General theory of geometrically nonlinear size dependent shells taking into account contact interaction. Part 1. Chaotic dynamics of geometrically nonlinear axially symmetric one-layer shells

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Abstract: A mathematical model of flexible flat rectangular in plan shells is proposed. A special case for spherical axisymmetric shells taking into account nanoscale effects has been studied. Shell structure material is homogeneous and isotropic, and the nanoscale factors has been taken into account. Partial differential equations for axisymmetric spherical shallow shells were reduced to the Cauchy problem by the method of finite differences of the second order of accuracy. The Cauchy problem was solved by the Runge-Kutta method of the 4th order. Convergence of the obtained results in dependence of the number of partitions along the radius was investigated. The system was studied as a system with an infinite number of degrees of freedom. The effect of the size-dependent parameter, which significantly affects the nonlinear dynamics of the shell, was taken into account.

Keywords: Mathematical model, rectangular shell, spherical axisymmetric shell, moment theory of elasticity, loss of stability, chaos, numerical experiment, geometric nonlinearity, distributed mechanical structure.

1. Introduction

In experimental studies of metals, polymers and metallic glass, a size dependent effect was observed when the thickness of mechanical structures in the form of rods, plates, and full shells was compressed to a micron [1, 2]. This effect plays an important role when taking into account the mechanics of the mentioned structures [3].

Experimental studies of the real microstructures are extremely complex and expensive. Chong and Lam [4] observed that the flexural rigidity increases by about 2.4 times with a decrease in thickness from 115 to 20 μm when testing the micro-rod from epoxy polymers for bending. From the works of these interesting experiments can be concluded that the size-caused behavior is an inherent property of materials that can not be neglected when designing optimal dynamic devices using MEMS [5], [6].

Young et al. [7] developed couple stress based strain gradient theory for elasticity using the theory of higher order of continuous media. The behavior of the pairs of forces was determined by an additional symmetrical equilibrium relation, at which only one additional parameter of the scale of the length of the material took place.

Based on the modified theory of moment stresses, static mechanical properties [2], elastic bending [1], fluid transfer [8], dynamic characteristics [9-11], nonlinear vibration [12-13] of micro-rods were studied.

Modified couple stress theory of moment stresses for computation the size dependent plates was applied. The theory of moment stresses of microstructurally dependent pairs of forces applied to functionally graded rods and the Timoshenko rod was investigated by Reddy et al. [14-15]. Ciata [6] studied the static analysis of isotropic microplates using the Kirchhoff plate model. Iain et al. [16] analyzed the types of dynamic behavior of the Kirchhoff microplate, based on a modified theory of moment stresses. Lazopoulos [17], adopting the Kirchhoff model for plates, investigated the stress gradient in the bending of thin plates to determine the size effect. Ke et al. [5] performed studies using the moment theory for plates of Mindlin plates. Reddy et al. [18] applied the theory of the third approximation (model of Sheremetyev-Pelekh) [19] taking into account piezo effects. Stress-strain state size dependence microstructures: plates, rods and shells take into account temperature effects for homogeneous materials was studied in papers [19-32]. In conclusion, it is important to note that the study of nonlinear dynamics of the size effect for rods, plates and shells is not done. The main goal of this paper is the construction of a general theory and study of nonlinear dynamics of size dependent plates and shells in a temperature field with account for couple of deformation fields and temperature. Algorithms and software complexes for analysis of nonlinear dynamics of size dependent effects of the flat in plan axisymmetric shells under the action of a transverse periodic load were created.

2. Mathematical background

In the classical theory of elasticity, the work of deformation and the strain energy depend on the stress tensor and do not depend on the rotation vector due to material independence. However, the gradient of the rotation vector can be an important factor in the equations of state. Based on the modified couple stresses theory of moment stresses presented by Yang et al. [7], the strain energy density is a function of both the couple stress tensor (conjugate to the strain tensor) and the curvature tensor (conjugate to the tensor of moment stresses). In deformed isotropic linear elastic material, located in the region Ω, strain energy Π is expressed by the following equations

$$\Pi = \int_{\Omega} (\sigma_{ij}\,\varepsilon_{ij} + m_{ij}\eta_{ij})d\Omega \quad (i,j = 1,2,3) \tag{1}$$

Here σ_{ij} is the Cauchy stress tensor, ε_{ij} is the stress tensor, m_{ij} is a deviator component of the stress tensor, a η_{ij} - symmetric curvature tensor. The parameter of the material length scale related to the microstructures of the material was developed for the purpose of interpreting the dimensional effect in the non-classical Kirchhoff-Love model. These tensors are defined by formulas

$$\sigma_{ij} = \lambda tr(\varepsilon_{ij)}I + 2\mu\varepsilon_{ij}, \quad (2)$$

$$\varepsilon_{ij} = \frac{1}{2}[\nabla u + (\nabla u)^T], \quad (3)$$

$$m_{ij} = 2l^2\mu\chi_{ij}, \quad (4)$$

$$\eta_{ij} = \frac{1}{2}\left[\nabla\varphi + \left(\nabla_{\varphi}\right)^T\right], \quad (5)$$

where: u - displacement vector; $\lambda = \frac{Ev}{(1+v)(1-2v)}$ and $\mu = \frac{E}{2(1+v)}$ - constants of Lamé; E, v represent the Young's modulus and Poisson's ratio for the shell material, respectively; l — this parameter is a scale of the length of the material, understood as a property of the material, characterizing the effect of the moment stress [15]. The latter parameter describes mathematically the square of the ratio of the curvature module to the shear modulus and it can be determined by experiments for thin torsion cylinders [33] or for thin rods for bending [34] on a micron scale; φ — this rotation vector, represented as $\varphi_{i=}\frac{1}{2}rot(u_i)$.

From the analysis of equations (3) and (5) it follows that the stress tensor ε_{ij} and the curvature tensor η_{ij} are symmetric, and, consequently, equations (2) and (4) yields the stress tensor σ_{ij} and deviator component of the stress tensor m_{ij} also symmetric. In deriving the equations of flexible, dimensionally dependent shallow shells, the following hypotheses are used:

- shell is homogeneous, isotropic, and elastic;
- shallow shells are defined by the Reissner [35] or by V.Z. Vlasov [36];
- shell is subjected to the hypothesis of Kirchhoff-Love;
- geometric nonlinearity is introduced by the Kármán model [37].

Let a shallow shell be considered in rectangular system of coordinates (see Fig.1) introduced in the following way: $\Omega = \{x, y, z | (x, y) \in [0; a] \times [0; b], z \in [-h; h]\}, 0 \leq t < \infty$.

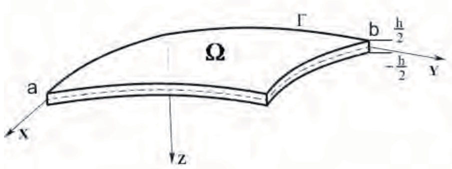


Fig. 1. Single-layer rectangular in plan shell

According to the principle of Hamilton-Ostrogradsky;

$$\int_{t_0}^{t_1} (\delta K - \delta \Pi + \delta' W) dt = 0, \tag{6}$$

where: K, П – kinetic and potential energy, respectively; $\delta' W$ - work of external forces.

The system of nonlinear PDEs governing dynamics of the flexible rectangular shells on the basis of couple stress theory has the following form:

$$(D_0 + D_l)\nabla^4 w - \Delta_k^2 F - L(w, F) + ph\varepsilon\dot{w} - \frac{q}{h} + ph\ddot{w} = 0, \tag{7}$$

$$\nabla_k^2 w + \frac{1}{2} L(w, w) + \frac{1}{Eh}\nabla^4 F = 0, \text{ где } D_l = \frac{El^2 h}{2(1+\mu)}, D_0 = \frac{Eh^3}{12(1-\mu^2)},$$

$$L(w \quad , F \quad) = 2\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 F}{\partial x \partial y}\right],$$

$$\nabla_k^2(\cdot) = K_y \frac{\partial^2(\cdot)}{\partial x^2} + K_x \frac{\partial^2(\cdot)}{\partial y^2},$$

where $\nabla_k^2(\cdot) -$ 4th order Laplace operator; K_x and K_y - curvature of the shell or can be interpreted small initial irregularities; t- time; ε - coefficient of resistance of the medium in which the shell moves; F - stress function; w - deflection function; h - shell thickness; μ - Poisson's coefficient; q - external load parameter; l – size-dependent parameter.

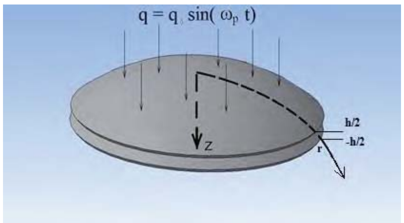


Fig. 2. Spherical axisymmetric shell.

To obtain the axial symmetric theory of size dependent shells, we employ the cylindrical coordinate system. The second equation of the system is multiplied by *r*, integrated and a new resolving function $\Phi \quad = \frac{\partial F}{\partial r}$ is introduced [1].

Equations for nano axisymmetric shells have the following form

$$\frac{\partial^2 w}{\partial t^2} + \varepsilon\frac{\partial w}{\partial t} = -\left(1 + \frac{\gamma\eta}{2(1+\mu \quad)}\right)\frac{\partial^4 w}{\partial r^4} - \frac{2}{r}\frac{\partial^3 w}{\partial r^3} + \frac{1}{r^2}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^3}\frac{\partial w}{\partial r} + \frac{\partial \Phi}{\partial r}\left(1 + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{\Phi}{r}\left(1 + \frac{\partial^2 w}{\partial r^2}\right) + 4q. \tag{8}$$

We introduce the following dimensionless quantities:

$$t = \omega_0 t;\ \bar{x} = b\frac{x}{c};\ \bar{y} = b\frac{y}{c};\ \omega_0 = \sqrt{\frac{Eg}{\gamma R^2}};\ \bar{\varepsilon} = \sqrt{\frac{g}{\gamma E}\frac{R}{h}\varepsilon};\ \bar{F} = \eta\frac{F}{Eh^3};\ \bar{w} = \sqrt{\eta}\frac{w}{h};\ \bar{r} = b\frac{r}{c};$$

$$\bar{q} = \frac{\sqrt{\eta}}{4}\frac{q}{E}\left(\frac{R}{h}\right)^2;\ \eta = 12(1-\mu^2);\ \gamma = \frac{l^2}{h^2};\ b = \sqrt{\eta\,\frac{c^2}{Rh}};$$

where: R, C - the main radius of curvature of the reference contour and the radius of the reference contour in the circumferential direction, respectively; b - parameter of flatness; r - distance from the axis of rotation to the point on the middle surface. In the given equations, the bars over dimensionless quantities are omitted for simplicity. For an axisymmetric problem, the boundary conditions are written in the following form.

1) Simple movable contour in the meridional direction:

$$\Phi = w = 0, \frac{\partial^2 w}{\partial r^2} + \frac{v}{b} w = 0, \qquad \text{for } r = \bar{r}. \tag{9}$$

2) Rigidly clamed contour

$$\frac{\partial \Phi}{\partial r} - v\frac{\Phi}{b} = 0, w = 0, \frac{\partial^2}{\partial r^2} + \frac{v}{r}\frac{\partial w}{\partial r} = 0, \quad \text{for } r = \bar{r}. \tag{10}$$

3) Sliding clamping of the contour:

$$\Phi = w = 0, \frac{\partial w}{\partial r} = 0, \quad \text{for } r = \bar{r}. \tag{11}$$

4) Simple nonmovable contour:

$$\frac{\partial \Phi}{\partial r} - v\frac{\Phi}{b} = 0, w = 0, \frac{\partial w}{\partial r} = 0, \quad \text{for } r = \bar{r}. \tag{12}$$

and the following initial conditions: $w = f_1(r,0) = 0, w' = f_2(r,0) = 0 \ \ 0 \le t < \infty$.

In addition, the following conditions in the vicinity of the shallow top are employed:

$\Phi \approx Ar;\ \Phi' \approx A; w \approx B + Cr^2; w' \approx 2Cr; w'' \approx 2C;\ w''' \approx 0.$

In order to reduce the problem (8) - (12) governing dynamics of the considered continuous system into a system with lumped parameters, the method of finite differences (FDM) with approximation $O(\Delta^2)$ is used. PDEs as well as the boundary and initial conditions (9) - (12) are recast to the following finite difference formulas with respect to the spatial coordinate *r* and time:

$$w'' + \varepsilon w' = -\frac{w_{i+1} - w_{i-1}}{2\Delta}\left(\frac{1}{r_i^3} - \frac{\Phi_{i+1} - \Phi_{i-1}}{2r_i\Delta}\right) + \frac{w_{i+1} - 2w_i + w_{i-1}}{r_i\Delta^2}\left(\Phi_i + \frac{1}{r_i}\right) +$$
$$+ \frac{\Phi_{i+1} - \Phi_{i-1}}{2\Delta} + \frac{\Phi_i}{r_i} - \left(1 + \frac{\gamma\eta}{2(1+\mu)}\right)\frac{w_{i+2} - 4w_{i+1} + 6w_i + 4w_{i-1} + w_{i-2}}{\Delta^2}$$
$$- \frac{w_{i+2} - 2w_{i+1} + 2w_{i-1} - w_{i-2}}{r_i\Delta^3} + 4q, \tag{13}$$

$$\Phi_{i+1}\left(-\frac{1}{\Delta^2}-\frac{1}{2r_i\Delta}\right)+\Phi_i\left(\frac{2}{\Delta^2}+\frac{1}{r_i^2}\right)+\Phi_{i-1}\left(-\frac{1}{\Delta^2}+\frac{1}{2r_i\Delta}\right)=\frac{w_{i+1}-w_{i-1}}{2\Delta}\left(1-\frac{w_{i+1}-w_{i-1}}{4r_i\Delta}\right),$$

where $\Delta = b/n$ and n denotes the number of modes of the shell radius.

The counterpart difference forms of the boundary conditions are as follows: If small terms are neglected and the differential operators are substituted by the central finite differences for $r = \Delta$, the following conditions are obtained in the shell top:

$$\Phi_0 = \Phi_2 - 2\Phi_1;\ w_0 = \frac{4}{3}w_1 - \frac{1}{3}w_2;\ w_{-1} = \frac{8}{3}w_1 - \frac{8}{3}w_2 + w_3 \tag{14}$$

The transverse load can be changed arbitrarily with respect to the spatial coordinate and time. In this work the harmonic transverse load of the form $q = q_0\sin(\omega_p t)$ where q_0 stands for an amplitude and $\omega_p = \frac{2\pi}{T}$ is a frequency of the excitation, is used.

After reduction of the problem (14) to the normal form, we solve the Cauchy problem by the Runge-Kutta method of the fourth order of accuracy. The time step is chosen from the stability condition of the solution ($\Delta t = 2.441 \cdot 10^{-4}$).

3. Results and discussions

Investigate complex vibrations shallow spherical shell with the boundary conditions: simple movable contour in the meridional direction (9), the parameter shallowness $b = 4$, $\gamma = 0$; 0.3; 0.7. When solving the problem by the method of finite differences $r \in [0; b]$ the interval of integration was divided into 120 parts. This number of partitions of the integration interval made it possible to treat the shell structure as with distributed parameters, rather than as a structure with lumped parameters, i.e. considered it as a system with an infinite number of degrees of freedom. Figures 3, 4, 5 show the dependence of the deflection at the center of the shell in dependence of the alternating transverse load q_0 (load on the shell uniformly distributed, changing according to law $q = q_0 sin(\omega_p t)$, where $\omega_p = 0.516$ is a frequency of the excitation, which is close to the fundamental frequency of linear vibration). Colored dots in Figures 3-5 denote the free vibrations (the dependence of the deflection at the top of the shell in time W(0), phase portraits W(W'), Fourier frequency power spectra, S(ω), and their characteristics are given in Table 1.

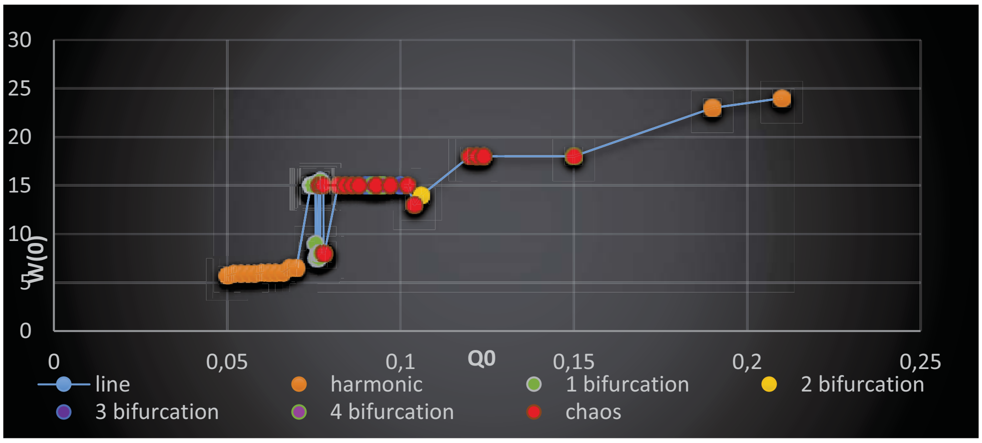


Fig. 3. Dependence W(q) for n=120, ɤ= 0.0

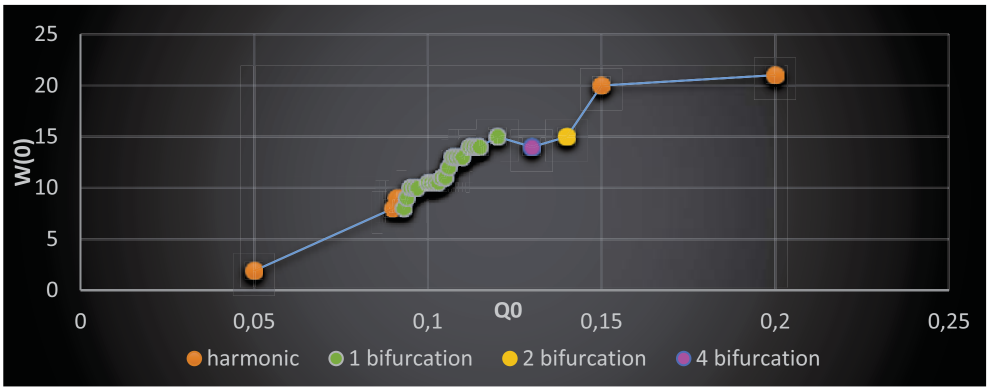


Fig. 4. Dependence W(q) for n=120, ɤ= 0.3

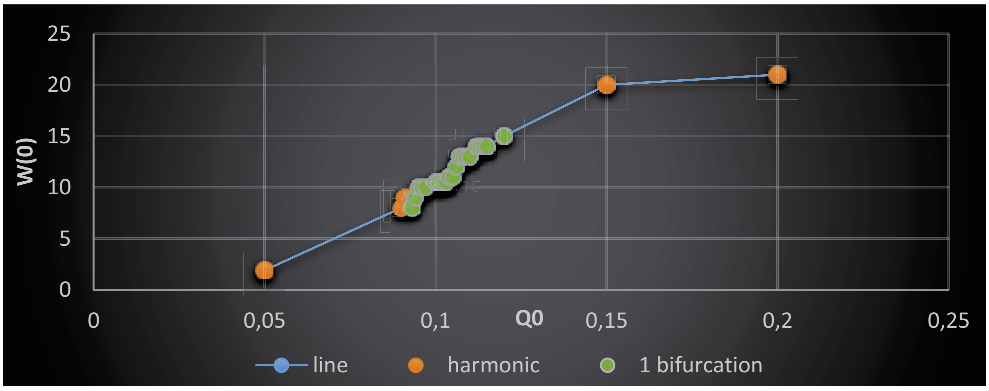
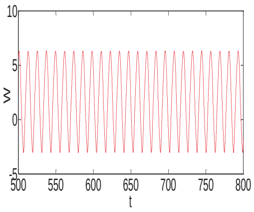
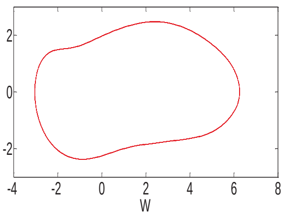
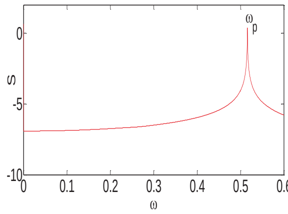
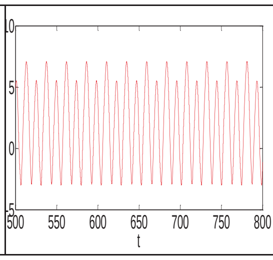
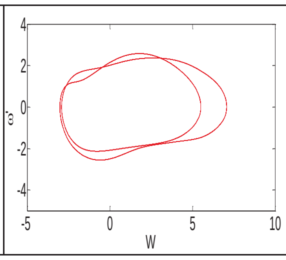
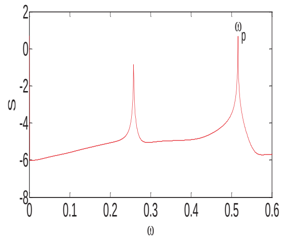
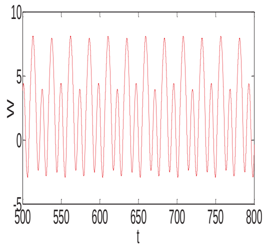
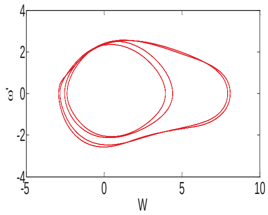
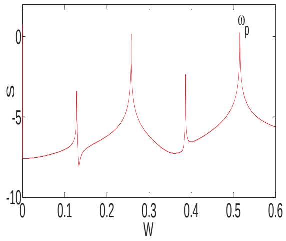
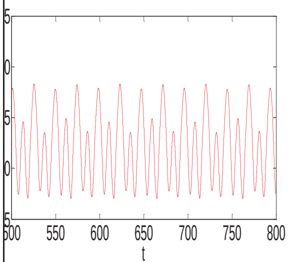
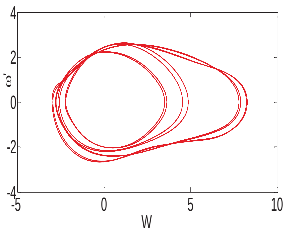
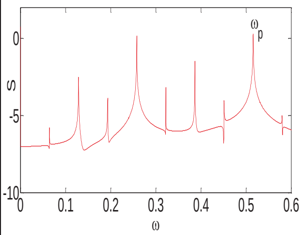
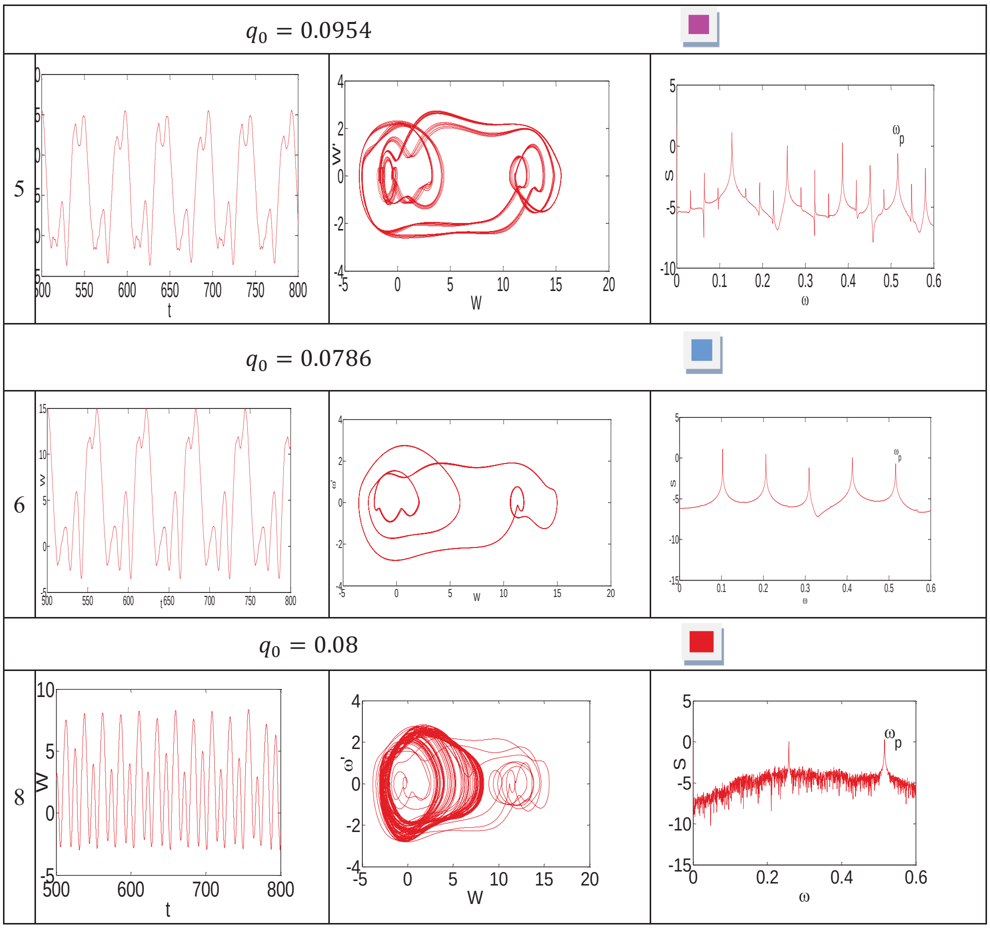


Fig. 5. Dependence W(q) for n=120, ɤ= 0.7

Table 1

Type of vibrations			
	Time history	Phase portrait	Power spectrum
	$q_0 = 0.07$		
1			
	$q_0 = 0.07351$		
2			
	$q_0 = 0.07822$		
3			
	$q_0 = 0.07933$		
4			



4. Conclusion

The analysis of the results shows that an increase in the value of the parameter γ is simplifies the shell vibrations and transition from chaotic vibrations to harmonic vibrations has been observed. Complex vibrations with the effect of loss of stability are characteristic for the shells with $\gamma = 0$. Increasing γ parameter does not yield loss of stability. In this case, the vibrations become periodic. The amplitude gradually increases together with increase of the load ($\gamma = 0.7$). For MEMS devices, this effect is of great importance, as no chaotic vibration MEMS devices results in greater system reliability and durability.

Acknowledgements

This work has been supported by the grants the Russian Science Foundation, RSF 16-19-10290.

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