Numerical and experimental investigations of dynamics of magnetic pendulum with an aerostatic bearing

Ewelina Ogińska, Krystian Polczyński, Dariusz Grzelczyk, Jan Awrejcewicz

Abstract: In this paper, both numerical and experimental results of the dynamics of a magnetic pendulum with an aerostatic bearing are presented. The experimental stand consists of the physical pendulum with a neodymium magnet at its end, whereas two electric coils are placed underneath. The pivot of the pendulum is supported by aerostatic bearing, therefore dry friction can be negligible, and it has only a viscous character. The electric current that flows through the coils is of a square waveform with a given frequency and duty cycle. Mathematical and physical models with the system parameters confirmed experimentally, are presented. The magnetic interaction is characterized as a moment of force as a function of the electric current and angular position of the pendulum. The results of the simulation and experiment showed the rich dynamics of the system, including various types of regular motion (multi-periodicity) and chaos.

1. Introduction

Pendulums are the objects of different studies in numerous scientific works due to their simplest construction and nonlinear character of motion. It is known that mechanical energy can be produced by an interaction between electric and magnetic fields with a high level of efficiency. This phenomenon is used, for example, in electric motors, which means that it is a developmental topic. In this paper we analyzed an original construction of a physical pendulum with magnetic interactions, and its axis of rotation coincides with the shaft's axis suspended in the pressured air generated by an aerostatic bearing. Electric coils, working as an excitation source, were introduced into the system, in order to repulse the neodymium magnet attached to the end of the pendulum. As a result, the considered case is a dynamical system in which electromagnetic forces affect the mechanical system. This system is coherent, so the pendulum's movement is closely dependent on the force generated by the electromagnetic field, but also on the distance from the coil. The presented dependency is the object of the study in this paper.

The investigations in which the pendulum behaviour depends on the electromagnetic field were carried out, for instance, by Kraftmakher [1,2]. In those papers, two magnets were placed on opposite sides of the pendulum's rod at different distances from the point of the rotation. The external magnetic field was driving the pendulum motion and could be used to modify the torque. Chaotic behaviour and nonlinear oscillations (forced and free) were detected and studied numerically. The Poincaré sections, phase plane graphs, histograms and Fourier's spectra were also presented.

In another paper Wojna et al. [3] analyzed numerically and experimentally the behaviour of a system containing a double physical pendulum with two permanent magnets forced by alternating magnetic field comes from the coils. They presented extended bifurcation diagrams for different frequencies of excitation signal as a control parameter, obtained both experimentally and numerically.

Berdahl and Lugt [4] investigated pendulum driven by rotating permanent magnet, using the power spectra, Poincaré maps and time-delay plots of the system. They observed that, depending on driving frequencies, some behaviour of Poincaré maps were periodic, and another chaotic. The time-lag plots for both periodic and chaotic motion were also presented in that study.

Polczyński et al. [5,6] described the behaviour of a two-degree-of-freedom system consists of two pendulums with magnets and elastic element coupling their pivots. Tests were conducted both numerically and experimentally. By using time histories, phase portraits, Poincaré sections and bifurcation graphs, they presented rich nonlinear dynamics of the considered system. Moreover, the results obtained experimentally were in good agreement with the simulation ones. The uniqueness of that work lies in the mechatronic system and the original way of the excitation source.

The modification of the Duffing equation of periodically driven iron pendulum in a magnetic field was analyzed by Donnagáin and Rasskazov [7]. Studies led to creating a special Poincaré section, time histories and phase portraits for the determined values of the parameters.

Kadjie and Woafo [8] presented a model of energy harvester consisting of an electromechanical pendulum subjected to nonlinear springs. The investigations showed that the suitable range of control parameters of the device vary led to the more efficient power generation than the case without springs. The transition from periodic to chaotic states were clearly noticed.

Concluding, in contrast to the above mentioned articles, this paper describes the periodic and chaotic behaviour of an asymmetrically forced physical pendulum system. One electric coil is mounted exactly under the pendulum, i.e. when the non-forced pendulum is in a stable position. The axis of rotation of the second electric coil is inclined at the angle of 45 degrees to the first one. Both coils generate an electromagnetic field, which defines the behavior of magnetic pendulum. An aerostatic bearing is the next new aspect, since it eliminates dry friction force and provides only viscous resistance.

2. Experimental rig and the electric excitation signal

The experimental setup of the considered physical pendulum system is presented in Fig. 1. The stand is equipped with the pendulum (1), which has a neodymium magnet (2) at the end of the rod. The axis of rotation has coincided with the shaft's axis suspended in the pressured air produced by an aerostatic bearing. Two electric coils (3) are mounted on the textolite board (4), whereas the angle between them is 45 degrees. The aluminium disk (5) with a diameter of 65 mm is attached to the shaft. Distance between the coils and the neodymium magnet during the experiment was equal to 2 mm. The test stand is made of non-magnetic materials such as aluminium alloys, brass and polymer composites, due to diminishing the interaction with the magnetic elements of the investigated system.

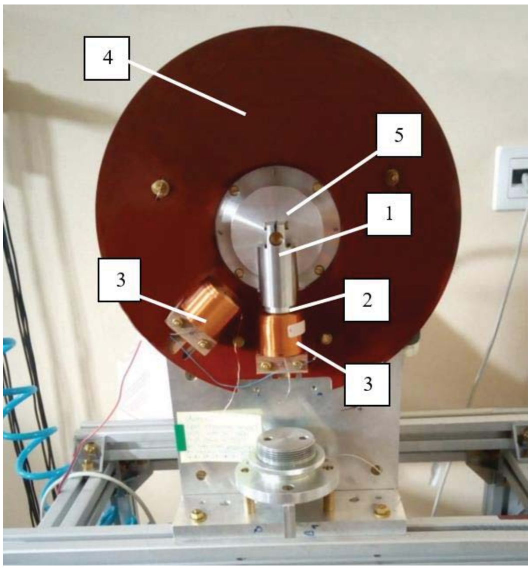


Figure 1. Experimental rig: 1 – physical pendulum, 2 – neodymium magnet, 3 – electric coils, 4 – textolite board, 5 – aluminium disk.

The shape of the electric current signal inside the electric coils is presented in Fig. 2 and flows through both coils at the same time. The parameters of the signal such as frequency and duty cycle can be controlled independently, whereas the amplitude i_a of the electric current was fixed to 0.5 A for the experimental tests.

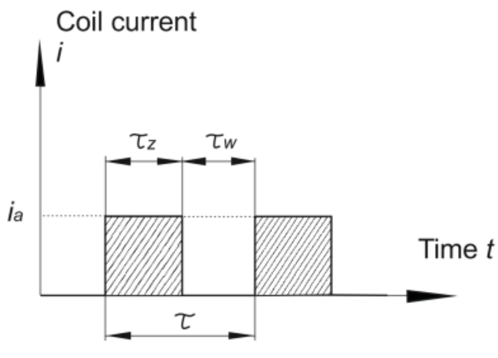


Figure 2. Excitation current signal: τ_z – switched on current; τ_w – switched off current; $\tau = \tau_z + \tau_w$ – the period of the signal; $w = \frac{\tau_z}{\tau} \cdot 100\%$ – duty cycle).

3. Mathematical model

In this section, the physical and mathematical models of the system are developed and presented. The physical model of the considered pendulum system is shown in Fig. 3.

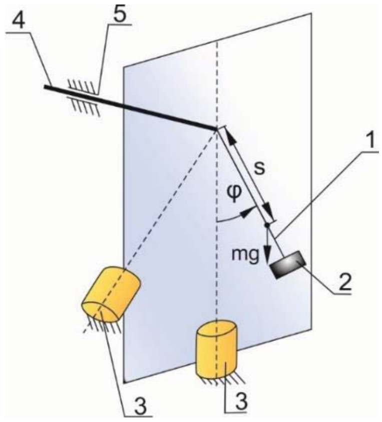


Figure 3. Physical model of the system: 1 – pendulum; 2 – neodymium magnet; 3 – electric coils; 4 – shaft; 5 – aerostatic bearing.

The mathematical model has been carried out according to classical mechanics laws. General equation of motion is as follow

$$I\ddot{\varphi} + c\dot{\varphi} + mgs\sin\varphi = M_{1mag}(\varphi, i) + M_{2mag}(\varphi, i), \tag{1}$$

where *I* stands for the moment of inertia of the pendulum, *mg* is the weight of the pendulum, *s* is the length between the pivot and centre of mass of the pendulum, and *c* stands for coefficient of viscous damping. The term $M_{1mag}(\varphi,i)$ describes the magnetic interaction between the magnet and coil placed under the pendulum, whereas $M_{2mag}(\varphi,i)$ concerns the inclined coil case. The argument *i* is the value of the current signal dependent on time.

4. Experiments versus numerical simulations

We started our studies from the identification of the system parameters. In order to reduce the number of the parameters which we had to find using numerical methods, some of them were identified experimentally. First of all, the value of the product of *m* and *s* parameters was obtained experimentally. While the electric coils were switched off (i.e. $M_{1mag}(\varphi,i) = M_{2mag}(\varphi,i) = 0$), the moment of gravity *mgs* was balanced by the torque generated by force F. This torque was generated by tensometric beam connected with the aluminium disk with diameter D (attached to the shaft, see Fig. 1) by the string. This dependence could be written by using the equilibrium equation which yields

$$ms = \frac{FD}{2g \sin \varphi}. \quad (2)$$

Assuming values of the parameters g = 9.81 N/kg and D = 0.065 m, the values of the force F was measured for angles φ larger than zero. Taking into consideration Eq. (2) and measured values of force F, we received a constant value of $ms = 4.4 \cdot 10^{-3}$ kg·m.

In the next step, we identified the values of I and c based on time histories of the pendulum displacement obtained experimentally. The parameters I and c were obtained numerically, by fitting the Eq. (1) with neglected terms $M_{1mag}(\varphi,i)$ and $M_{2mag}(\varphi,i)$ to the experimental data of time histories of angular position of the pendulum during free oscillations. Figure 4 shows the time histories of angular position of the pendulum with the fitting process, where blue markers denote the experimental data and the red line is the fitted solution of Eq. (1). The fitting process was obtained by using *Mathematica* software. The best fit was obtained for $I = 0.21179 \cdot 10^{-3}$ kg·m² and $c = 9.28868 \cdot 10^{-6}$ N·m·s/rad.

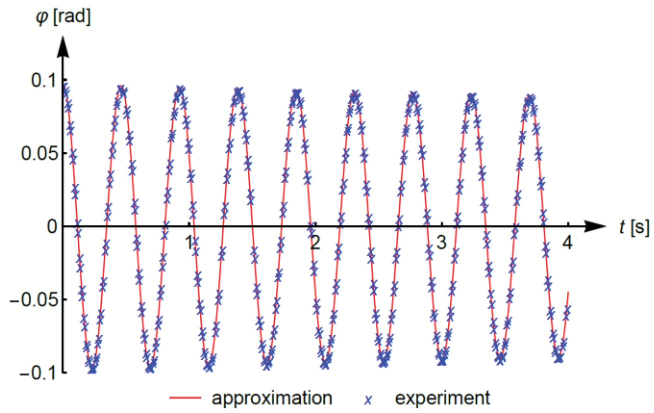


Figure 4. Time histories of free oscillations obtained experimentally (blue markers) and fitted numerical solution of Eq. (1) (red line).

In the last step, we modelled and identified the magnetic interaction between the magnet and coils based on the experimental data. For this purpose, to obtain experimental data, we used only the bottom coil of mentioned interaction and assume that the excitation of the second coil (located at an angle $\pi/4$ from the bottom coil) has the same nature as the first one. Therefore, we used modified equation of motion in steady state by adding the magnetic interaction term $M_{1mag}(\varphi,i)$. For a fixed value of $i(t) = i_a = 0.5$ A the coil produces a steady torque $M_{1mag}(\varphi,i_a)$ for each φ. The torque $M_{1mag}(\varphi,i_a)$ can be computed by using the following formula

$$M_{1mag}(\varphi, i_a) = F\frac{D}{2} + mg\sin\varphi. \tag{3}$$

In further studies we have modelled magnetic torque $M_{1mag}(\varphi,i)$ as an analytical approximation of the obtained experimental data comes from Eq. (3). Moreover, the torque $M_{2mag}(\varphi,i)$ has been described by this same approximation formula, whereas the angle argument is shifted by the fixed angle $\pi/4$. Both formulas have the following forms

$$M_{1mag}(\varphi, i) = Aie^{-\lambda\varphi^2}\varphi, \tag{4}$$

$$M_{2mag}(\varphi, i) = Aie^{-\lambda\left(\varphi+\frac{\pi}{4}\right)^2}(\varphi + \pi/4), \tag{5}$$

where φ is limited to the range $[-\pi, \pi]$, A and λ are constants coefficients for a given pair of magnet and coil as well as a current signal. Figure 5 presents experimental data of torque $M_{1mag}(\varphi,i_a)$ calculated from Eq. (3) and analytical approximation described by Eq. (4). The fitting process conducted via *Mathematica* has given the following coefficients: A = 0.943439 N·m/(rad·A) and λ = 14.6911 1/rad^2.

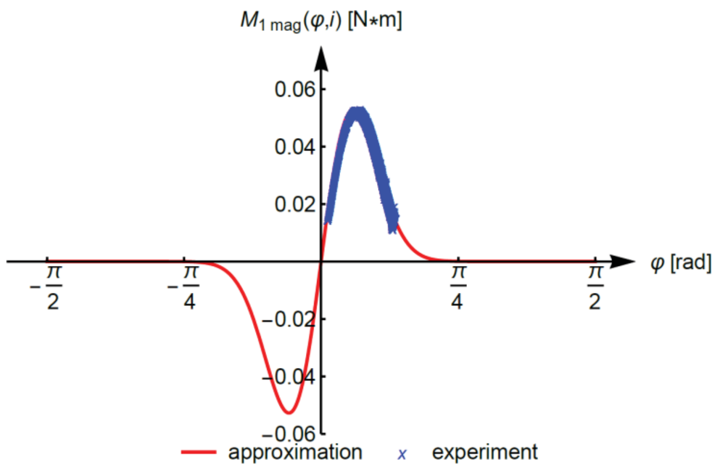


Figure 5. Comparison of experimental data (blue markers) and the torque $M_{1mag}(\varphi, i)$ obtained analytically from Eq. (4) (red line) for steady $i(t) = i_a = 0.5$ A.

Taking into account Eqs. (4) and (5), the total excitation torque acts on the pendulum can be expressed as the sum of both equations. The total value of magnetic interaction is presented in Fig. 6.

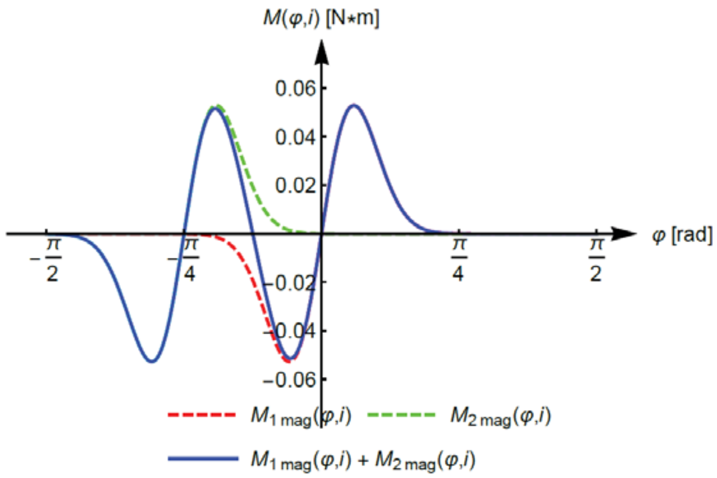


Figure 6. The total magnetic torque acting on the pendulum (blue line) as a sum of torques generated by both coils.

The developed mathematical model of the considered pendulum system subjected to the magnetic torque induces by two coils was verified experimentally. The angular position of the forced pendulum was recorded and confirmed with simulation. The current signal parameters during the experiment were fixed as follow: amplitude of current – i_a = 0.5 A, the frequency – f = 2.2 Hz, and the duty cycle – w = 50%. Furthermore, the formula describes the rectangular waveform of the current signal reads [5]

$$(f, w, t) = i_a \frac{1}{2}\left[1 - \tanh\left(200 \sin(\pi f t) \sin\left(\pi f t - \frac{\pi w}{100}\right)\right)\right], \qquad (6)$$

where f and w are frequency and duty cycle of the current signal, respectively, whereas t is time.

Fig. 7 shows time histories of angular position of the pendulum in different time intervals, obtained both experimentally and numerically.

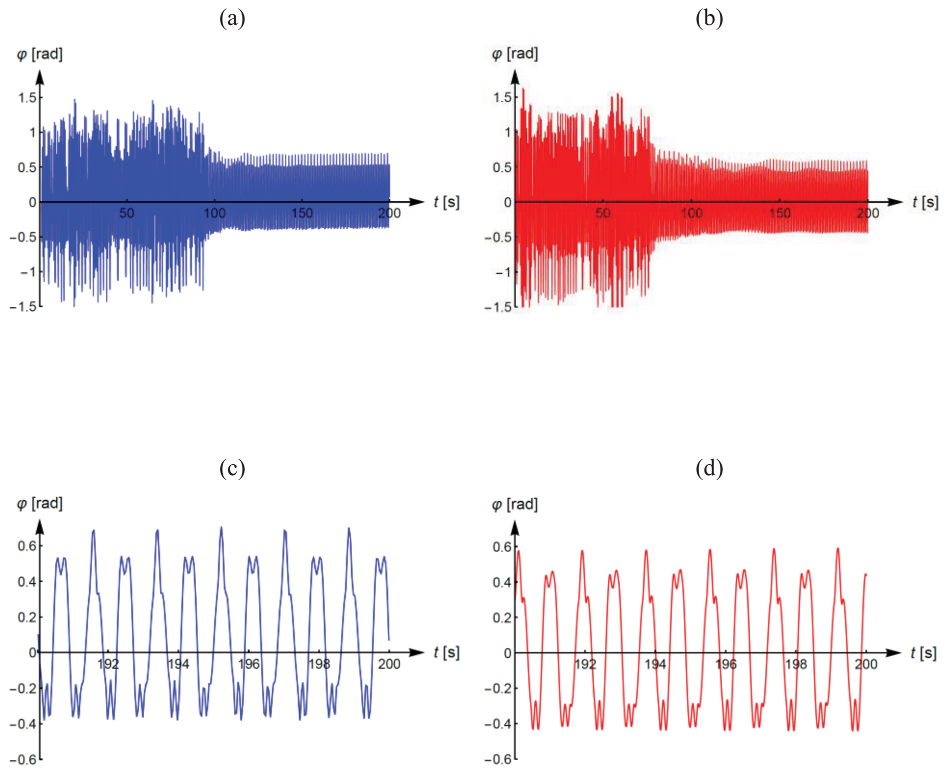


Figure 7. Comparison of experimental (a), (c) and numerical (b), (d) time histories of the angular positions of the pendulum for f = 2.2 Hz, w = 50% and i_a = 0.5 A.

As can be seen, the transient motion is clearly visible both in experimental and numerical investigations. Furthermore, the experimental transient behaviour is slightly longer than the simulation one. When the transient motion has vanished, the periodic oscillation has revealed for fixed parameters. The period and amplitude of the oscillation are in good agreement for both experimental and numerical analysis. In the considered case, the moment of impact of the pendulum on the magnetic barrier is clearly visible as a double amplitude peak, both in experimental and numerical results.

The bifurcation analysis has yielded a wider dynamical spectrum of the system motion. Figure 8 shows the numerical bifurcation diagrams with the frequency f as a control parameter, while the w = 50% and i_a = 0.5 A. Figure 8a displays the bifurcation for increasing value of frequency, and the windows of various multiperiodic motions as well as of chaotic motion can be recognized. In turn,

Fig. 8b displays the bifurcation diagram for decreasing frequency, and the coexisting attractors of periodic solution were exhibited.

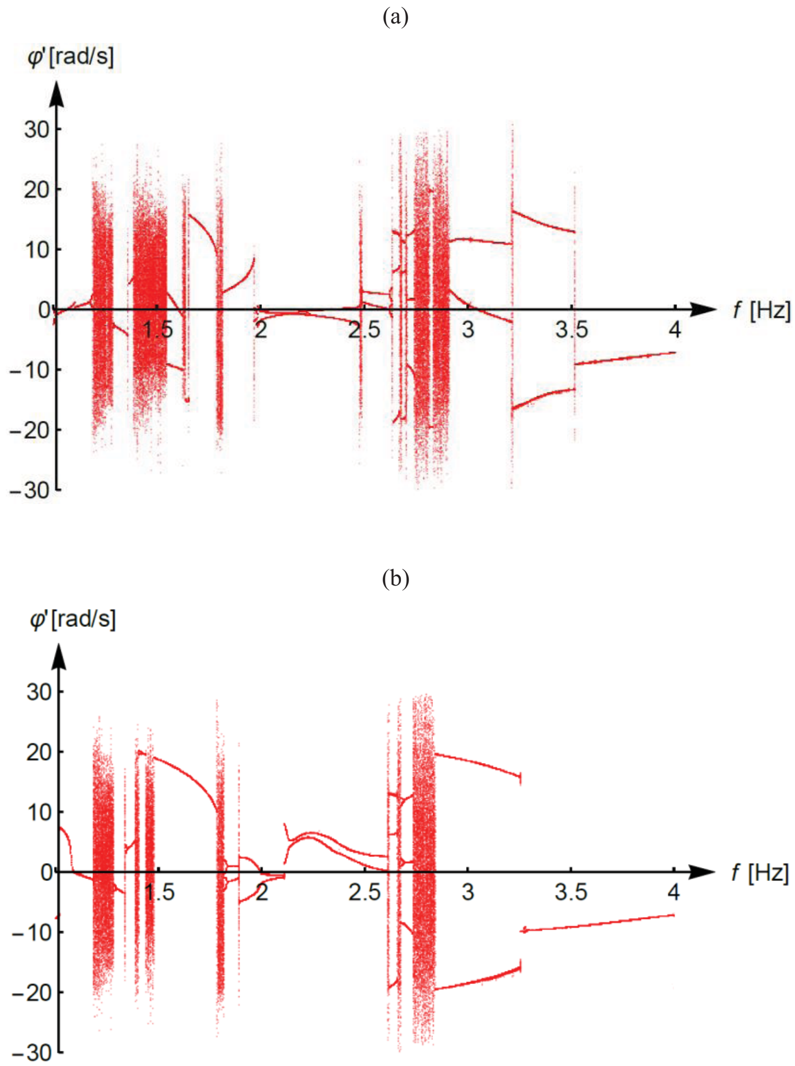
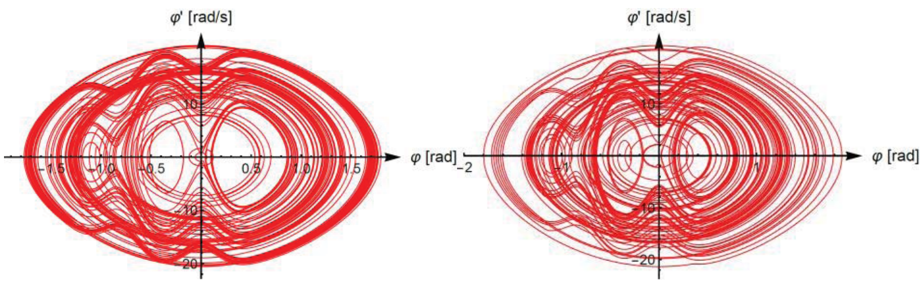


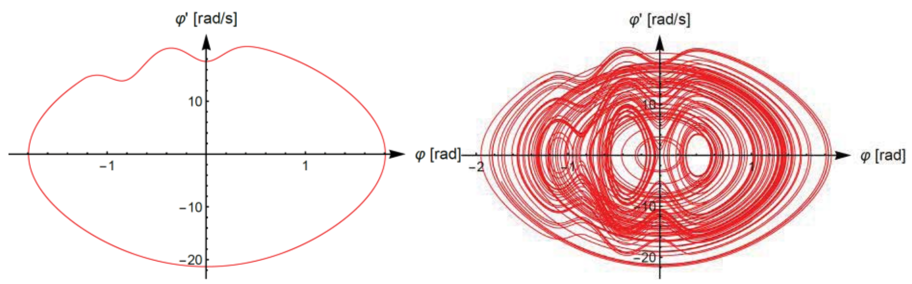
Figure 8. Comparison of bifurcation diagrams for increasing (a) and decreasing (b) value of frequency as a control parameter.

The phase plots for different regular and chaotic type of motion are presented in Fig. 9. The phase plots have been taken for a small increment of frequency to present the evolution of the trajectory. The rare trajectories of periodic motion are shown in Fig. 9c and 9i, the significant influence on the form of the trajectories has the asymmetric nonlinear character of the magnetic interaction. The double amplitude peaks in the form of loops of the trajectory can be seen in Fig. 9e, 9f and 9i. Above the frequency 3.5 Hz, the one well-oscillation are exhibited by the system (see Fig. 9j).

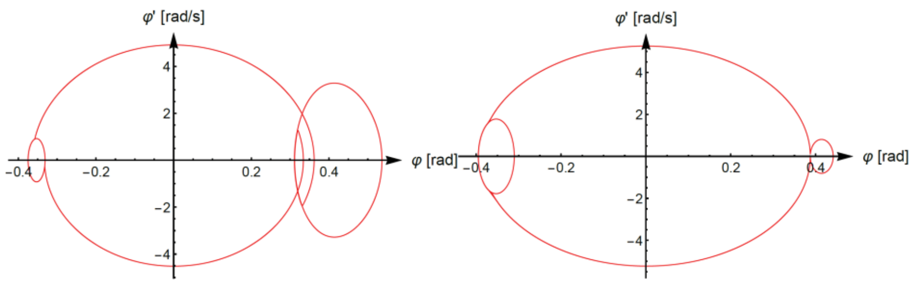
(a) $f = 1.2$ Hz (b) $f = 1.5$ Hz



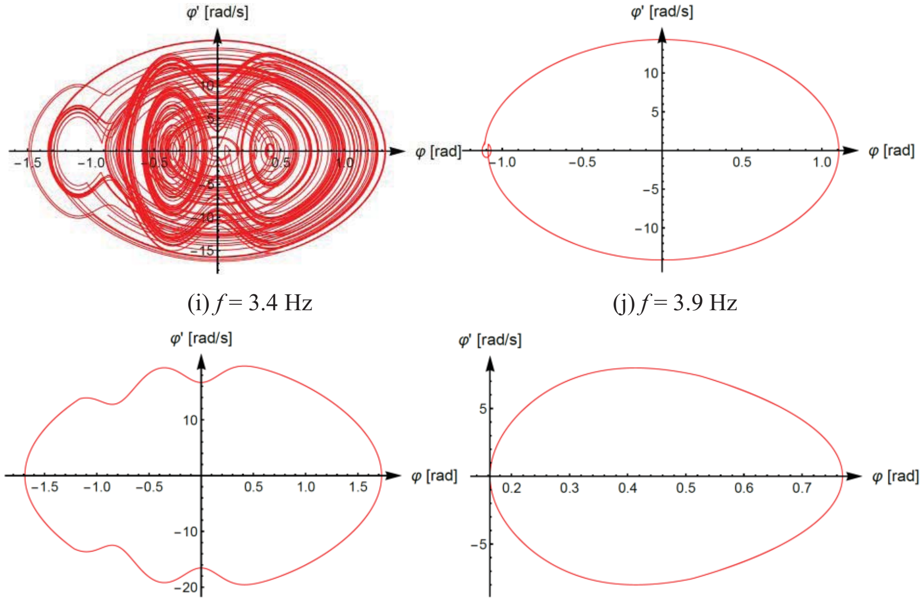
(c) $f = 1.7$ Hz (d) $f = 1.8$ Hz



(e) $f = 2.0$ Hz (f) $f = 2.4$ Hz



(g) $f = 2.9$ Hz (h) $f = 3.0$ Hz



(i) $f = 3.4$ Hz

(j) $f = 3.9$ Hz

Figure 9. Regular (c, e, f, h, i, j) and chaotic (a, b, d, g) dynamics detected by phase portraits plotted in the range $t = 960\text{-}1000$ s for different values of frequency f.

5. Conclusions

In this paper the system of a magnetic pendulum supported by aerostatic bearing and subjected to an asymmetric repulsive magnetic field has been studied both experimentally and numerically. The magnetic field was alternating and induced by electric coils powered by a rectangular current signal. The current signal has controlled values of frequency, duty cycle and amplitude. The physical and mathematical models of the considered system have been developed, where magnetic interaction has been applied as an approximation function of experimental data. The numerical time histories plots of periodic motion have been shown and their good agreement with the experimental data. The bifurcation analysis has been presented for increasing and decreasing paths of frequency as a control parameter. The multiperiodic and chaotic ranges of oscillation as well as the coexisting attractors have been reported and discussed. Especially, evolution of the chaotic motion has been shown in a set of phase plots. The simplicity of the mathematical model is important for developing analytical solutions which can be experimentally validated. Therefore, the developed mathematical model and the constructed experimental stand are a valuable source for further investigations.

Acknowledgements

This work has been supported by the National Science Centre of Poland under the rant OPUS 14 No. 2017/27/B/ST8/01330. Special thanks to Grzegorz Wasilewski, PhD, (Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology) for help with experimental investigations.

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Ewelina Ogińska, B.A. (M.Sc. student): Lodz University of Technology, Faculty of Mechanical Engineering, Department of Automation, Biomechanics and Mechatronics, Stefanowskiego 1/15, 90-924, Lodz, Poland (*e.j.oginska@gmail.com*). The author gave a presentation of this paper during one of the conference sessions.

Krystian Polczyński, M.Sc. (PhD student): Lodz University of Technology, Faculty of Mechanical Engineering, Department of Automation, Biomechanics and Mechatronics, Stefanowskiego 1/15, 90-924, Lodz, Poland (*krystian.polczynski@edu.p.lodz.pl*).

Dariusz Grzelczyk, PhD: Lodz University of Technology, Faculty of Mechanical Engineering, Department of Automation, Biomechanics and Mechatronics, Stefanowskiego 1/15, 90-924, Lodz, Poland (*dariusz.grzelczyk@p.lodz.pl*).

Jan Awrejcewicz, Professor: Lodz University of Technology, Faculty of Mechanical Engineering, Department of Automation, Biomechanics and Mechatronics, Stefanowskiego 1/15, 90-924, Lodz, Poland (*jan.awrejcewicz@p.lodz.pl*).