Double Physical Pendulum with Magnetic Interaction

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Abstract. The paper is devoted to experimental investigation of a system consisting of a double physical pendulum with magnetic interaction caused by a pair of permanent magnets repelling each other – one mounted on the end of a second link of a pendulum and one mounted in the body of the setup. To the experimental rig of double physical pendulum system with the first body periodically forced a constructed magnetic interaction forces measurement system is added. The work consists of investigation on how magnetic interaction forces depend on pendulum's links relative angle. Constructed measurement system consists of two links shafts encoders and one force sensor – extensometer beam with an output signal amplifier. Three analog signals are read by acquisition device, sent to connected PC and processed in LabVIEW measurement software programs. The dynamics of the system is also observed. Regular and chaotic behaviors of the system are recorded depending on the frequency of the motor. The work is the first approach to electromechanical system of the double physical pendulum with magnets. Such system is a unique construction. The magnetic forces measurement system is constructed and tested. The regular and chaotic behaviors of the system are shown and discussed.

Keywords: Magnetic and mechanical system · Double pendulum · Magnets · Experiment and construction

1 Introduction

1.1 Object of Investigations

This paper is a first approach to dynamics of a unique electromechanical system. An object of presented investigations is a double physical pendulum with magnetic interaction caused by a pair of permanent magnets: one mounted at the end of second link of the pendulum and one mounted in the body of an experimental rig. The magnets are oriented so that repelling force exists between them. The considered system is unique for a few reasons. Firstly, there is no paper which describes a double physical pendulum with magnets or electromagnets. Additionally pendulum is constructed so that each link is able to do multiple revolutions around its axis of revolution. Furthermore, a motion of the first link of pendulum is forced by a specially constructed motor.

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As mentioned above the considered system has unique features. In the literature may be found dynamic analysis of simpler systems which consist of single pendulum with magnetic interaction. Simple example of EMS is described in [1]. In this paper a system consisting of a pendulum, a permanent magnet and an AC electromagnet is considered. A magnet is mounted at the end of pendulum and additionally on its body. There is presence and relative relations of chaos and parametric resonance investigated. Analytical results are confronted with numerical simulations and experimental studies.

In [2] a system with a single physical pendulum placed in the magnetostatic field (two repelling magnets) and forced sinusoidally is considered. A system dynamics is experimentally studied. Computer simulations results are shown. A measurement system consists of two encoders. Both the experimental studies and the numerical simulations indicate the presence of chaotic and regular behavior of the system (depending on parameters of the system). The following parameters are defined: forcing frequency, relative distance of magnets and relative magnets orientation in horizontal plane. Additionally there peaks of magnitude, hysteresis and bistable states are observed.

Paper [3] considers a system with physical pendulum. An the end of the pendulum a permanent magnet is mounted. A magnet interacts with another one placed in the body. The magnets are close when a pendulum is placed downwards. In this work physical and mathematical models are presented. The correctness of models has been proved by simulations of bifurcations generated by the estimated coefficients for several frequencies. Simulations show also that estimated coefficients of one frequency can be used to predict system behavior in case of other forcing frequencies.

The authors of [4–6] describe experimental and numerical analysis of a system consisting of triple physical pendulum. Such construction with a possibility of recording multiple revolutions of every link of triple pendulum is a unique construction. The system is strongly nonlinear and there can be observed phenomena among others: regular and irregular behaviors, bifurcations, coexisting periodic, quasiperiodic, chaotic and hiperchaotic solutions. In considered system the motion of first link is forced by a specially constructed motor. The motor consists of two immovable stators and two rotors. Both parts of motor are symmetric and electrically coupled, but optical commutator is placed only on the one stator's plate. Such drive eliminates possible misalignments of construction. More detailed description of forcing motor and an entire measuring system can be found in [4]. Parameters of the system have been identified by the sum of squares of deviations minimalization of signals from experimental and numerical data. A high compatibility of experimental and simulation results has been obtained.

2 Experimental Rig

Existing experimental rig of double physical pendulum with magnets is augmented with moment of force of magnetic interaction measurement system. The system enables to measure how the force depends on links relative angle. Entire setup is shown in Fig. 1.

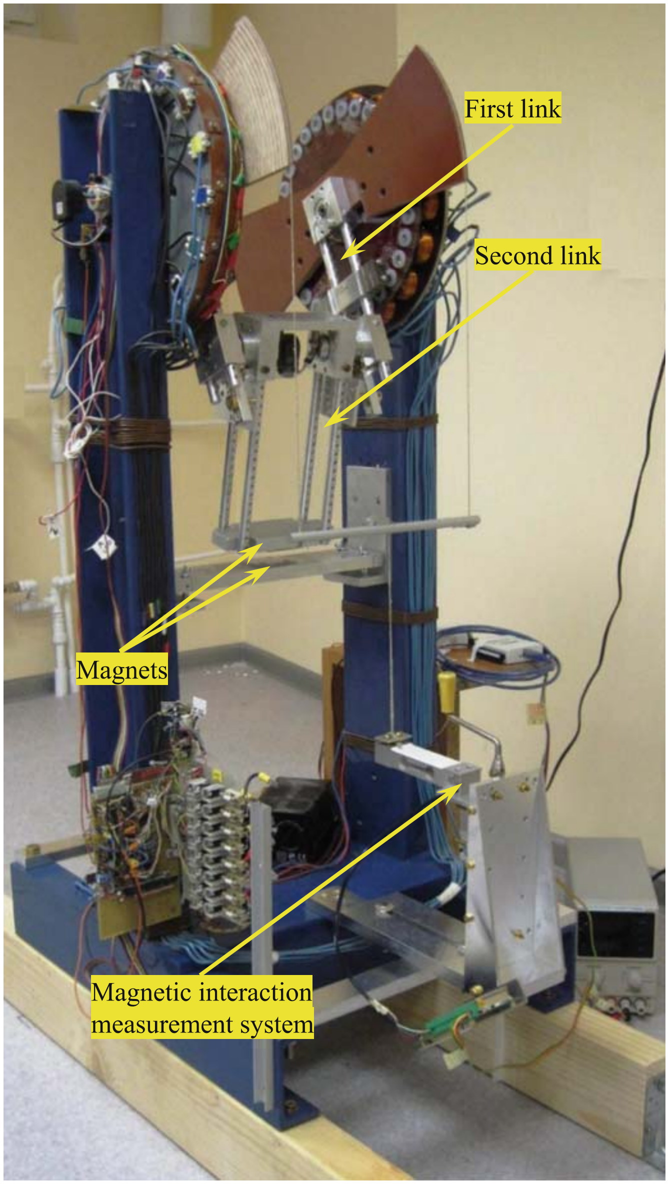

Fig. 1. Experimental setup with magnetic interaction measurement system.

A physical model and a scheme of the setup is presented in Fig. 2. The first link of pendulum is mechanically coupled with an extensometer beam by a ties system. To measure a force acting on the extensometer beam and calculate the moment of force the second link has been stiffly coupled relatively to the first one in known angular position. Constructed measurement system enables us to couple the pendulum's links together, which is shown in Fig. 3. Thanks to such construction there are 19 possibilities for links angular relative coupling in the range of ±40°. During the measurement the extensometer beam has been moved down pulling the tie at the same time. In this way a moment of force acts on the first link of pendulum trying to overcome the repelling force of approaching magnets. Such measurements have been carried out by approaching the magnets from the right and the left side of the experimental setup, and including every possible relative angular coupling of links.

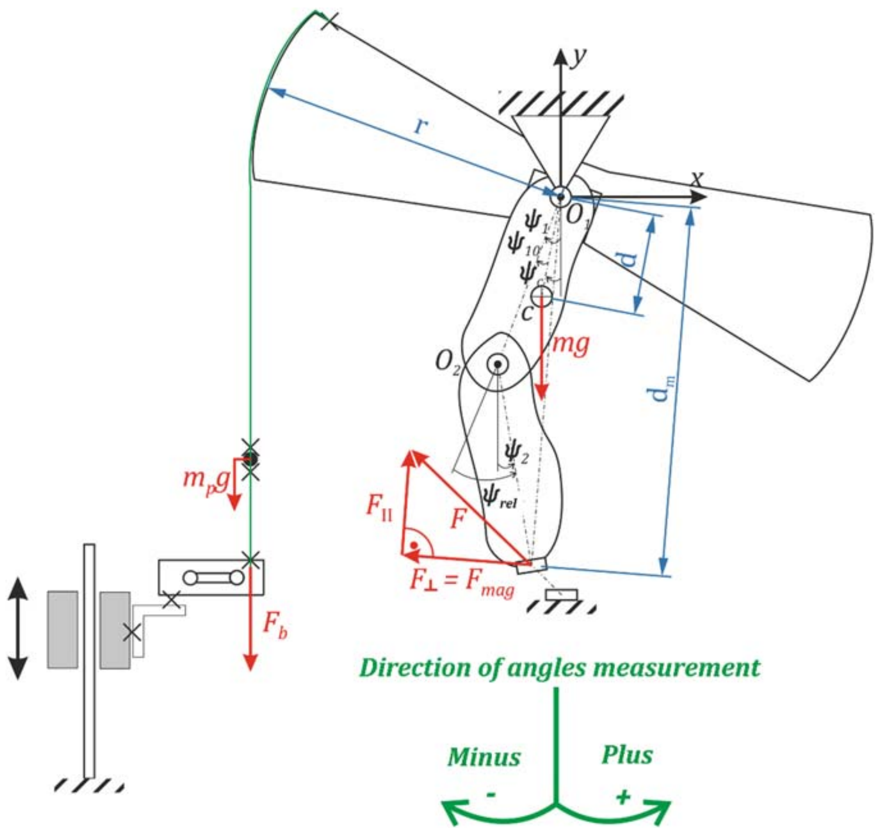

Fig. 2. Physical model of the setup.

The force acting on the extensometer beam has a constant arm r. Thanks to it the moment of force acting on the first link of pendulum due to magnets interaction has been easily obtained. Read values of moment of force have been modified and supplemented by known values of weight of the tie system and the gravity force of pendulum. Thus, only moment of force due to magnets interaction has been considered.

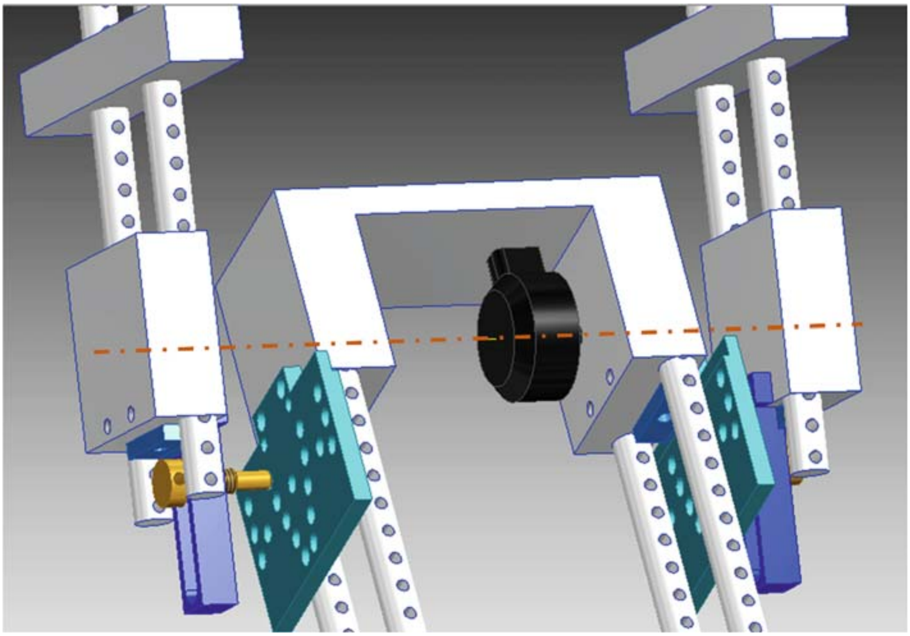

Fig. 3. Relative angular pendulum's links coupling system.

2.1 Sensory System

During the studies three signals from three sensors have been used and processed. Following sensors have been implemented: an extensometer beam (a force measurement) and two rotary-pulsed transducers (an angular positions measurement). The sensors have been connected to NI data acquisition device and read by PC computer. Recording, readout and data processing have been led in LabVIEW environment.

3 Experiment

Conducted experiments include two main parts. Firstly, approximations of moment of force caused by magnets as a function of first link absolute angle for possible relative coupling have been obtained. Furthermore, time series of pendulum's links have been recorded for a few different frequency of forcing.

3.1 Moment of Force of Magnets Characteristics

Figure 4 illustrates polynomial approximations curves of moment of force caused by magnets obtained during the experimental study. To have a full view of moment's course for one specific relative coupling the characteristics of measurements form right and left setup side should be combined. It may be noticed that the highest values of moment exist for the smallest relative coupling angles absolute values, i.e. for

$\psi_{rel} = 0°, \pm 3°$. The highest values are about 13.5 Nm. For comparison purposes it can be noticed that for $\psi_{rel} = \pm 35°$ moment M_{mag} reaches the value about 5 times lower.

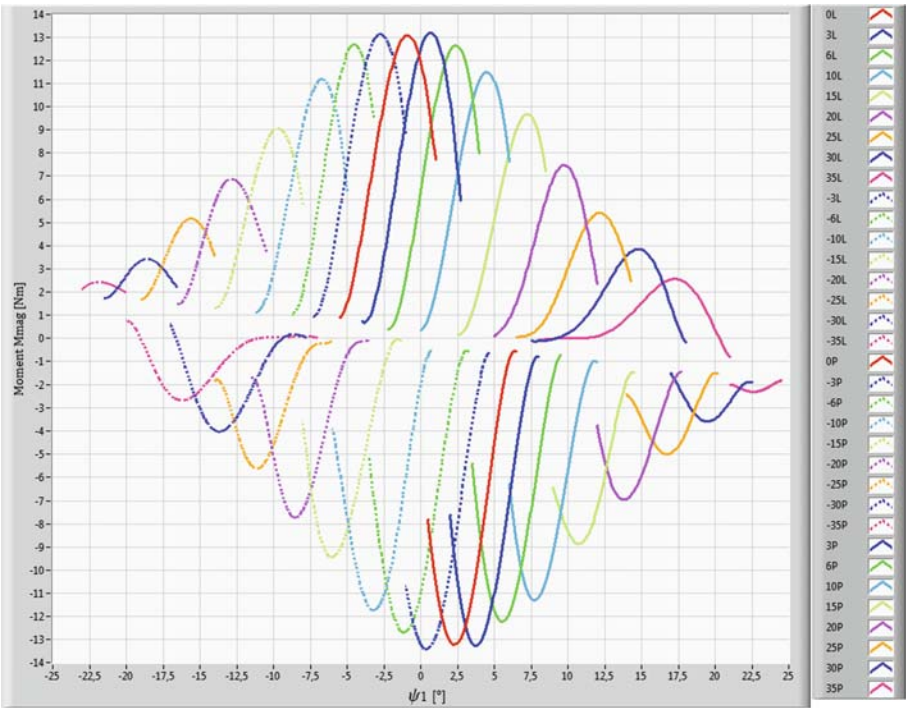

Fig. 4. Moment of force due to magnets characteristics in function of an angle of the first pendulum's link (e.g. 15*L* label in legend indicates the measurement conducted from Left side of setup with $\psi_{rel} = +15°$).

Not approximated values measured form right and left side of the setup with $\psi_{rel} = 0°$ are presented in Fig. 5. It can be easily noticed that the moment values are symmetric and change from positive to negative values nearby $\psi_1 = 0°$.

The obtained characteristics will be necessary in the further stage of studies. In dynamic analysis a mathematical model of the system will be presented. The crucial component of the equations of motion is reserved for the moment of magnetic interaction. The considered characteristics will enable us to obtain the coefficients of function describing magnetic interaction.

3.2 Pendulum's Time Series Recording

In this chapter exemplary time series of double pendulum motion with magnets are presented. External pendulum forcing is realized by rectangular function of torque (see Fig. 6) set by generator connected to the pendulum motor control circuit.

Below there are time series of regular and irregular behavior provided. The preset parameter was frequency f.

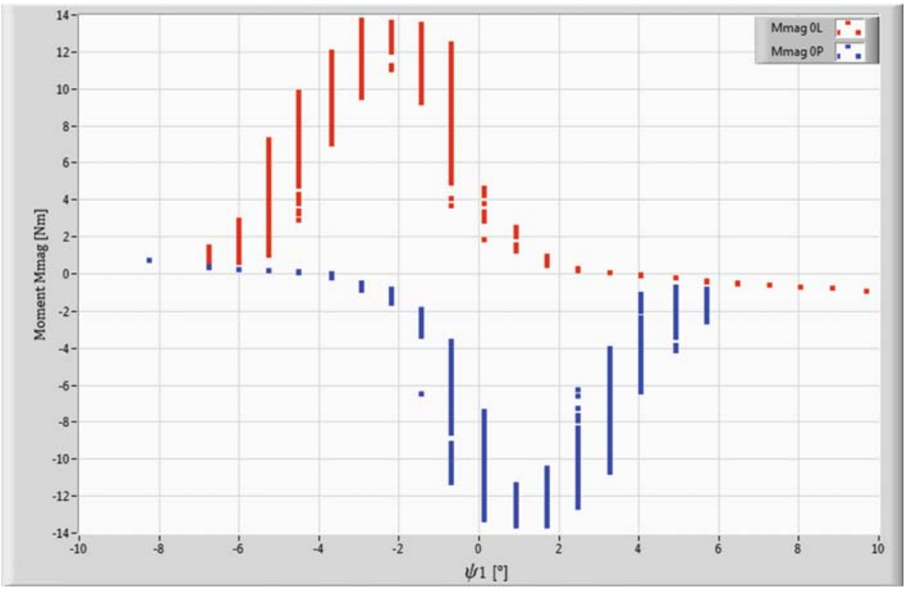

Fig. 5. Not approximated moment of force due to magnets in function of an angle of the first pendulum's link for $\psi_{rel} = 0°$

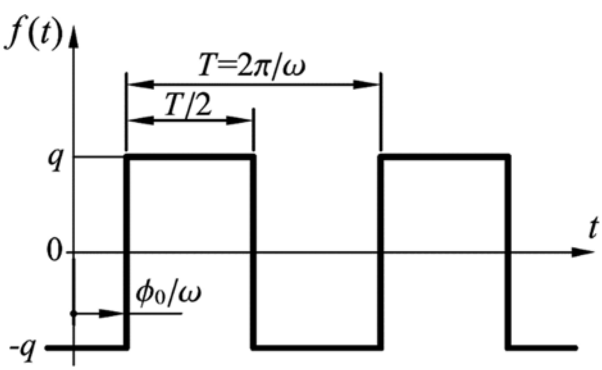

Fig. 6. External forcing motor torque function

Figure 7a presents regular motion of both links with the frequency $f = 0.1$ Hz. Enlarged part of these time series is shown in Fig. 7b. There a kind of regularity may be easily noticed. With the value of $f = 0.5$ Hz chaotic motion can be observed (see Fig. 7c). For such frequency multiple revolutions do not occur. The behavior, where multiple revolutions of each link can be observed is presented in Fig. 7d, e with the forcing frequency $f = 1$ Hz. The second link of pendulum does about 20 revolutions while the first one does only one revolution. The motion is chaotic. With frequency $f = 1.8$ Hz the motion with quite long temporary irregular stage is detected (Fig. 7f and g). After firstly 90 s of temporary stage, a regular behavior of a system exists. The amplitudes are small and links are in the opposite phases.

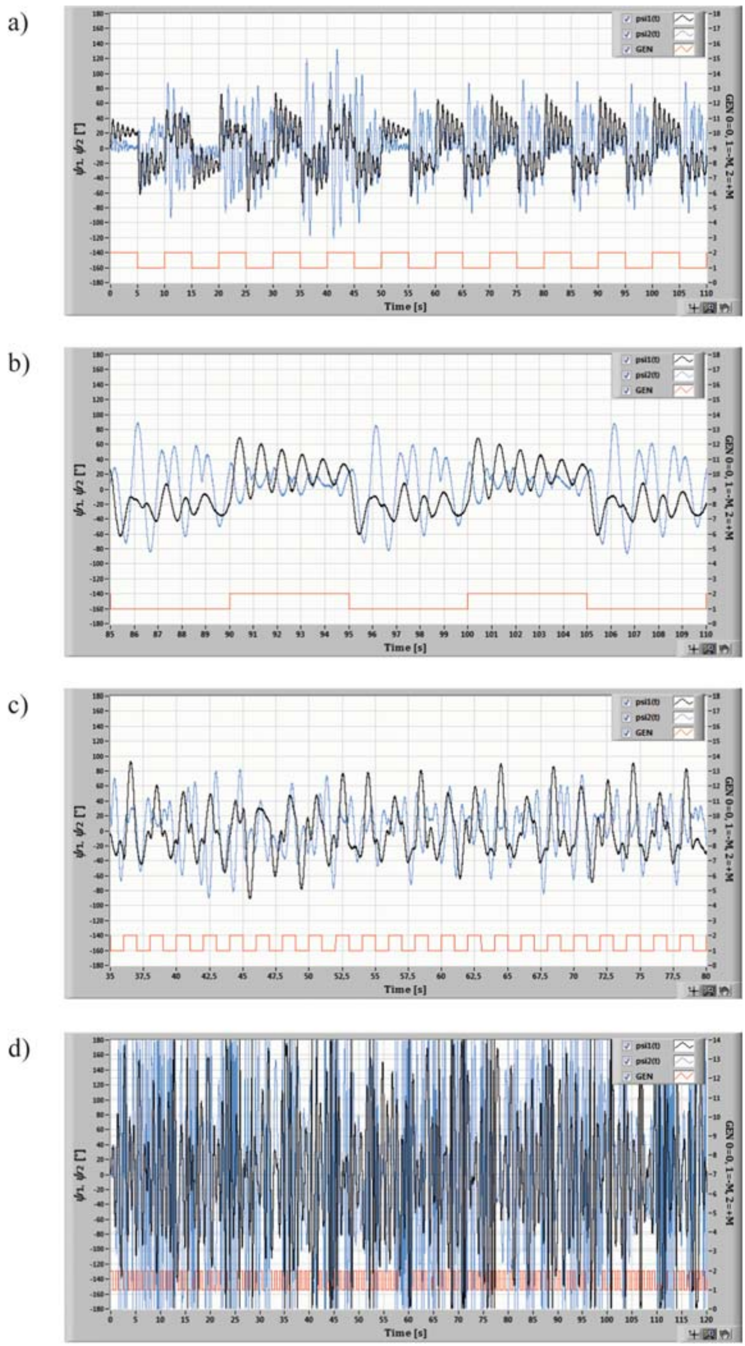

Fig. 7. Exemplary time series of motion of pendulum, (a) and (b) forcing frequency $f = 0.1$ Hz; (c) $f = 0.5$ Hz; (d) and (e) $f = 1$ Hz; (f) and (g) $f = 1.8$ Hz.

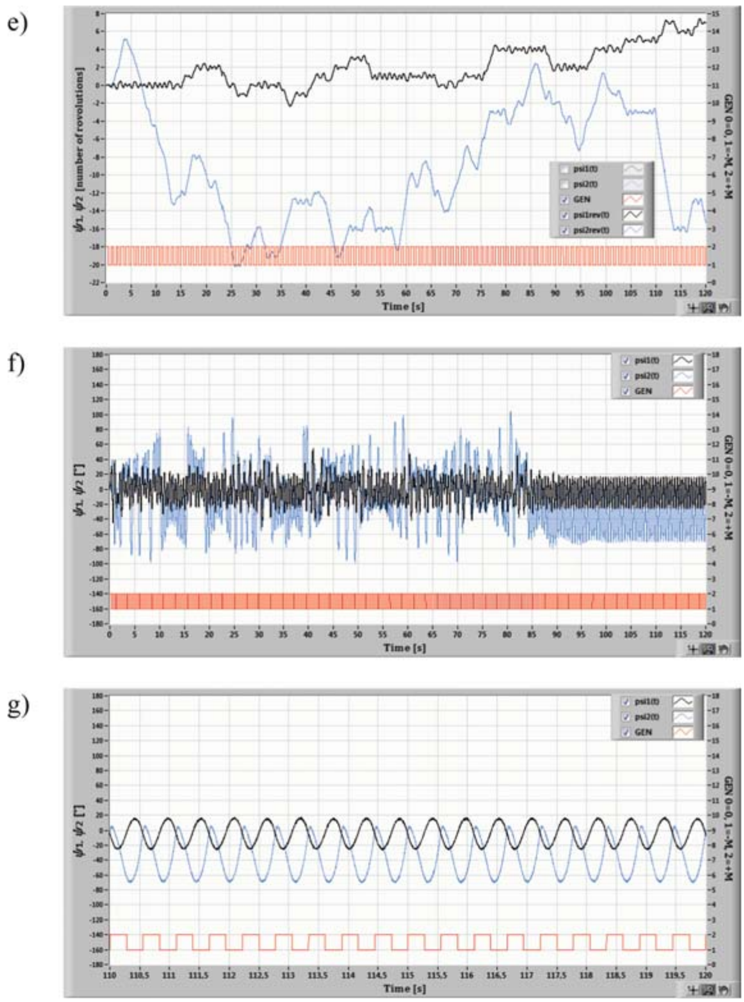

Fig. 7. (*continued*)

4 Conclusions

The constructed magnetic interaction measurement system enables to measure force caused by two repelling permanent magnets. It allows to obtain polynomial approximation curves of moment of force, which will be used for mathematical modeling of the system in further papers. Numerical and experimental analysis of system dynamics will be the purpose of next authors' articles.

The conducted experimental analysis proves that considered system is strongly non-linear. Recorded time series of angular positions show the regular or chaotic motion of the pendulum depending on the forcing frequency.

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