Theory of coupled deformation and temperature fields for three-layer nano-mechanical structures

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Abstract: In this work a mathematical model of a mechanical structure consisting of two nanoplates is developed, and between these nanoplates there is a nanobeam, and there are gaps between the elements. The resolving equations of this mathematical model are obtained using kinematic hypotheses of the first approximation (for the plates - Kirchhoff's conjecture, for beams - Bernoulli-Euler). The contact interaction is taken into account by the theory of Kantor. As a result, the obtained model takes into account the parabolic heat conduction equation. There are no restrictions on the temperature fields distribution in height for nanoplates and nanobeams (for nanoplates, the temperature field is three-dimensional, for a beam it is two-dimensional). The resulting system of partial differential equations is hyperbolic-parabolic and of different dimension. In addition, the equations are highly nonlinear and integro-differential, since the contact interaction between the elements of the structure is taken into account. To obtain reliable results, we reduce the resulting system to the Cauchy problem by two methods: the Faedo-Galerkin method in higher approximations and the finite difference method with the approximation $0(h^2)$ and $0(h^4)$ with respect to the spatial coordinates. Next, the Cauchy problem is solved by the Runge-Kutta methods of the 4th, 6th, 8th accuracy orders regarding time. Such a variety of methods of solution is necessary to obtain true results as a system with an infinite number of freedom degrees.

Keywords: mathematical model, temperature, strain field, contact interaction, plate-beam structures, Faedo-Galerkin method, Runge-Kutta type methods, nonlinear dynamics.

1. Introduction

At present, the production and application of miniature sensors of inertial and external information, micromotors and converters are in great demand. Modern micromechanical and microelectromechanical systems find their application for a wide range of mobile objects, i.e. in navigation equipment, automotive industry, military equipment, aircraft construction and rocket engineering [1-5]. Compound elements of micro- and nano electromechanical systems (MEMS, NEMS), such as vibration sensors [6], micro-drives [7], microswitches [8], are micro- and nanoscale beams and plates. Due to their nanosize and the presence of small gaps between the elements, an extremely important issue is the study of the contact interaction of nanoplates and nanobeams, taking into account the connectivity of the temperature and deformation fields. In references [9, 10], the

nonlinear dynamics of a three-layer microplate has been studied. On the basis of Kirchhoff plate theory and von Kármán nonlinear deformations, nonlinear size-dependent transverse and plane equations of motion have been derived. The model takes into account the nonconservative damping force of a viscous type, as well as the external exciting load. In references [11-13], according to the momentum theory of elasticity, mathematical models have been constructed and the layered beam nanostructures have been studied, but in these works the connection between the fields of temperature and deformation have not been taken into account.

2. Problem statement

In this paper we study the nonlinear dynamics of beam and plate nanostructures with allowance for their contact interaction and connection between the fields of temperature and deformation. The multilayer structure consists of two parallel nanoplates, and there is a nanobeam located centrally between them. There is a small gap h_k between all elements and they are connected only through the boundary conditions (Fig.1).

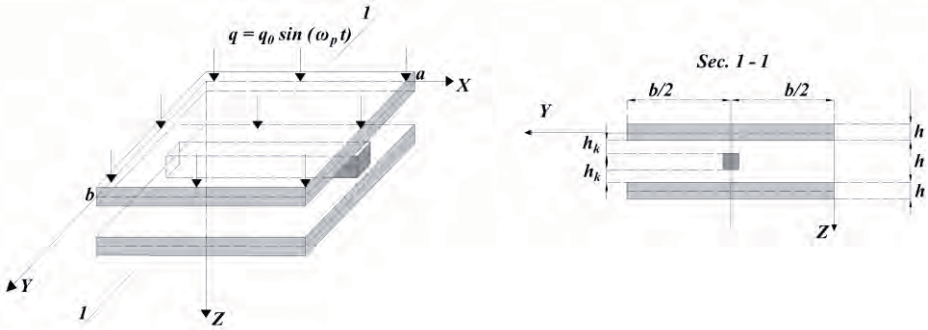

Figure 1. Scheme of the studied plate–beams structure.

For nanoplates the Kirchhoff kinematic model is employed, whereas for the nanobeam we use for the Euler-Bernoulli theory. The relationship between strain and deformations in the nanoplate ($\beta_1 = 1$) and nanobeam ($\beta_1 = 0$) can be represented in the following form [14]:

$$\varepsilon_1^z = \frac{1}{E}(\sigma_1 - \beta_1 \nu \sigma_2) + \alpha_T T_m \,, \overset{\leftrightarrow}{(1,2)} \,, \varepsilon_{12}^z = \beta_1 2 \frac{1+\nu}{E} \sigma_{12} \,, \overset{\leftrightarrow}{(1,2)} \tag{1}$$

where: $E = E(x, \beta_1 y, z, \varepsilon_0, \varepsilon_i, \theta)$, $\nu = \nu(x, \beta_1 y, z, \varepsilon_0, \varepsilon_i, \theta)$. This representation is based on the variable elasticity method [15]. The following notation is employed: ε_0 - volume deformation, ε_i -

strain intensity, α_T - coefficient of linear thermal expansion, $\theta(x,\beta_1 y,z)$ – temperature increment for plate ($\beta_1 = 1$) and for beam ($\beta_1 = 0$). The resolving differential equations for nanoplates and nanobeam are yielded by the variation principle [16]:

$$\delta V + \delta D^* - \delta K^* = \iint_A (\overline{F_0}\delta\overline{u_0} - \theta\bar{n}\delta\bar{S})dA, \tag{2}$$

where $\delta V, \delta D^*, \delta K^*$ stand for the variations of the generalized free energy, the dissipative function and the kinetic energy, respectively; $F_0, \overline{u_0}$ — the surface force and displacement; $\bar{n}$ — an outer normal; A – the bounding body surface.

After a few transformations according to the variational calculus, taking into account of (2) and according to the momentum theory of elasticity [17], the following differential equations for a three-layer nanostructure are obtained

$$\begin{cases}
\frac{\partial^2}{\partial x^2}[-\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)(\frac{\partial^2 w_1}{\partial x^2}+\nu\ \frac{\partial^2 w_1}{\partial y^2})dz - \alpha_1\int_{-h/2}^{h/2}\theta\ z(1+\nu\ )dz] + \\
+\frac{\partial^2}{\partial y^2}[-\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)(\frac{\partial^2 w_1}{\partial y^2}+\nu\ \frac{\partial^2 w_1}{\partial x^2})dz - \alpha_1\int_{-h/2}^{h/2}\theta z(1+\nu\ )dz] - \\
-2\frac{\partial^2}{\partial x\partial y}(\frac{\partial^2 w_1}{\partial x\partial y}\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)dz) + q_1 - \frac{\gamma}{g}h\frac{\partial^2 w_1}{\partial t^2} - \varepsilon\frac{\gamma}{g}h\frac{\partial w_1}{\partial t} - q_{k1} = 0, \\
\nabla^2(\theta\ ) = \frac{T_0}{K_T}\frac{\partial}{\partial t}(\frac{c\theta}{T_0} + G^*\alpha_T e_1), \qquad (\beta_1 = 1), \\
-\frac{\partial^2}{\partial x^2}\left(\int_{-h/2}^{h/2}(\gamma_3+\gamma_4)dz\frac{\partial^2 w_2}{\partial x^2}\right) - \frac{\partial^2 M_t}{\partial x^2} + q_2 - \frac{\gamma}{g}h\frac{\partial^2 w_2}{\partial t^2} - \varepsilon\frac{\gamma}{g}h\frac{\partial w_2}{\partial t} + q_{k1} - q_{k2} = 0, \\
\nabla^2(\theta\ ) = \frac{T_0}{K_T}\frac{\partial}{\partial t}(\frac{c\theta}{T_0} + G^*\alpha_T e_2), \qquad (\beta_1 = 0), \\
\frac{\partial^2}{\partial x^2}[-\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)(\frac{\partial^2 w_3}{\partial x^2}+\nu\ \frac{\partial^2 w_3}{\partial y^2})dz - \alpha_3\int_{-h/2}^{h/2}\theta\ z(1+\nu\ )dz] + \\
+\frac{\partial^2}{\partial y^2}[-\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)(\frac{\partial^2 w_3}{\partial y^2}+\nu\ \frac{\partial^2 w_3}{\partial x^2})dz - \alpha_3\int_{-h/2}^{h/2}\theta z(1+\nu\ )dz] - \\
-2\frac{\partial^2}{\partial x\partial y}(\frac{\partial^2 w_3}{\partial x\partial y}\int_{-h/2}^{h/2}(\gamma_1+\gamma_2)dz) + q_3 - \frac{\gamma}{g}h\frac{\partial^2 w_3}{\partial t^2} - \varepsilon\frac{\gamma}{g}h\frac{\partial w_3}{\partial t} + q_{k2} = 0, \\
\nabla^2(\theta\ ) = \frac{T_0}{K_T}\frac{\partial}{\partial t}(\frac{c\theta}{T_0} + G^*\alpha_T e_3), \qquad (\beta_1 = 1),
\end{cases} \tag{3}$$

where: $M_t = \alpha_2 \int_{-h/2}^{h/2} E\theta_2 z dz$, $q_{k1} = K(w_1 - w_2 - h_k)\Psi_1$, $\Psi_1 = \frac{1}{2}[1 + sign(w_1 - h_k - w_2)]$,

$q_{k2} = K(w_2 - w_3 - h_k)\Psi_2$, $\Psi_2 = \frac{1}{2}[1 + sign(w_2 - h_k - w_3)]$, $e_1 = \varepsilon_{11}^z + \beta_1 \varepsilon_{22}^z$, ($\nabla^2(\theta)$ — a three-dimensional ($\beta_1 = 1$) and $\nabla^2(\theta)$ — two-dimensional (β_1=0) Laplace operators). Furthermore, (γ_1, γ_3) and (γ_2, γ_4) are the coefficients of the general theory of elasticity and the coefficients of the moment theory of elasticity, respectively. The contact interaction is taken into account according to the Winkler model [18]. If there is a contact between the upper or lower nanoplate and nanobeam, then $\Psi_1 = 1$ and $\Psi_2 = 1$ if there is no contact, then $\Psi_1 = 0$ and $\Psi_2 = 0$, respectively. The expressions q_{k1} and q_{k2} represent Winkler connection between compression and contact pressure. The appearance of clutch zones is unlikely, since the contact pressure between layers is small. The conditions of contact between layers can depend on the coordinates and include all kinds of imperfect one-sided contact. In the system (3) w_1, w_3, w_2 denote the deflection functions of the upper, lower nanoplate and nanobeam, respectively; K is the stiffness factor of the transversal compression of the plate in the contact area; h_k – the gap between elements; g – the gravity acceleration; γ – the specific weight of material; c - the specific heat; G^* - bulk compression modulus. The nanoplates thickness h and beam height h are the same, the nanobeam width is 1; a denotes the nanobeam and nanoplate length, b stands for nanoplates width, whereas $q_i(x, \beta_1 y, t)$ is the transverse load acting on the nanoplates (β_1=1), and acting on the nanobeam (β_1=0).

As a result, the obtained model takes into account the parabolic heat conduction equations. There are no restrictions on the temperature fields distribution over the thickness for nanoplates and in height for nanobeams (for nanoplates, the temperature field is three-dimensional, for a nanobeam it is two-dimensional). The heat exchange between the elements is not taken into account. It is possible to consider different diagrams $\sigma_i(\varepsilon_i, \theta)$ describing the dependence of stress on deformation and temperature for several chosen materials. The boundary conditions of the first type and the initial conditions for the heat equations must be added to the system (3).

Boundary conditions of the first type. The temperature distribution is set on the body surface:

$$\begin{aligned} &S: \quad (0 \le x \le a), (-h/2 \le z \le h/2), \\ &\theta = \varphi(x, z, t) : (x, z) \in S. \end{aligned} \tag{4}$$

As initial conditions, we take the distribution of deflections, deflection velocities, and temperature increment at the initial moment of time $t = 0$:

$$w(x,\beta_1 y,t)|_{t=0}=\varphi_1(x),,\ \dot{w}(z,t)|_{t=0}=\varphi_2(x),\ \theta(x,\beta_1 y,z)|_{t=0}=\varphi_3(x,\beta_1 y,z). \tag{7}$$

Also, the boundary conditions for nanoplates (β_1=1) and nanobeam (β_1=0) are taken.

Hinged support on the contour for nanoplates and at the ends of the nanobeam:

$$w_{\bar{n}}=M_{\bar{n}}=0. \tag{8}$$

System (3) and boundary conditions create a system of integro-differential equations of different dimensions of hyperbolic-parabolic type, describing nonlinear oscillations and contact interaction of the related deformation and temperature fields for structure with physical nonlinearity, and also it exhibits the dependence of material properties on temperature. In addition, the equations are highly nonlinear and integro-differential, since the contact interaction between the elements of the structure is taken into account. To obtain reliable results, we reduce the resulting system to the Cauchy problem by two methods: the Faedo-Galerkin method in higher approximations and the finite difference method with the approximation $0(h^2)$ and $0(h^4)$ with respect to the spatial coordinate. In this case, an iterative procedure is constructed. The heat conductivity equations are solved by the finite differences method, the temperature moments are calculated, and the found values of the temperature field are substituted into the motion equations, and then we find deformations. For this aim, the Cauchy problem is solved by the methods of the Runge-Kutta type: the fourth-order Runge-Kutta-Felberg method (rkf45), and the eighth-order Runge-Kutta Prince-Dormand method (rk8pd) [19]. Such a variety of solution methods is necessary for obtaining true results for a system with an infinite number of freedom degrees, since the solution essentially depends on the method and the solution time step, i.e. of the initial conditions.

3. Concluding remarks

1. Mathematical model of a three-layer package of distributed mechanical structures, consisting of two parallel plates, and a beam has been derived. Between elements there are gaps. Each of the elements of the structure is described by kinematic models of the first approximation. Coupling of temperature and strain fields by Fourier theory has been taken into account. Equations from the Biot functional has been yielded. The mechanical structure is described by the modified couple stress theory.
2. The algorithm for nonlinear dynamics of a three-layer nanostructure, taking into account the contact interaction between the elements, is developed. The algorithm is based on the application of the Faedo-Galerkin method in higher approximations for reduction to the Cauchy problem, which is solved by the Runge-Kutta type methods. The developed software package allows to consider various options of heating the structure (preheated only the beam, or only one or another plate, or co-heating). The contact interaction between the elements yield a high nonlinearity effects of the studied system.

3. The proposed algorithm allows to solve constructively-nonlinear stationary problems.

4. Chaotic vibrations have been detected and studied.

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