General theory of geometrically nonlinear size dependent shells taking into account contact interaction.
Part 2. Contact interaction of two-layer axially symmetric shells

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Abstract: In this paper a mathematical model of the nonlinear dynamics of flexible two-layer axisymmetric spherical shells of equal curvature is proposed. The geometric nonlinearity is taken into account by the model of Theodore von Karman. The shell material is isotropic and homogeneous. For each layer, the Kirchhoff-Love hypothesis is applied. The contact interaction between them is taken into account according to the Cantor model. The problems are considered as systems with an infinite number of degrees of freedom. The method of finite differences of the second order of accuracy, and the Runge-Kutta type methods are used. The impact of the size-dependent parameter, amplitude and frequency of the forcing load on the contact interaction of shells is studied.

Keywords: spherical shell, contact interaction, nonlinear dynamics, chaos, power spectra, phase portrait, Poincaré map, wavelet analysis, phase synchronization.

1. Introduction

The study of the nonlinear dynamics of mechanical systems with contact interaction is a necessary direction of research for many areas of life and human activity. Multi-layer systems are elements of structures in engineering construction, consumer equipment, medical equipment, military and aerospace engineering, and nuclear power engineering. Questions of studies of nonlinear vibrations of mechanical systems are discussed in [1-4]. Method of solving a differential equation with a nonlinear relationship between components, based on replacement of non-linear terms by integrals from their derivatives is proposed in these works. The obtained solutions allowed to improve the accuracy of a gyroscope by analytical error compensation. One-dimensional mathematical models of beams, panels of infinite length and shells are constructed into account geometric, physical, constructive kinematic nonlinearity and their different combinations. Many problems were solved by various methods: finite differences method, Bubnov-Galerkin method, Rayleigh-Ritz method. Scenarios of transition of mechanical systems from periodic vibrations to temporal and space-temporal chaos are obtained. By analogy with the phenomenon of the universality of the onset of chaos in simple systems, the existence of a certain universality of the turbulence transition in the spatial problems of the theory of one-dimensional mechanical structures is shown. Studies of nonlinear dynamics of multilayer structures are devoted in references [5-10]. The features of analysis of complex vibrations of a two-layer mechanical structures in the form of beams, rectangular plates supported by beams, cylindrical shells are considered. An analysis of the modern literature shows that the problems considered in this paper have not been investigated previously.

2. Problem statement

Mathematical model of nonlinear dynamics of flexible two-layer spherical round in plan hinged-supported shells, taking into account their size properties has been built. The geometric nonlinearity is taken into account by the von Kármán model. Shells material is elastic, isotropic, and homogeneous with constant density. The contact interaction between them is taken into account according to the Cantor [11] model. For each layer the Kirchhoff-Love hypothesis is applied. Between the shells there is a gap, and hence the shells are connected via boundary conditions.

According the modified couple stress theory we consider the two-layer flexible spherical shell on a rectangular plane under the action of transverse dynamic loading. Load is evenly distributed on the surface of the first shell $q(t) = q_0 \sin(\omega_p t)$ (Fig. 1). Each layer system satisfies the Kirchhoff hypotheses.

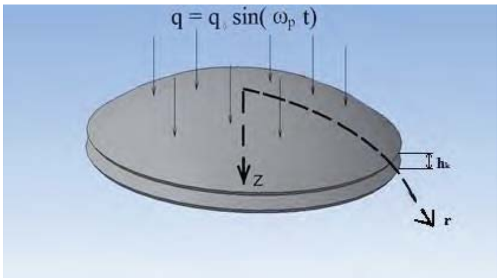

Fig. 1. The construction of two axisymmetric spherical shells of equal curvature connected through boundary conditions

The system of nonlinear PDEs control dynamics of a design from nano-axisymmetric shells has the following form

$$\frac{\partial^2 w_i}{\partial t^2} + \varepsilon \frac{\partial w_i}{\partial t} = -\left(1 + \frac{\gamma \eta_i}{2(1+\mu_i)}\right)\frac{\partial^4 w_i}{\partial r^4} - \frac{2}{r}\frac{\partial^3 w_i}{\partial r^3} + \frac{1}{r^2}\frac{\partial^2 w_i}{\partial r^2} - \frac{1}{r^3}\frac{\partial w_i}{\partial r} + \frac{\partial \Phi_i}{\partial r}\left(1 + \frac{1}{r}\frac{\partial w_i}{\partial r}\right) + \frac{\Phi_i}{r}\left(1 + \frac{\partial^2 w_i}{\partial r^2}\right) + 4q + (-1)^i K_i (w_1 - w_2 - h_k)\Psi, \quad (1)$$

$$\frac{\partial^2\Phi_i}{\partial r^2}+\frac{1}{r}\frac{\partial^2\Phi_i}{\partial r^2}-\frac{1}{r^2}\Phi_i=-\frac{\partial w_i}{\partial r}\left(1+\frac{1}{2r}\frac{\partial w_i}{\partial r}\right),$$

$$D_0=\frac{Eh^3}{12(1-\mu^2)},D_1=\frac{El^2h}{2(1+\mu)},\quad L(w_i,F_i)=2\left[\frac{\partial^2 w_i}{\partial x^2}\frac{\partial^2 F_i}{\partial y^2}+\frac{\partial^2 w_i}{\partial y^2}\frac{\partial^2 F_i}{\partial x^2}-2\frac{\partial^2 w_i}{\partial x\partial y}\frac{\partial^2 F_i}{\partial x\partial y}\right].$$

For the simple movable contour in the meridional direction we have:

$$\Phi=w=0,\frac{\partial^2 w}{\partial r^2}+\frac{v}{b}w=0,\ \text{for}\ r=\bar{r}, \tag{2}$$

the following initial conditions are taken

$$w=f_1(r,0)=0,w'=f_2(r,0)=0,0\le t<\infty, \tag{3}$$

and the following conditions in the vicinity of the shallow top are employed

$$\Phi\approx Ar,\Phi'\approx A;w\approx B+Cr^2;w'\approx 2Cr,w''\approx 2C;\ w'''\approx 0.$$

The following nondimensional quantities (with bars) are introduced:

$$\bar{t}=\omega_0 t;\ \bar{x}=b\frac{x}{c};\ \bar{y}=b\frac{y}{c};\ \omega_0=\sqrt{\frac{Eg}{\gamma R^2}};\ \bar{\varepsilon}=\sqrt{\frac{g}{\gamma R^2}}\frac{R}{h}\varepsilon,\bar{F}=\eta\frac{F}{Eh^3};\ \bar{w}=\sqrt{\eta}\frac{w}{h};$$

$$\bar{r}=b\frac{r}{c};\ \bar{q}=\frac{\sqrt{\eta}}{4}\frac{q}{E}\left(\frac{R}{h}\right)^2;\ \eta=12(1-\mu^2);\ \gamma=\frac{l^2}{h^2};b=\sqrt{\eta}\frac{c^2}{Rh},\bar{K}=\frac{b^4K}{h^5},$$

where: t - time; ε- coefficient of viscous-type external damping in which the shell moves; F- stress function; w- displacement function; R, C - main radius of the shell curvature and the radius of the shell contour, respectively; h - shell thickness; b- parameter of flatness; μ- Poisson's ratio; r - distance from the axis of rotation to the point on the middle surface; q - external load parameter; l - size-dependent parameter; h_k - casing gap; K - bulk modulus of elasticity.

In order to reduce the problem (1)-(3) governing dynamics of the considered continuous system into a system with lumped parameters, the method of finite differences (FDM) with approximation $O(\Delta^2)$ is used. PDEs as well as the boundary and initial conditions (2)-(3) are recast to the following finite difference formulas with respect to the spatial coordinate r and time:

$$w''+\varepsilon w'=-\frac{w_{j+1,i}-w_{j-1,i}}{2\Delta}\left(\frac{1}{r_j^3}-\frac{\Phi_{j+1,i}-\Phi_{j-1,i}}{2r_j\Delta}\right)+\frac{w_{j+1,i}-2w_{j,i}+w_{j-1,i}}{r_j\Delta^2}$$

$$\left(\Phi_j+\frac{1}{r_j}\right)+\frac{\Phi_{j+1,i}-\Phi_{j-1,i}}{2\Delta}+\frac{\Phi_{j,i}}{r_j}-\left(1+\frac{\gamma_i\eta}{2(1+\mu_i)}\right),$$

$$\frac{w_{j+2,i}-4w_{j+1,i}+6w_{j,i}-4w_{j-1,i}+w_{j-2,i}}{\Delta^4}-\frac{w_{j+2,i}-2w_{j+1,i}+2w_{j-1,i}-w_{i-2}}{r_j\Delta^3}+ \tag{4}$$

$$+4q_i+(-1)^iK\left(w_{i,1}-w_{i,2}-h_k\right)\psi,$$

$$\Phi_{j+1,i}\left(-\frac{1}{\Delta^2}-\frac{1}{2r_j\Delta}\right)+\Phi_{j,i}\left(\frac{2}{\Delta^2}+\frac{1}{r_j^2}\right)+\Phi_{j-1,i}\left(-\frac{1}{\Delta^2}+\frac{1}{2r_j\Delta}\right)$$
$$=-\frac{w_{j+1,i}-w_{j-1,i}}{2\Delta}\left(1-\frac{w_{j+1,i}-w_{j-1,i}}{4r_j\Delta}\right),$$

where: $\Delta= {}^{b}/_{n}$; n - denotes the number of modes of the shell radius.

Boundary conditions for the shell is pivotally-movable in the meridian direction supporting contour:

$$\Phi_n=0; w_{i+1}=\frac{v\Delta-2b}{2b+v\Delta}w_{i-1}\; w_n=0 \text{ for } r_n=b \quad (5)$$

and the following initial conditions are taken

$$w_n=f_1(r_k,0), w'_n=f_2(r_k,0), (0\le k\le n), 0\le t\le\infty. \quad (6)$$

If small terms are neglected and the differential operators are substituted by the central finite differences for $r=\Delta$, the conditions are obtained in the shell top. If we neglect the small terms and replace the central differential operators with finite-difference we obtain the conditions at the vertex:

$$\Phi_0=\Phi_2-2\Phi_1;\; w_0=\frac{4}{3}w_1-\frac{1}{3}w_2;\; w_{-1}=\frac{8}{3}w_1-\frac{8}{3}w_2+w_3. \quad (7)$$

The transverse load can be changed arbitrarily with respect to the spatial coordinate and time. In this work the harmonic transverse load of the form $q=q_0\sin(\omega_p t)$, where q_0 stands for an amplitude and ω_p is a frequency of the excitation, is used.

After reducing the task (1) – (3) to the normal form, we solve the Cauchy problem by the Runge-Kutta method of the fourth order of accuracy. The time step is chosen from the stability condition of the solution($\Delta t=1.2207\cdot10^{-4}$).

3. Numerical results

We study the vibrations of axisymmetric for the simple movable contour in the meridional direction (2) construction consists of two nano shells (b = 4) under the action of an alternating load on the upper shell. Signals, phase portraits, Poincaré section, autocorrelation function, Fourier spectrum, sign changes the highest Lyapunov exponent in time was analyzed for each of the shells. Impact magnitudes the size dependent parameter γ between the shells was studied. Change the contact area in time has also been studied.

The method of phase chaotic synchronization of mechanical dynamical systems on the basis of wavelet analysis is used. To describe and analyze phase chaotic synchronization, the phase of the chaotic signal is introduced. Phase chaotic synchronization means that the phase of chaotic signals is captured. Time as the amplitudes of these signals remain unrelated together and look chaotic. The phase capture entails coincidence of frequencies signals. The dark zones of the wavelet spectrum correspond to phase synchronization of the beam vibrations.

The influence of magnitude the size-dependent parameter γ on the vibration character of a two-layer packet shells has been studied. Particular attention is drawn on initial joint vibrations, i.e. from the moment of contact of the shells. In Table 1 are given signals, phase portraits and power spectra for the construction from shells with a small gap $\delta = 0.01$ and $\gamma = 0;\ 0.7$. In both cases, before the contact, the first shell experiences harmonic vibrations, and the second shell is on rest. Increase parameter γ yield the system more rigid and resistant to loads, and deflection at depending on ascending loads increases slower. Contact for shells with the parameter γ = 0 comes at an amplitude of the sign of the variable load $q_0 = 0.0002$, and for shells with the parameter $\gamma = 0.7$, at the load amplitude $q_0 = 0.0005$. In Tables 2 and 3, the results are in the following way: a) the signal of joint vibrations of the two shells; b) the phase portrait for the first shell; c) the phase portrait for the second shell; d) the power spectrum based on the fast Fourier transform for the first shell; e) the power spectrum for the second shell; e) the phase difference.

Table 1.

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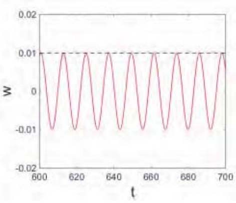	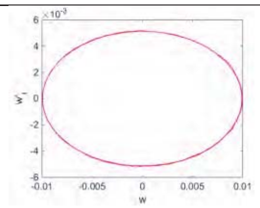	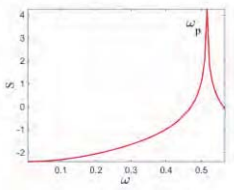
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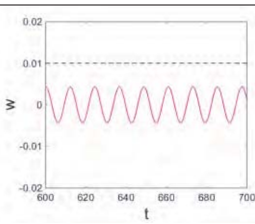	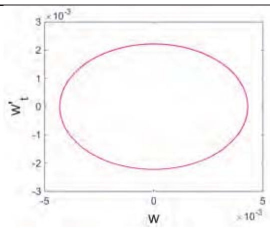	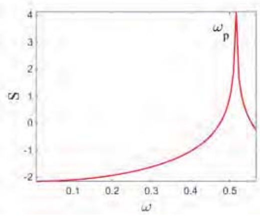

Table 2.

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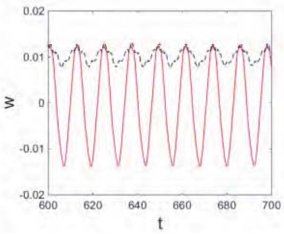
<td>
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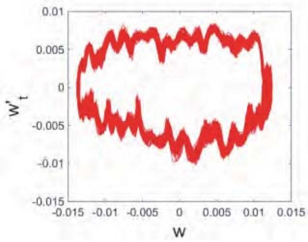
<td>
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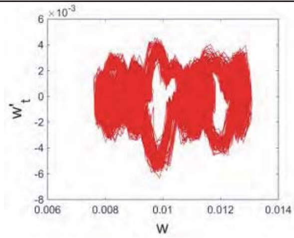
<td>
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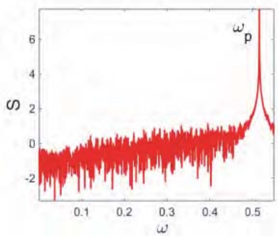
<td>
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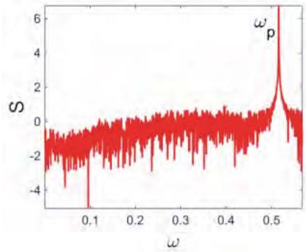
<td>
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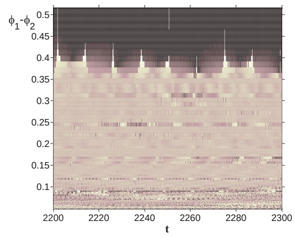
<td>
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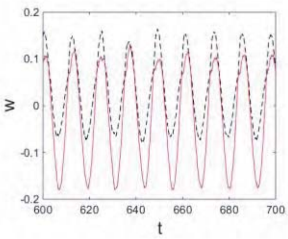
<td>
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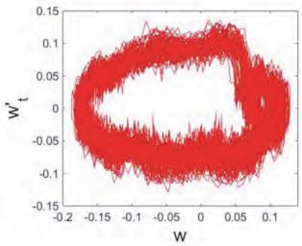
<td>
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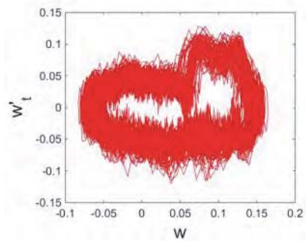
<td>
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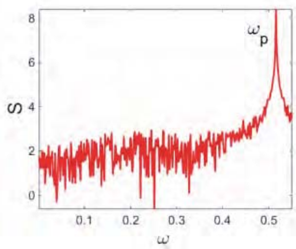
<td>
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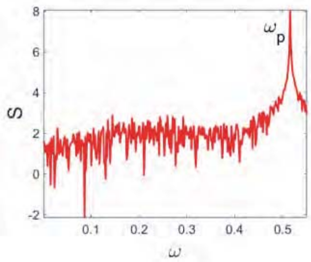
<td>
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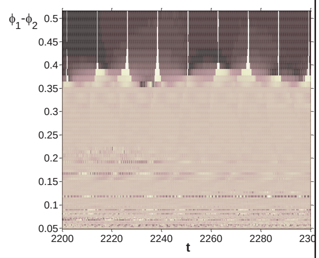
<td>
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Table 3.

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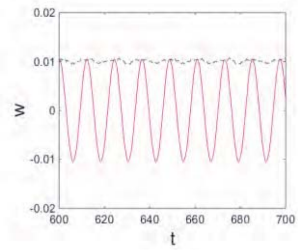
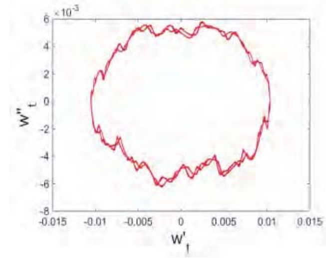
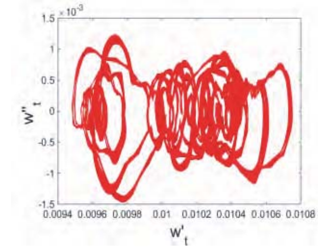
<td></td>
<td></td>
<td></td>
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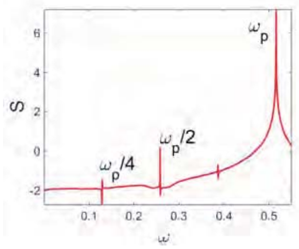
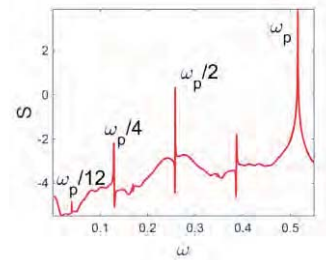
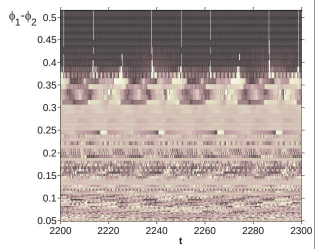
<td></td>
<td></td>
<td></td>
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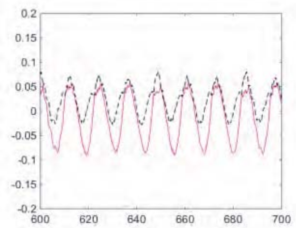
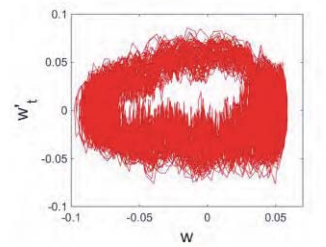
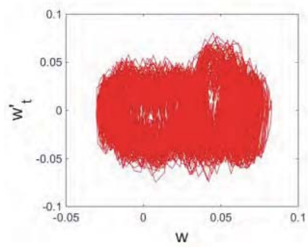
<td></td>
<td></td>
<td></td>
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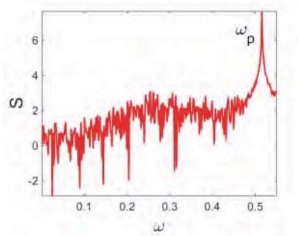
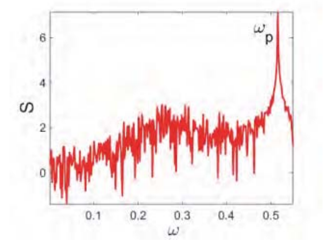
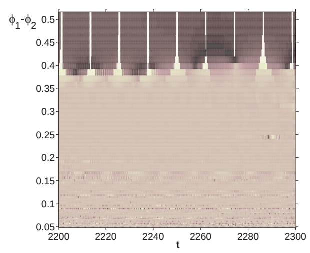
<td></td>
<td></td>
<td></td>
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Consider the vibrations of a two-layer package with a gap $h_k = 0.01$ between the shells and with parameter $\gamma = 0$. At increase amplitude of the load there is a contact of the shells. Both shells vibrate chaotically. The phase portrait for the first envelope represents a thickened orbit. In the phase portrait of the second shell three centers of attraction of phase trajectories are visible. On the power spectrum of the first shell, chaos is observed at low frequencies, and for a second shell, on a solid pedestal. The phase difference indicates that the frequencies present in the signal are not synchronized. Further increase of the amplitudes of excitation also generates chaotic vibrations of the shells. Phase portraits of shells have a similar shape. Power spectra have noisy components. On the graph, the phase difference increased the number of dark spots, which means synchronization of some frequencies.

Now we will analyze the situation when the vibrations of a two-layered packet with a gap $h_k = 0.01$ and parameter $\gamma = 0.7$ between shells are studied. At initial joint vibrations of a two-layer package shells with an amplitude of the driving force $q_0 = 0.0005$ shell power spectrum of the first shell demonstrates frequencies ω_p, $\frac{\omega_p}{2}$, and $\frac{\omega_p}{4}$ and in the signal of the second shell there are frequencies: $\omega_p, \frac{\omega_p}{2}, \frac{\omega_p}{4}. \frac{\omega_p}{12}$. In the phase portrait of the first shell two orbits are visible, and the second shell has 12 thickened orbits. When increasing the amplitude excitation to $q_0 = 0.005$, the vibrations become chaotic.

4. Concluding remarks

A mathematical model of the contact interaction of two spherical axisymmetric circular nano-shells has been constructed. The nonlinear dynamics of the contact interaction of two axisymmetric nano-shells has been investigated. Comparison vibrations depending on from increase the size-dependent parameter has been carried out. It is revealed that with the increase in the dimension-dependent parameter, the stability of the system increases. On the other hand, a contact between shells implies chaotic vibrations.

Acknowledgements

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