Theory of size-dependent physically nonlinear Euler-Bernoulli beams in an aggressive medium with taking into account the coupling of temperature and deformation fields

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Abstract: In this paper a size-dependent theory of physically nonlinear beams described by the kinematic theory of the first approximation is constructed. The basis of the developed theory is the moment theory of elasticity. The physical nonlinearity is taken into account following the Birger method of variable elasticity parameters, according to which the physical parameters of the beam material are not constant, but are functions of coordinates and a stress-strain state of the structure. The input partial differential equations of motion are obtained from the Hamilton variation principle. Equations take into account the relationship between deformation and temperature fields, material dependence on temperature and the aggressive medium properties in which the beam is located. The governing equations are nonlinear of the hyperbolic-parabolic type and exhibit different dimension. The equation of beam motion is one-dimensional, and the equation of thermal conductivity is two-dimensional. It means that no any restrictions for temperature distribution over beam thickness are employed. A calculation algorithm with nested iterations is developed in order to solve the problem in a reliable and validated way.

1. Introduction

The study of the effects associated with corrosion, wear and dynamic thermal force phenomena on the behavior of mechanical systems is an extremely complex but promising direction of the scientific research. The reorganization of the dynamic system modes may depend not only on the change in the parameters of the force (mechanical) loading, but also on the change in the thickness of the structure due to the action of the corrosive medium, as well as it is influenced by the temperature effects. Interest in such tasks is related to a need to develop mechanical structures capable of operating in corrosive environments under conditions of uneven non-stationary heating (for example, in aviation and rocketry industries, gyroscopes fabrication, nuclear reactor protection systems, micromechanical systems, etc.). Engineering practice constantly requires increasing the accuracy of mathematical models describing the vibrations of structural elements. Investigation of the effect of corrosion and wear on the vibrations of mechanical systems in the form of beams located in temperature fields is an actual and interesting problem.

Thin-walled and thick-walled spherical shells subjected to mechanical-chemical of corrosion under the action of external and internal pressure are considered in the series of papers Pronina and Sedova [1-4]. Analysis and comparison of the results obtained on the basis of analytical solutions is done in the works. A mathematical model of uniform corrosion of a thick-walled long flexible cylindrical tube subjected to the internal and external pressure at different temperatures has been constructed in reference [5]. The influence of corrosion is taken into account according to the Dolinsky model [7] with an exponential decay in time. The problems of calculating the tensile rod being inhomogeneous along its length, taking into account corrosion wear, and using geometric nonlinear theory, are considered in [6]. The necessity of taking into account the nonlinearity in the problems under consideration is justified in this paper. Fridman solved the problem of determining the dimensions of the cross-section of the truss elements of the ring section constructions (for a given period of their operation), subject to corrosion, using the Dolinsky model. The influence of two-sided and one-sided corrosion on the frequency of natural vibrations of freely supported plates has been studied in [8]. With the help of the finite element analysis, the influence of the corrosion degree on the value of the natural frequency and on the bending shape of a plate has been investigated. The papers [9, 10] are devoted to the study of the loss of stability of thin-walled cylindrical pipes (circular and non-circular cross section) of the Kirchhoff-Love model. The pipes are subjected simultaneously to the action of transverse compression forces and uniform unilateral corrosion on the outside or from the inside. The critical time of loss of stability of pipes has been found. Also, the authors considered the problems of stability loss of thin-walled spherical shells [11, 12] under the influence of external pressure and internal corrosion in temperature fields. It was shown that an increase in temperature leads to an increase in the corrosion rate. In the papers [13-17] it has been shown that to obtain more accurate results it is necessary to take into account the coupling of the temperature and deformation fields.

In recent decades, the interest in micro-dimensional mechanical structures has increased since in most cases they are the most important elements in MEMS [18, 19].

Many properties of the elastic bodies are associated with the characteristic dimensions, these properties are different [20-22]. Despite a large number of works on this subject, where linear models are used for numerical analysis, we note that it is necessary to take into account the influence of nonlinearity on the dynamics of micro and nano mechanical systems [23]. The resolving linear equations, initial and boundary conditions for the size-dependent Euler-Bernoulli model (the first-approximation model) have been obtained in [24, 25] using the modified moment theory of elasticity. The influence of the size parameter on the static deformation and the magnitude of the natural frequencies have been investigated.

For a static problem, a linear equation of the fourth order for longitudinal displacement is considered. The natural frequencies are investigated for small deflections using a linear equation of the

6th order for the function of deflection. To reduce the partial differential equations to the ordinary differential equations with respect to time, the Bubnov-Galerkin method has been empoyed in the first approximation. In reference [26], the equations for the geometrically nonlinear Euler-Bernoulli beam have been obtained on the basis of the Kármán relations has been used. To get a numerical solution, the Bubnov-Galerkin method in the first approximation. The effect of the size coefficient on the value of the natural frequency of nonlinear vibrations has been investigated.

The linear problems for the determination of natural frequencies and the static problems for investigating the influence of dimension-dependent parameters are considered in many papers. The effect of corrosion wear along with the temperature effect was considered for macro-dimensional mechanical systems. It is necessary to study in more detail the nonlinear deformations of size-dependent beams under the influence of static and dynamic loads, taking into account mechano-chemical corrosion and the related problem of thermodynamics. To study the dynamics of size-dependent beams, it is necessary to involve the apparatus of nonlinear dynamics on the basis of Fourier analysis and wavelet spectra, the phase portraits, the Poincaré sections, the change of the largest Lyapunov exponent (LLe) in time, the autocorrelation functions, amongst others [27-31]. The mentioned problems have been analysed with an account of three types of nonlinearity: physical, geometric and constructive (contact interaction in time). However, in these papers, the results have been obtained on the basis of the classical theory of elasticity, without considering the size-dependent behavior of structures [32-36].

At the moment, there are no mathematical models of vibrations of size-dependent beam structures including effects of corrosion wear, temperature and strain field connectivity, physical and geometric nonlinearity. In this paper we consider the interplay of all factors on the example of the Euler-Bernoulli beam.

2. Main hypotheses and assumptions

A mathematical model of non-linear vibrations of a beam of variable thickness under the influence of a normal distributed load is derived.

We make the following assumptions about the beam geometry, the material properties and the operating conditions for formulate the mathematical models: 1) the Euler-Bernoulli hypothesis [37]; 2) the inertia of rotation of beam elements is not taken into account; 3) external forces do not change their direction when the beam is deformed; 4) the longitudinal size of the beam considerably exceeds its lateral size; 5) to describe the size-dependent properties of the system, the modified momentum theory of elasticity is employed [38]; 6) the geometric nonlinearity is taken into account in the form of Kármán [39]; 7) the physical nonlinearity is taken into account on the basis of the Bierger's variable elasticity method [40,41]; 8) normal stresses in the direction of the normal to the middle surface can be neglected in comparison to the main stresses. Basic stresses mean normal and tangential stresses in the middle surface itself and in layers parallel to it; 9) the influence of corrosion wear is taken into account according to the Dolinsky model; it is assumed that the corrosion rate depends linearly on the maximum stress and decays exponentially with time [42]; 10) there are no restrictions on the propagation of temperature over the thickness of the beams, that is, two-dimensional heat conduction equations are considered; 11) we consider isotropic homogeneous beams of variable thickness; 12) dissipative systems are considered.

3. Employment the moment theory of elasticity for a beam

In the modified couple stress based gradient theory [38], the potential deformation energy U in an elastic body occupying the domain $\Omega = \{0 \le x \le a; 0 \le y \le b;\ -h \le z \le h\}$, for infinitely small deformations is $U = \frac{1}{2}\int_{\Omega} (\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dv$, where $i,j = \overline{x,y,z}$; $\varepsilon_{ij} -$ the components of the deformation tensor and $\chi_{ij} -$ are the components of the symmetric tensor of the gradient of curvature $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \sum_{m=1}^{3}\frac{\partial u_m}{\partial x_j}\frac{\partial u_m}{\partial x_j}\right)$, $\chi_{ij} = \frac{1}{2}\left(\frac{\partial \theta_i}{\partial x_j} + \frac{\partial \theta_i}{\partial x_i}\right)$, $\theta_i = \frac{1}{2}(rot(u))_i$. Here, u_i represent the components of the displacement vector u, θ is an infinitesimal rotation vector with the components. θ_i and δ_{ij} are the Kronecker symbols. For a linear isotropic elastic material, the stresses caused by the kinematic parameters included in the expression for the energy density of deformation are determined by the following state equations [38]: $\sigma_{ij} = \lambda\varepsilon_{mm}\delta_{ij} + 2\mu\varepsilon_{ij}$, $m_{ij} = 2\mu l^2\chi_{ij}$, where σ_{ij}, ε_{ij}, m_{ij} and χ_{ij} denote the components of the classical stress tensor σ, the strain tensor ε, the deviator part of the symmetric moment tensor of higher order m and the symmetric part of the curvature tensor χ, respectively; $\lambda = \frac{Ev}{(1+v)(1-2v)}$, $\mu = \frac{E}{2(1+v)}$ are the Lamé parameters; $E(x,y,z)$, $v(x,y,z)$ are the Young's modulus and Poisson's ratio, respectively; $\rho(x,y,z)$ is the density of the beam material; e_i is the intensity of deformation. The parameter l, appearing in the higher order moment m_{ij}, is an additional independent material length parameter associated with the symmetric rotational gradient tensor.

In this paper, the mathematical model of vibrations of a size-dependent geometrically and physically nonlinear beam exposed to unilateral corrosion wear will be constructed on the basis of the Euler-Bernoulli model (the hypothesis of the first approximation). The model reflects only the bending of the beam without turning and curving the cross section. The beam occupies the domain $\Omega = \{0 \le x \le a; 0 \le y \le 1;\ -h \le z \le h - \delta\}$, where $\delta = \delta(x,t)$ is the negative thickness increment function, due to corrosive wear. The displacement of an arbitrary point in a certain layer of a beam parallel to the median line away from it by a distance $z \ne 0$ will have the form: $u_x(x,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x}$, $u_y(x,t) = 0$, $u_z(x,t) = w(x,t)$, where $u(x,t)$ is the axial displacement of an arbitrary point of the middle line of the beam, and $w(x,t)$ is the transverse deviation.

We consider the inhomogeneous theory of elasticity. The physical constants are assumed to depend on the coordinates and the intensity of the deformations. We shall carry out the model studies, taking into account the physical nonlinearity with the dependence $E(x, y, z, e_i)$ on the coordinates, using the deformation theory of plasticity and employing the Bierger variable elasticity parameter [43], as is done for flexible physically nonlinear shells [44].

We consider an isotropic inhomogeneous rectilinear beam, under the action of the distributed transverse intensity force $q(x, t)$. The median line is located in the plane $z = 0$. Taking into account the Euler-Bernoulli hypothesis, we can write the expression for the deformation of the elongation in the x direction, taking into account the geometric nonlinearity according to the von Kármán model, the influence of the temperature field, the variable beam thickness and the corrosion wear:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2} w \frac{\partial^2 h}{\partial x^2} - z\frac{\partial^2 w}{\partial x^2} - \alpha_t T(x, z, t). \tag{1}$$

The total deformation of an arbitrary point on a layer located from the median line by a distance *z*, where ε_{xx} is composed of the deformation of the median line $\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2} w \frac{\partial^2 h}{\partial x^2}$, the deformation of the bend $-z\frac{\partial^2 w}{\partial x^2}$ and the temperature deformation $-\alpha_t T(x, z, t)$. Here $h = h(x)$ is the law of the beam thickness variation along its length, α_t is the coefficient of thermal expansion of the beam material, and $T(x, z, t)$ is the function of the temperature field.

We write the expressions for the nonzero components θ, the symmetric part of the curvature tensor χ, the normal stress σ_{xx} and the nonzero components of the higher order moments:

$$\theta_2 = -\frac{\partial w}{\partial x}, \chi_{12} = \chi_{21} = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2},$$

$$\sigma_{xx} = (\lambda + 2\mu)\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2} w \frac{\partial^2 h}{\partial x^2} - z\frac{\partial^2 w}{\partial x^2} - \alpha_t T(x, z, t)\right), m_{12} = m_{21} = -\mu l^2 \frac{\partial^2 w}{\partial x^2}. \tag{2}$$

4. Variational formulation of the problem: mathematical modelling of the flexible physically nonlinear and size-dependent Euler-Bernoulli beams

The potential energy *U*, obtained on the basis of the addition of higher-order forces, the kinetic energy *K*, the external work *W* associated with the distributed forces and energy dissipation will take the following form:

$$U = \frac{1}{2}\int_0^a \int_{-h}^{h-\delta} (\sigma_{11}\varepsilon_{11} + 2m_{12}\chi_{12}) dz dx = \tag{3}$$

$$= \frac{1}{2}\int_0^a\int_{-h}^{h-\delta}\left((\lambda+2\mu)\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}-z\frac{\partial^2 w}{\partial x^2}-\alpha_t T(x,z,t)+\right.\right.$$

$$\left.\left.\mu l^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right)^2\right)dzdx,$$

$$K=\frac{1}{2}\rho\int_0^a\int_{-h}^{h-\delta}\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dzdx\ ,$$

$W=\int_0^a(q(x,t)w+\varepsilon\frac{\partial w}{\partial t}w)dx$, $\varepsilon -$ is the dissipation coefficient.

The equations of beams motion, as well as the boundary and initial conditions, are obtained from the Hamilton-Ostrogradskiy principle. According to this principle, a comparison is made of the close motions that lead the system of material points from the initial position at time t_0 to the final position at time t_1. For true motions, the condition: $\int_{t_0}^{t_1}(\delta K-\delta\Pi+\delta W)dt=0$ should be satisfied. Varying over the variables u, w, integrating by parts, and equating the expressions for δu and δw to zero, we obtain the resolving equations of motion and add to the resulting system the equations for corrosive wear:

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)C_{00}+\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)dC_{00}-$$

$$-\frac{\partial^3 w}{\partial x^3}C_{10}-\frac{\partial^2 w}{\partial x^2}dC_{10}+dN_t=\frac{\gamma(2h-\delta)}{2gp_1}\frac{\partial^2 u}{\partial t^2}; \qquad (4)$$

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)\frac{\partial w}{\partial x}C_{00}+\left[\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)C_{00}-\right.$$

$$\left.\frac{\partial^2 w}{\partial x^2}C_{10}-N_t\right]\frac{\partial^2 w}{\partial x^2}+\left[\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)dC_{00}-\frac{\partial^3 w}{\partial x^3}C_{10}-\frac{\partial^2 w}{\partial x^2}dC_{10}+dN_t\right]\frac{\partial w}{\partial x}+$$

$$\left(\frac{\partial^3 u}{\partial x^3}+\left(\frac{\partial^2 u}{\partial x^2}\right)^2+\frac{\partial u}{\partial x}\frac{\partial^3 u}{\partial x^3}+\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x^3}-\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 h}{\partial x^2}-\frac{\partial w}{\partial x}\frac{\partial^3 h}{\partial x^3}-\frac{1}{2}w\frac{\partial^4 h}{\partial x^4}\right)C_{10}+2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\right.$$

$$\left.\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)dC_{10}+\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)d^2C_{10}-\frac{\partial^4 w}{\partial x^4}\left(C_{20}+\right.$$

$$\left.\frac{p_2}{2p_1}l^2C_{00}\right)-2\frac{\partial^3 w}{\partial x^3}dC_{20}-\frac{\partial^2 w}{\partial x^2}d^2C_{20}+d^2M_t=\frac{1}{p_1}\left(\frac{\gamma(2h-\delta)}{2g}\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{\gamma(2h-\delta)}{2g}\frac{\partial w}{\partial t}-q\right); \qquad (5)$$

$$\frac{\partial\delta}{\partial t}=\left(\delta_0+K\left[(\lambda+2\mu)\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}-z\frac{\partial^2 w}{\partial x^2}-\alpha_t T(x,z,t)\right)\right]\right)\exp(-bt); \qquad (6)$$

where: $C_{00}=\int_{-h(x)}^{h(x)-\delta(x)}E(x,z,e_x)dz$, $C_{10}=\int_{-h(x)}^{h(x)-\delta(x)}E(x,z,e_x)zdz$,

$C_{20}=\int_{-h(x)}^{h(x)-\delta(x)}E(x,z,e_x)z^2dz$, $N_t=\alpha_t\int_{-h(x)}^{h(x)-\delta(x)}E(x,z,e_x)T(x,z,t)dz$,

$M_t=\alpha_t\int_{-h(x)}^{h(x)-\delta(x)}E(x,z,e_x)T(x,z,t)zdz$, $p_1=\frac{1-v}{(1+v)(1-2v)}$, $p_2=\frac{1}{2(1+v)}$, where γ is the specific gravity of the beam material; g is the acceleration of free fall. The effect of corrosion wear is taken into account according to the Dolinsky model, and it is assumed that the corrosion rate linearly depends on the maximum stress and decays exponentially with time [42]. The constants K and b are determined experimentally [45], and δ_0 is the initial corrosion rate.

No restrictions are imposed on the propagation of temperature over the thickness of the beam, and therefore a two-dimensional heat equation for a nonstationary field is considered, taking into account the coupling of deformation fields and temperatures:

$$\frac{C_0}{T_0}\frac{\partial T}{\partial t}-\frac{\lambda}{T_0}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=\frac{E\alpha_t}{1-\nu}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}-z\frac{\partial^2 w}{\partial x^2}-\alpha_t T(x,z,t)\right), \quad (7)$$

where: C_0 is the specific heat of the beam material; T_0 is the beam temperature in the initial undeformed state.

We add initial conditions to the systems of differential equations (4)-(6):

$$w(x,t)=\varphi_{30}(x);\ u(x,t)=\varphi_{10}(x);\ T(x,z,t)=\varphi_4(x,z);\ \delta(x,t)=\varphi_5(x);\ t=0;$$
$$\frac{\partial w(x,t)}{\partial t}=\psi_{30}(x);\ \frac{\partial u(x,t)}{\partial t}=\psi_{10}(x);\ t=0. \quad (8)$$

As well as one of the boundary conditions to the system of equations of motion (4)-(5) is taken, and to the heat conduction equation (7) one of the conditions I, II or III type are employed:

$$w(x,t)=u(x,t)=\frac{\partial w(x,t)}{\partial x}=0, x=0, x=l;$$
$$w(x,t)=u(x,t)=\frac{\partial^2 w(x,t)}{\partial x^2}=0, x=0, x=l;$$
$$w(x,t)=u(x,t)=0, x=0, x=l, \frac{\partial w(0,t)}{\partial x}=\frac{\partial^2 w(l,t)}{\partial x^2}=0; \quad (9)$$
$$w(x,t)=u(x,t)=\frac{\partial w(x,t)}{\partial x}=0, x=0, x=l, M_x(x,t)=N_x(x,t)=0.$$

Here $\varphi_{10}(x), \varphi_{30}(x), \psi_{10}(x), \psi_{30}(x), \varphi_4(x,z), \varphi_5(x)$ are known functions that determine the initial state of the beam. The equation of motion of the beam element contains a fourth-order derivative, which is extremely important in proving the existence of a solution of the studied governing equations and the convergence of various methods for their solution.

The system of governing PDEs supplemented by boundary and initial conditions is reduced to the counterpart dimensionless form using the following variables:

$$x=a\bar{x}, \delta=h_0\bar{\delta}, \delta_0=\frac{\alpha}{h_0}\overline{\delta_0}, b=b_0\bar{b}, h=h_0\bar{h}, l=h_0\bar{l}, w=h_0\bar{w},$$
$$u=\frac{h_0^2}{a}\bar{u}, q=\frac{h_0^4E_0}{a^4}\bar{q}, t=\frac{h_0^2}{\alpha}\bar{t}, E=E_0\bar{E}, \varepsilon=\frac{\alpha}{h_0^2}\bar{\varepsilon}, \lambda=\frac{\alpha}{h_0}, T=\frac{h_0^2}{a^2\alpha_{t0}}\bar{T}, \alpha_t=\alpha_{t0}\overline{\alpha_t}, \quad (10)$$
$$C_{00}=E_0h_0\overline{C_{00}}, C_{10}=E_0h_0^2\overline{C_{10}}, C_{20}=E_0h_0^3\overline{C_{20}}.$$

The system of equations of motion (4-5), corrosion wear (6), and the heat equation (7), with allowance for the dimensionless parameters, will have the following form (bars over the non-dimensional quantities are omitted):

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)C_{00}+\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)dC_{00}-$$

$$-\frac{\partial^3 w}{\partial x^3}C_{10}-\frac{\partial^2 w}{\partial x^2}dC_{10}+dN_t=\frac{K}{\lambda^2}\frac{(2h-\delta)}{2p_1}\frac{\partial^2 u}{\partial t^2}; \quad (11)$$

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)\frac{\partial w}{\partial x}C_{00}+\left[\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)C_{00}-\right.$$
$$\left.\frac{\partial^2 w}{\partial x^2}C_{10}-N_t\right]\frac{\partial^2 w}{\partial x^2}+\left[\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)dC_{00}-\frac{\partial^3 w}{\partial x^3}C_{10}-\frac{\partial^2 w}{\partial x^2}dC_{10}+dN_t\right]\frac{\partial w}{\partial x}+$$
$$\left(\frac{\partial^3 u}{\partial x^3}+\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial u}{\partial x}\frac{\partial^3 u}{\partial x^3}+\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x^3}-\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 h}{\partial x^2}-\frac{\partial w}{\partial x}\frac{\partial^3 h}{\partial x^3}-\frac{1}{2}w\frac{\partial^4 h}{\partial x^4}\right)C_{10}+2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\right.$$
$$\left.\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial^2 h}{\partial x^2}-\frac{1}{2}w\frac{\partial^3 h}{\partial x^3}\right)dC_{10}+\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}\right)d^2C_{10}-\left(C_{20}+\right.$$
$$\left.\frac{p_2}{2p_1}l^2\right)\frac{\partial^4 w}{\partial x^4}-2\frac{\partial^3 w}{\partial x^3}dC_{20}-\frac{\partial^2 w}{\partial x^2}d^2C_{20}+d^2M_t=\frac{1}{p_1}\left(\frac{K(2h-\delta)}{2}\frac{\partial^2 w}{\partial t^2}+\varepsilon\frac{K(2h-\delta)}{2}\frac{\partial w}{\partial t}-q\right); \quad (12)$$

$$\frac{\partial \delta}{\partial t}=\left(\delta_0+Pp_1\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}-z\frac{\partial^2 w}{\partial x^2}-\alpha_t T(x,z,t)\right)\right)\exp(-Bt); \quad (13)$$

$$\frac{\partial T}{\partial t}-L\left(\frac{\partial^2 T}{\partial x^2}+\lambda^2\frac{\partial^2 T}{\partial z^2}\right)=D\alpha_t\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}w\frac{\partial^2 h}{\partial x^2}-z\frac{\partial^2 w}{\partial x^2}-\alpha_t T(x,z,t)\right). \quad (14)$$

Here $\frac{\lambda_g h_0^2}{a^2 C_0 \alpha}=L$, $\frac{E_0\alpha_{t0}^2 T_0}{(1-\nu)C_0}=D$, $KE_0\frac{h_0^3}{a^2\alpha}=P$, $b_0\frac{h_0^2}{\alpha}=B$, $\frac{\gamma}{g}\frac{a^4\alpha^2}{h_0^2 E_0 h_0^4}=K$ are dimensionless physical and geometric parameters; λ_g is the coefficient of thermal conductivity of the beam material, and α is the thermal diffusivity of the beam material.

5. Methods of solution

The finite difference method is used for solving the resulting system of equations (5)-(8). When integrating the equations of motion with boundary and initial conditions, a uniform grid with the number of nodes n along the length and m along the thickness has been superimposed on the beam. Partial derivatives with respect to spatial coordinates, to improve the accuracy of the design scheme, have been replaced by central finite-difference approximations:

$$\Lambda_x(\square_i)=\frac{(\square)_{i+1}-(\square)_{i-1}}{2c},\ \Lambda_{x^2}(\ \ _i)=\frac{(\square)_{i+1}-2(\square)_i-(\square)_{i-1}}{c^2},$$
$$\Lambda_{x^4}(\square_i)=\frac{(\square)_{i+2}-4(\square)_{i+1}-6(\square)_i-4(\square)_{i-1}+(\square)_{i-2}}{c^4},\ \Lambda_{x^2}(\square_{i,k})=\frac{(\square)_{i+1,k}-2(\square)_{i,k}-(\square)_{i-1,k}}{c^2}, \quad (15)$$
$$\Lambda_{z^2}(\square_{i,k})=\frac{(\square)_{i,k+1}-2(\square)_{i,k}-(\square)_{i,k-1}}{p^2},\ i=\overline{0,n},\ k=\overline{0,m},$$

where: $c-$ is step in the spatial coordinate x, $c=\frac{1}{(n-1)}$; step along the thickness of the beam is $p=\frac{1}{(m-1)}$.

The resulting system of the ordinary differential equations of the second order with the corresponding boundary and initial conditions reduces to a system of ordinary differential equations of the first order. The obtained system is solved by the Runge-Kutta method of the fourth order of accuracy. The choice of the method is due to the fact that the results obtained by the methods of the 4th and 6th order of accuracy completely coincide, but the counting time for Runge-Kutta of the 4th order is half the size of the 6th order Runge-Kutta method [46].

At each step in time for the node x_i the value of the function $\delta(x_i)$, which corresponds to the change of the thickened beams due to corrosion, the values of the stiffnesses C_{00}, C_{10}, C_{20} and their derivatives, as well as the temperature moments and stresses, are calculated. After that, the obtained parameters are substituted into the equations of motion. The thickness of the beam $h(x_i)$ is recalculated taking into account the corrosive component from the previous layer. Based on the displacement and deflection of the beam obtained from the equations of motion, a total deformation is calculated for each points $\varepsilon_{11}(x_i, z_k)$. Substituting it into the expression for the corrosion function δ, we obtain its new value on the time layer under consideration. Substituting the values of the total deformation into the heat equation, we obtain the values of the temperature field function $T^j(x_i, z_k)$ at each point of the grid. Integrating over the thickness, we will have T_i in the middle line of the beam, which will allow us to obtain the values of the temperature moments and stresses.

6. Conclusions

In the presented work, the mathematical model of vibrations of the Euler-Bernoulli size-dependent beam with the taking into account the corrosive wear, temperature and strain field connectivity, physical and geometric nonlinearity has been worked out for the first time. The calculation algorithm is under development.

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