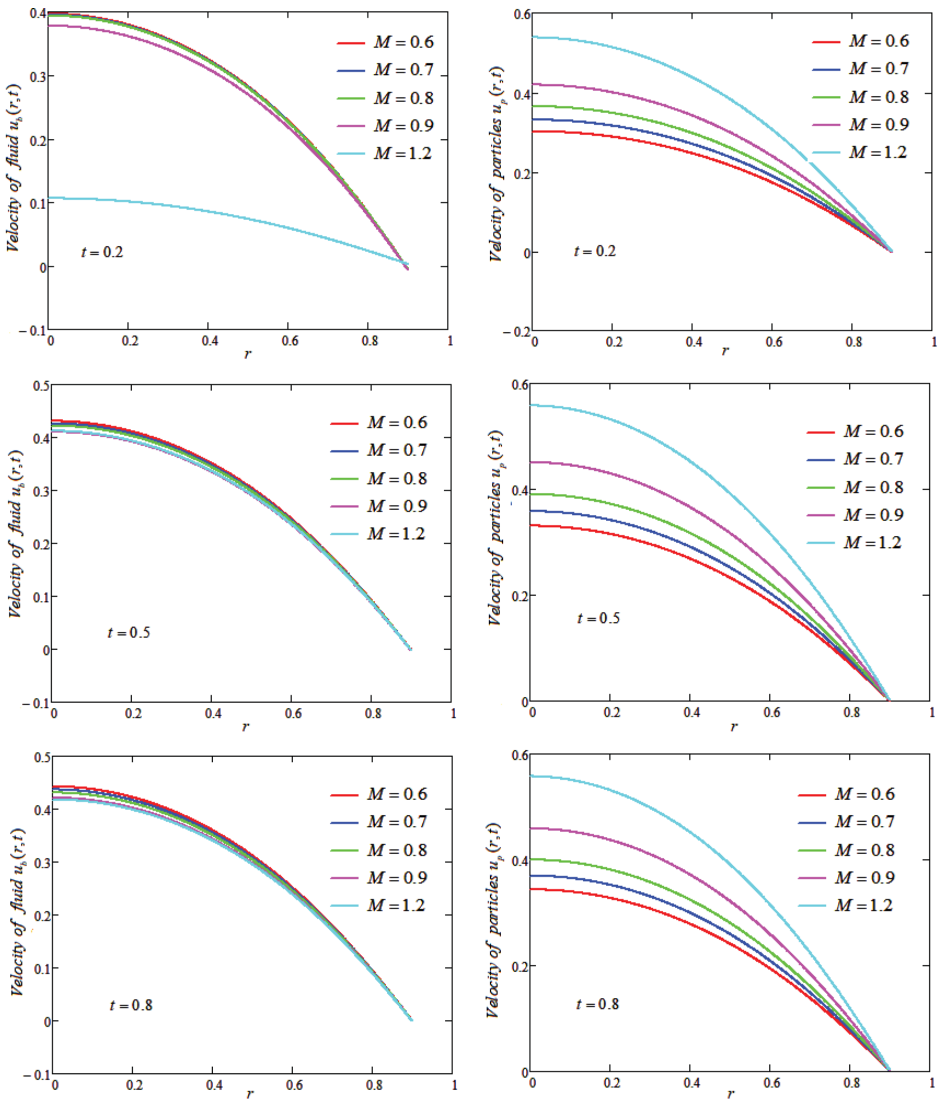

Figure 4. Profiles of velocities for different values of particles mass parameter and times.

In all the diagrams we have chosen the values of the parameters ψ_0, ψ_1 and ω to be 1 , 0.8 and $\frac{\pi}{6}$ respectively.

5. Conclusions

The purpose of this investigation was to study the dynamics of proposed blood model with suspended magnetic particles flowing through a cylindrical tube under the influence of magnetic field and oscillatory pressure gradient. The exact solutions for the dimensionless form of the fluid velocity and magnetic particles velocity are obtained and expressed in terms of steady state and transient parts. Furthermore, influence of the external magnetic field, particles concentration and mass parameters on the dynamics of fluid and particles is investigated via numerical simulations and graphical illustrations

The noteworthy conclusions of the investigation are as under:

- Strength of Hartmann number Ha , retarded the flow of fluid as well as particles.
- The velocities of fluid as well as of particles are decreasing functions of particles concentration parameter.
- Influence of concentration parameter is significant on fluid velocity as compare to particles velocity.
- The particles mass parameter influence inversely on particles velocity, while its effect on the fluid velocity is insignificant.

Acknowledgments

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