# Size-dependent nonlinear vibrations of micro-plates subjected to in-plane magnetic field

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*Abstract:* Nonlinear vibrations of the microplates subjected to the influence of a longitudinal magnetic field are considered. Size-depended model based on a modified couple stress theory is employed. The governing equations for geometrically nonlinear vibrations use the von Karman plate theory. Effect of the magnetic field is taken into account due to the Lorentz force deriving from the Maxwell's equations. Developed approach is based on applying of the Bubnov-Galerkin method and reducing partial differential equations to an ordinary differential equation. Some calculations are performed to validate the proposed algorithm in comparison with the known from literature results. Influence of the magnetic field, material length scale-parameter, plate aspect ratio on the system behavior is studied.

It is clear that problems of micro and nano sized elements have been increasingly studied because of the widespread use of microplates, microbeams, microshells in high-tech industries. It is often the microelements are subjected to various loads which can significantly effects on its behavior. Investigation of plate under magnetic field in-plane influence is of great importance due to using as elements of NEMS, MEMS, resonators, sensors etc. The experimental and theoretical investigations allow to conclude that a size-dependent effect appears when thickness is in a micro or nano scale [1] and for accurate analysis classical elasticity theory can be not enough. Various theories have been applied to study of micro and nano structures, theory of micropolar elasticity by Cosserat and Cosserat [2], couple stress theory by Mindlin and Tiersten [3], Toupin [4] , Koiter [5], the nonlocal elasticity theory by Eringen [6], strain gradient theory by Lam et al. [1] . In this paper we use modified couple stress theory (MCST) proposed by Yang et al [7], which contains only one additional material length scale parameter and a symmetric couple stress tensor.

Recently, MCST was used in linear vibrations, buckling and bending plate analysis [8-13], nonlinear vibrations of micro-plates [14,15], FG Mindlin microplates [16], viscoelastic plates [17], chaotic vibrations of nano-shells [18]. Influence of magnetic field on micro and nano plates is studied in [19-22] using nonlocal elasticity theory. Analysis of published results has shown that geometrically nonlinear vibrations of small-sized plates subjected to magnetic influence in framework MCST has not been investigated yet.

In the paper we present an analytical method for small-sized geometrically nonlinear vibrations of plates. The investigation is based on the modified couple stress theory, the von Karman plate theory, Kirchhoff-Love hypotheses and Maxwell's relations. The governing PDEs is reduced to ODE by applying of the Bubnov-Galerkin method. The present results contain study of the magnetic field effect and material length scale parameter influence on the frequencies and backbone curves.

### 1.1. Formulation

Geometrically nonlinear vibrations of isotropic plate (see Fig.1) in magnetic field are considered. According to the modified couple strain theory [7] the strain energy unlike the classical elasticity theory depends on stress tensor and curvature tensor and it is presented as

$$U = \frac{1}{2}\int_V (\sigma_{ij}\,\varepsilon_{ij} + m_{ij}\chi_{ij})dV \tag{1}$$

where $\sigma_{ij}, \varepsilon_{ij}, m_{ij}\chi_{ij}$ are components of stress tensor, strain tensor, diviatory part of the couple stress tensor, symmetric curvature tensor, that are defined as

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij},\;\; \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{m,i}u_{m,j}), \tag{2}$$

$$m_{ij} = 2l^2\mu\chi_{ij},\;\; \chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}), \tag{3}$$

where $\lambda, \mu$ are Lame constants

$$\lambda = \frac{Ev}{(1+v)(1-2v)}, \mu = \frac{E}{2(1+v)}.$$

$\delta_{ij}$ is Kronecker delta, $l$ is a material length scale parameter, $v$ is Poisson's ratio, $E$ is Young's modulus, $u_i$ are displacements, $\theta_i$ are components of rotation vector, which have form

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j}, \tag{4}$$

here $e_{ijk}$ is permutation symbol.

The nonlinear dynamics of the plate is derived by the equations based on the von Karman theory. Mixed form of the governing equations in propagation of elastic waves in longitudinal equations are neglected is presented [18]

$$(D + D_L)\Delta^2 w = L(w, F) - \rho h\frac{\partial^2 w}{\partial t^2} + q_l, \tag{5}$$

$$\frac{1}{2}L(w, w) = -\frac{1}{Eh}\Delta^2 F, \tag{6}$$

$$L(w, w) = 2\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right),\;\; L(w, F) = \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2} - 2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}.$$

In (5), (6) $F$ is the stress function [18,23], $h$ is thickness of the plate, $D = \frac{Eh^3}{12(1-v^2)}$, $D_L = \frac{Elh^2}{2(1+v)}$, $q$ is load.

System of equations (see Eq. 5, 6) is supplemented with the boundary conditions:

simply supported movable edges:

$$w = 0, \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0, \frac{\partial^2 F}{\partial x \partial y} = 0, \int_0^b \frac{\partial^2 F}{\partial y^2} = 0, x = 0, a, \tag{7}$$

$$w = 0, \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0, \frac{\partial^2 F}{\partial x \partial y} = 0, \int_0^a \frac{\partial^2 F}{\partial x^2} = 0, y = 0, b. \tag{8}$$

simply supported immovable edges:

$$w = 0, \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0, \frac{\partial^2 F}{\partial x \partial y} = 0, u = 0, x = 0, a, \tag{9}$$

$$w = 0, \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0, \frac{\partial^2 F}{\partial x \partial y} = 0, v = 0, y = 0, b. \tag{10}$$

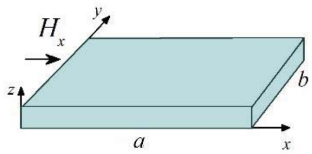



**Figure 1**. Microplate subjected to in-plane magnetic field

### 1.2. Influence of magnetic field

Changing the vibrational characteristics of the small-sized plates by an appropriate external influence can be effectively used in the structure design, vibration control etc. One of the significant effects is the use of a magnetic field. We consider the plate exposed to the uniaxial magnetic field [19,21,22], defined by the vector of magnetic field strength

$$\vec{H} = (H_x, 0,0). \tag{11}$$

From Maxwell's relations distributing vector of the magnetic field $\vec{h}$ has the form

$$\vec{h} = [\nabla, [\vec{U}, \vec{H}]], \tag{12}$$

where vector $\vec{U} = (u_x, u_y, u_z)$ is vector of displacements. After substitution (see Eq. 11) into (see Eq. 12) it can be obtained

$$\vec{h} = (-H_x \frac{\partial u_y}{\partial y} - H_x \frac{\partial u_z}{\partial z}, H_x \frac{\partial u_y}{\partial x}, H_x \frac{\partial u_z}{\partial z}). \tag{13}$$

Thus, current density $\vec{J}$ is written as

$$\vec{J} = [\nabla, \vec{h}] = ( H_x \frac{\partial^2 u_z}{\partial x \partial y} - H_x \frac{\partial^2 u_y}{\partial x \partial z}, -H_x \frac{\partial^2 u_z}{\partial x^2} - H_x \frac{\partial^2 u_y}{\partial y \partial z} - H_x \frac{\partial^2 u_z}{\partial z^2}, H_x \frac{\partial^2 u_y}{\partial x^2} + H_x \frac{\partial^2 u_y}{\partial y^2} + H_x \frac{\partial^2 u_z}{\partial y \partial z}). \tag{14}$$

The Lorentz force is defined as

$$f = (f_x, f_y, f_z) = \eta[\vec{J}, \vec{H}]. \tag{15}$$

In (see Eq. 15) $\eta$ is the magnetic permeability.

It should be noted that the transverse vibrations are considered and only $f_z$ is taken into account [22]. Formula (see Eq. 15) gives

$$f_z = \eta H_x^2 (\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_y}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial z^2}). \tag{16}$$

For Kirchhoff-Love plate transverse component $f_z$ takes form

$$f_z = \eta H_x^2 (\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}). \tag{17}$$

where $w$ is mid-plane displacements of the plate along $z$ directions. As a result, force produced by magnetic field can be presented as

$$q_l = \int_{-h/2}^{h/2} f_z dz = \eta h H_x^2 (\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}). \tag{18}$$

### 1.3. Linear vibrations of microplate in magnetic field

In the case of linear vibrations, system (see Eq. 5,6) is reduced and we have following equation

$$(D + D_L)\Delta^2 w = -\rho h \frac{\partial^2 w}{\partial t^2} + q_l. \tag{19}$$

Solution of such equation is taken as $w(x, y, t) = X(x, y) cos\omega_{mn} t$, where $X = sin\frac{m\pi}{a} sin\frac{n\pi}{b}$ is shape function, that allows to obtain linear frequency of plate vibrations under in-plane magnetic field

$$\omega_{mn}^2 = \frac{(D+D_L)\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 + \eta h H_x^2 \left(\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)}{\rho h}. \tag{20}$$

For first mode (1,1) this formula is reduced to

$$\omega_{11}^2 = \frac{(D+D_L)\pi^4 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 + \eta h H_x^2 \pi^2 \left(\frac{1}{a^2} - \frac{1}{b^2}\right)}{\rho h}. \tag{21}$$

### 1.4. Nonlinear vibrations of microplate in magnetic field

Now let us consider system of equations (see Eq. 5,6). The deflection $w(x, y, t)$ is presented as

$$w(x, y, t) = w_0(t) sin\frac{\pi x}{a}\ sin\frac{\pi y}{b}. \tag{22}$$

Substitution (see Eq. 22) into the equation in (see Eq. 6) leads to

$$\frac{1}{Eh}\Delta^2 F = \frac{1}{2} {w_0}^2 \left(\frac{\pi}{a}\right)^2 \left(\frac{\pi}{b}\right)^2 (cos\frac{2\pi x}{a} + cos\frac{2\pi y}{b}). \tag{23}$$

The solution of the last equation (see Eq. 23) [23] is

$$F = \frac{Eh{w_0}^2}{32}\left(\frac{a^2}{b^2} cos\frac{2\pi x}{a} + \frac{b^2}{a^2} cos\frac{2\pi y}{b}\right) + p_1 x^2 + p_2 y^2. \tag{24}$$

Coefficients $p_1$ and $p_2$ can be found from boundary conditions, for case (see Eq. 9, 10) $p_1 = 0, p_2 = 0$, for case (see Eq. 7, 8)

$$p_1 = \frac{\pi^2 Eh(a^2 + \mu b^2){w_0}^2}{16(1-\mu^2)a^2 b^2}, p_2 = \frac{\pi^2 Eh(b^2 + \mu a^2){w_0}^2}{16(1-\mu^2)a^2 b^2}. \tag{25}$$

Next step is substitution expressions (see Eq. 22,24) into the first equation (see Eq. 5) of governing system and applying the Bubnov-Galerkin approach that leads to the following Duffing type equation

$$y'' + \omega_L^2 y + \alpha y^3 = 0, \tag{26}$$

where

$$\alpha = \frac{\pi^4 E h^2}{16\rho}\left(\frac{1}{a^4} + \frac{1}{b^4}\right) + \frac{\pi^4 E h^2}{8(1-\nu^2)a^2 b^2}\left(\frac{b^2}{a^2} + \frac{a^2}{b^2} + 2\nu\right), \omega_L = \omega_{11}, y = \frac{w_0}{h}. \tag{27}$$

Equation (see Eq. 26) can be solved by the Bubnov-Galerkin approach, presenting the solution as $y(t) = A\cos\omega_N t$, where $A$, $\omega_N$ are amplitude and frequency of nonlinear vibrations. Thus, it can be obtained frequency ratio

$$\left(\frac{\omega_N}{\omega_L}\right)^2 = 1 + \frac{3}{4}\beta A^2, \beta = \frac{\alpha}{\omega_L^2}. \tag{28}$$

## 2. Validation

To verify presented method the results are compared with available ones, we considered size-dependent vibrations of rectangular simply supported plate without magnetic action. Dimensionless natural frequencies $\bar{\omega} = \omega_L \frac{a^2}{h}\sqrt{\frac{\rho}{E}}$ are presented in Table 1,2 for various values of $l/h$. The material properties for considered nanoplate are taken as $\rho = 1220 Kg/m^3, E = 1.44 GPa, \nu = 0.38$.

**Table 1**
Dimensionless natural frequencies $\bar{\omega}$ for isotropic simply supported square plate (*b*/*a*=1) depending on thickness ratio $l/h$

| $l/h$ | 1/6 | 1/5 | 1/4 | 1/3 | 1/2 | 1 |
|---|---|---|---|---|---|---|
| | | | b/a = 1 | | | |
| [17] | 6.471 | 6.602 | 6.839 | 7.323 | 8.558 | 13.383 |
| Present | 6.471 | 6.603 | 6.839 | 7.323 | 8.558 | 13.383 |

**Table 2**
Dimensionless natural frequencies $\bar{\omega}$ for isotropic simply supported rectangular plate (*b*/*a*=0.5) depending on thickness ratio $l/h$

| $l/h$ | 0.1 | 0.5 | 1 |
|---|---|---|---|
| [17] | 15.68 | 21.39 | 33.45 |
| Present | 15.685 | 21.396 | 33.459 |

In Table 3 nonlinear frequency ratios $\omega_N/\omega_L$ for plate found by proposed method with $l = 0$ (classical theory) are presented. Material parameters are $\rho = 1220 Kg/m^3, E = 1.44 GPa, \nu = 0.3$. Boundary conditions are supposed of type (see Eq. 9, 10). The results compared with ones obtained by another approaches [24-27].

**Table 3**

Nonlinear frequency ratio $\omega_N/\omega_L$ for isotropic simply supported plate ($b/a = 1$)

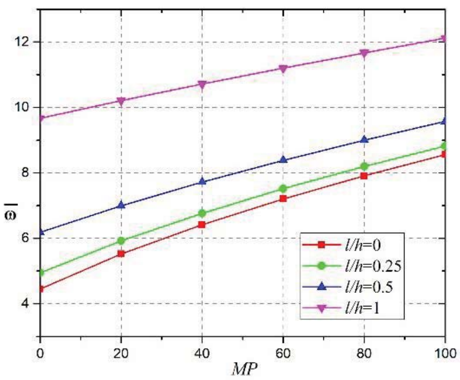
| $\frac{w}{h}$ | [25] | [26] | [27] | [24] | Present |
|---|---|---|---|---|---|
| 0.2 | 1.0195 | 1.0197 | 1.0195 | 1.0197 | 1.0195 |
| 0.4 | 1.0757 | 1.0768 | 1.0765 | 1.0767 | 1.0760 |
| 0.6 | 1.1625 | 1.1662 | 1.1658 | 1.1659 | 1.1641 |
| 0.8 | 1.2734 | 1.2813 | 1.2796 | 1.2813 | 1.2773 |
| 1 | 1.4024 | 1.4173 | 1.4163 | 1.4168 | 1.4095 |

Comparison of results allows to conclude about good agreement with the known ones in the literature.

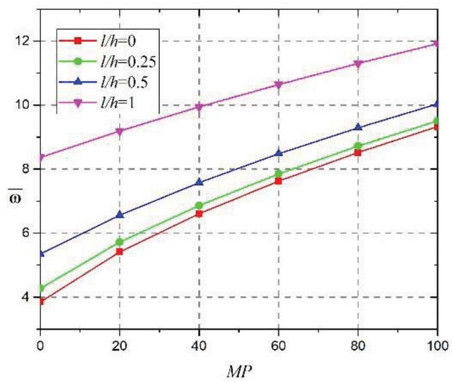
## 3. Numerical results

To investigate the influence of magnetic field on vibration process dimensionless linear frequencies $\bar{\boldsymbol{\omega}}$ for various values of magnetic parameter *MP* (here dimensionless magnetic parameter is introduced as $MP = \frac{\eta h H_x^2 a^2}{D}$) and thickness ratio *l/h* are calculated. It is assumed that plate has the following characteristics

$$\rho = 1220 \mathrm{Kg/m^3}, \mathrm{E} = 1.44 \mathrm{GPa}, \nu = 0.38, \mathrm{a} = 10 \mathrm{mm}, b/a = 1.5(a), b/a = 2(b), h/a = 0.01.$$




a)




b)

**Figure 2.** Dimensionless frequency for various values of material scale parameter, magnetic parameter, a) *b/a*=1.5, b) *b/a*=2.

Analyzing it can be seen that increasing of magnetic parameter leads to increasing of dimensionless frequency as well as it is observed the similar influence of length scale parameter on frequency in both cases of the plate aspect ratio. Also we can conclude that the aspect ratio has a small effect (especially when *l/h* is close to 1) on the frequency parameter at large values of the magnetic parameter.

Dimensionless frequency parameters $\bar{\omega}$ in terms of magnetic parameter and thickness ratio are calculated and presented on Figure 3. It can be found that frequency parameter generally increasing with increasing of magnetic parameter and material scale length parameter. Changing of magnetic parameter has smaller effect on vibration frequency when material length scale parameter is close to thickness of the plate. The minimum of dimensionless frequency achieves when *MP* and *l* vanish.

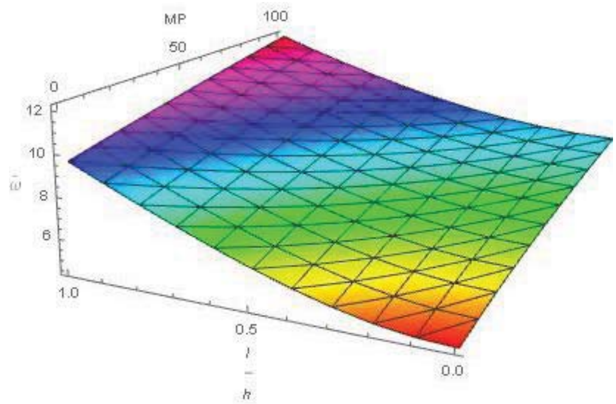
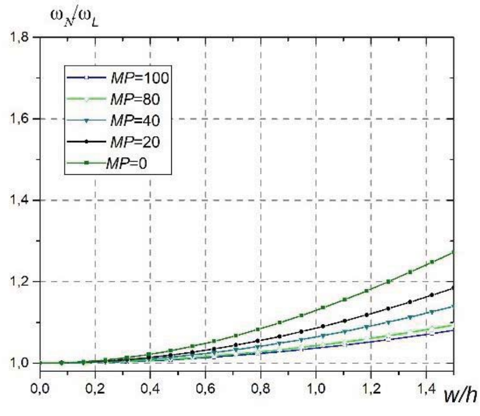
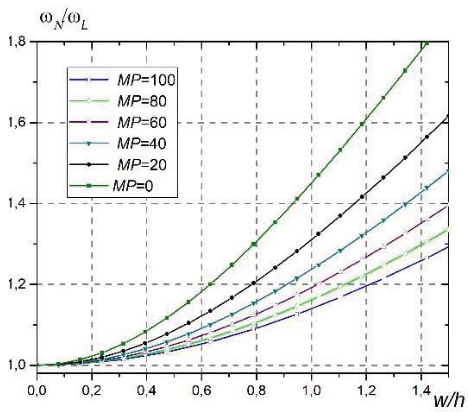



**Figure 3.** Dimensionless frequency parameter $\bar{\omega}$ in terms of MP and *l/h*

The effect of geometric nonlinearity is demonstrated on Figure 4, 5. The backbone curves (see Eq. 28) for rectangular plate specified by the aspect ratio *b/a*=1.5, thickness ratio *l/h*=0 and various *MP* are provided on Figure 4. According to obtained results frequency ratio decreases with increasing of magnetic parametric value *MP*. Further we fixed *MP*=0 to investigate the influence of material length scale parameter *l* on backbone curves, these results are presented on Figure 5. The size effect is more meaningful when *l/h*>2.5 and the difference in results obtained by classical theory and modified couple stress theory is insignificant when the thickness ratio *l/h* is small. Action of magnetic field as well as scale parameter is more significant in the case of immovable edges (see Eq. 9,10), in case of movable edges (see Eq.7,8) the backbone curves are closer each other (see Fig. 4, 5).

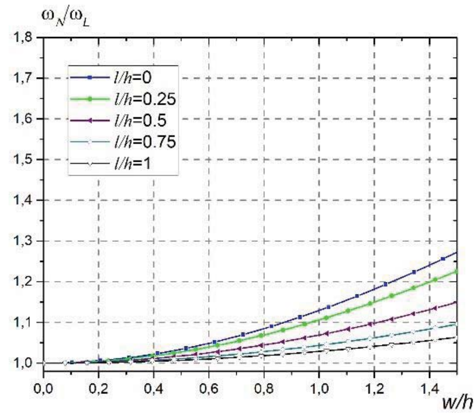



a)

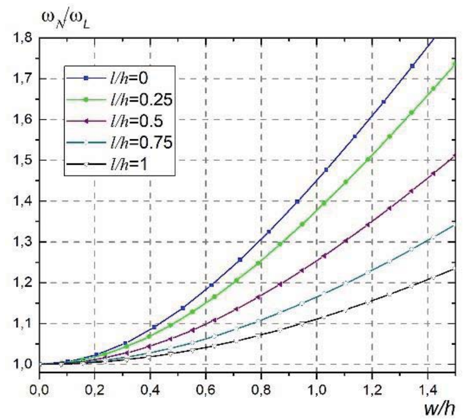



b)

**Figure 4.** Frequency ratio (see Eq. 28) for two types of boundary conditions: a)-conditions (see Eq. 7,8), b) – conditions (see Eq. 9,10), *l/h*=0




a)




b)

**Figure 5.** Frequency ratio (see Eq. 28) for two types of boundary conditions: a)-conditions (see Eq. 7,8), b) – conditions (see Eq. 9,10), *MP*=0

## 4. Conclusions.

The size-dependent nonlinear vibrations of microplates in magnetic field are studied. Governing PD equations are based on the modified couple stress theory, the Kirchhoff hypothesis, the von Karman theory. The influence of the material length scale parameter, the magnetic parameter, boundary conditions, aspect ratio on the linear frequency, nonlinear ratio is investigated. It has been shown that the linear frequency increases with increasing of length scale parameter and magnetic parameter unlike the ratio of nonlinear frequency to linear frequency, which decreases. Also, the small-size effect and magnetic action are more significant for immovable simply supported plates.

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