Gyroscopic forces and asymptotic stability for mechanical systems with partial energy dissipation

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Abstract: We study the stability problem for autonomous non - conservative mechanical system in presence of potential, gyroscopic, and dissipative forces. The matrix of dissipative forces is semi-positive, so Kelvin - Chetaev theorems cannot be applied. The significance of gyroscopic forces (GF) and their contribution to the overall phenomenon is discussed. The fact that energy dissipation is incomplete is essential, because the influence of gyroscopic terms in this case may be significantly different from the full dissipation case. It is shown that this influence may be both positive and negative (there are some sets in space of parameters where the asymptotic stability of the motion is broken). As an example, the problem of passive stabilization of permanent rotations of Lagrange gyroscope is considered. It is proved that adding a dashpot to gyro with stretched inertia ellipsoid stabilizes its permanent rotations with the exception of some "critical" values. The last may be found analytically from special conditions.

1. Introduction

In 1879 W. Thomson and P. Tait [1] put their attention on the fact that equations of motion of the system, in which the gyroscopes are present, contain terms linear with respect to velocities with a skew-symmetric coefficient matrix. When these terms are treated as forces, then their work on the actual displacement of the system will be zero $\sum_{i=1}^{n} \Gamma_i \, dq_i = 0$. This property was accepted by Thomson and Tait for the general definition of gyroscopic forces and, using it, they proved several theorems on the stability of the motion of gyroscopic systems. Gyroscopic forces can be found not only in systems containing gyroscopes, but also in various mechanical, electrical and other systems in which gyroscopes are absent. Therefore, for the systems of the most diverse physical nature, one can draw far-reaching analogies that can be used in various constructions. Non-conservative systems with both dissipative and gyroscopic force are widely presented in numerous publications from physical viewpoint [2 - 6], as well as for application purposes [7, 8].

Let the equilibrium position of the conservative mechanical system be unstable. Is it possible to stabilize it by adding dissipative forces, i.e. to select the force in such a way that the equilibrium position which is unstable in the presence of potential forces only becomes stable or asymptotically stable? The answer to this question is negative. Also it is known that such an equilibrium position can be stabilized by a certain combination of dissipative and circulation forces, but can this goal be achieved in the absence of the latter? In the case when the dissipation is complete (the matrix D is positive) the answer to this question is given by classical Kelvin-Chetaev [1, 9, 10] theorems:

Theorem 1. If the equilibrium of the mechanical system is stable under the action of potential forces only, it becomes asymptotically stable while adding dissipative forces with full dissipation.

Theorem 2. If the isolated equilibrium is unstable under the action of potential forces only, it cannot be stabilized by adding arbitrary dissipative forces with full dissipation.

Theorem 3. If the isolated equilibrium is unstable under the action of potential forces only, it remains unstable while adding arbitrary gyroscopic forces and dissipative forces with full dissipation.

At the same time, concerning Theorem 1, as noted in a number of works (see, for instance [11 - 13]), the requirement that the matrix characterizing dissipative forces should be positive is in some cases superfluous. In particular, a semi-positive matrix as a rule (with the exception of a set of measure zero) makes the stable equilibrium position of the conservative system asymptotically stable. However, how important is the influence of the gyroscopic forces in this case? Does the statement of the theorems 2, 3 extend to the case of partial energy dissipation?

In this paper, we would like to draw attention to two points: 1) In contradiction to Theorem 3, partial dissipative forces can make the gyroscopically stabilized motion of the system asymptotically stable; 2) Gyroscopic forces can "spoil" the asymptotic stability of the system. Namely, a motion that is asymptotically stable with potential forces and partial dissipative forces can become marginally stable when the gyroscopic forces are added.

2. Main results

We consider the motion of a holonomic mechanical system subject to stationary, ideal constraints. The position of this system is specified by n positional and m cyclic generalized coordinates. If such a system has stationary motion, then stability problem may be solved by consideration the linearized system which may be presented in the following form

$$\boldsymbol{M}\ddot{\boldsymbol{\xi}} + \boldsymbol{B}\dot{\boldsymbol{\xi}} + \boldsymbol{K}\boldsymbol{\xi} = \mathbf{0}, \tag{1}$$

where $\boldsymbol{M}, \boldsymbol{K}, \boldsymbol{B}$ are square real matrices (two first of them are symmetric and positive), $\boldsymbol{B}$ is semi-positive and always may be separated on symmetric (dissipative) and skew-symmetric (gyroscopic) components $\boldsymbol{B} = \boldsymbol{D} + \boldsymbol{G}$, $\boldsymbol{\xi} \in \mathbb{R}^n$.

Below we use the block notation for square matrix $\boldsymbol{P}$ of $s+l$ order in the form

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{pmatrix},$$

where $\boldsymbol{P}_{11}, \boldsymbol{P}_{22}$ are square matrices of s and l orders respectively, and $\boldsymbol{P}_{12},\ \boldsymbol{P}_{21}$ stand for the corresponding rectangle matrices. Also we split the vector $\boldsymbol{\xi}$ on sub-vectors

$$\boldsymbol{\xi} = col(\boldsymbol{x}, \boldsymbol{y}), \quad \boldsymbol{x} \in \mathbb{R}^s, \quad \boldsymbol{y} \in \mathbb{R}^l.$$

We suppose that matrix $\boldsymbol{D} = \boldsymbol{0}_s \bigoplus diag(d_1, d_2, \cdots, d_l)$. In other words the right lower block $\boldsymbol{D}_{22}$ is diagonalized, and three other blocks are zero matrices. Matrix $\boldsymbol{D}_{22}$ is positive. Similarly, we denote differential operators

$$L = \boldsymbol{M}\frac{d^2}{dt^2} + \boldsymbol{B}\frac{d}{dt} + \boldsymbol{K}, \quad L_{11} = \boldsymbol{M}_{11}\frac{d^2}{dt^2} + \boldsymbol{G}_{11}\frac{d}{dt} + \boldsymbol{K}_{11},$$

and the corresponding lambda-matrices (matrix polynomials)

$$\boldsymbol{\Lambda}(\lambda) = \boldsymbol{M}\lambda^2 + \boldsymbol{B}\lambda + \boldsymbol{K}, \quad \boldsymbol{\Lambda}_{11}(\lambda) = \boldsymbol{M}_{11}\lambda^2 + \boldsymbol{G}_{11}\lambda + \boldsymbol{K}_{11}. \tag{2}$$

Let λ_0 be some eigenvalue of $\boldsymbol{L}_{11}$, and $\boldsymbol{\beta}_{10}-$ the corresponding eigenvector, i.e.

$$\boldsymbol{\Lambda}_{11}(\lambda_0)\boldsymbol{\beta}_{10} = \boldsymbol{0}_s.$$

Here $\boldsymbol{0}_s$ means the matrix-column with s zero elements. Introduce the equality

$$\boldsymbol{\Lambda}_{21}(\lambda_0)\boldsymbol{\beta}_{10} = \boldsymbol{0}_l. \tag{3}$$

For our purposes we shall use the following theorem [14]:

Theorem 4. Let us consider a mechanical system which motion equations are described by (1) and suppose that none of the eigenvectors of operator $\boldsymbol{L}_{11}$ satisfies condition (3). Then adding to system an arbitrary dissipative force, which provides full dissipation on $\dot{\boldsymbol{y}}$ leads to the following results:

I) If all eigenvalues of matrix $\boldsymbol{K}$ are positive, then equilibrium of (1) becomes asymptotically stable. Stability is exponential and uniform.

II) If matrix $\boldsymbol{K}$ has some negative eigenvalues – then equilibrium is unstable, even if it was stabilized before by gyroscopic forces. Among particular solutions of the system at least one has negative Lyapunov characteristic number.

Comparing with the statements of theorems 1 - 3, in case of incomplete dissipation results of Kelvin - Chetaev theorems mostly persist, excluding some special relations between quantitative values that characterize the forces (some surfaces in space of mechanical parameters). A way of finding these relations is proposed by formulas (3). The key difference, we believe, is that case of full dissipation allows to solve the problem in qualitative manner – by analysis only potential (or potential and gyroscopic) forces, and conclusion does not depend on quantitative nature of them. In other words, only signs of matrix eigenvalues are important, not their exact values or connections between them. When dissipation is partial, this feature is lost, as it follows from (3). In particular, if matrix D is positive and system (1) is asymptotically stable or unstable, varying the GF cannot change this. The case of partial energy dissipation is somewhat opposed to this circumstance. More precisely, this property is inherent only with respect to "pure dissipative" component $\boldsymbol{y}$ of state vector, and variation of the other gyroscopic terms related to $\boldsymbol{x}-$ component can change the result.

2.1. Example 1

Let system (1) is given with matrices

$$\boldsymbol{M} = \begin{pmatrix} 5 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{G} = \begin{pmatrix} 0 & g_1 & 0 & g_2 \\ -g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_3 \\ -g_2 & 0 & -g_3 & 0 \end{pmatrix}, \quad \boldsymbol{K} = \begin{pmatrix} 4 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix},$$

$$\boldsymbol{D} = h \ \mathrm{diag}(1, 1, 0, 0), \tag{4}$$

and g_1, g_2, g_3 are unknown parameters.

Suppose that there exist λ_0 and $\boldsymbol{\beta}_0$ which satisfy (3). This means that all minors of second order of the matrix

$$\boldsymbol{\Lambda}^\star = \begin{pmatrix} 2\lambda^2 + 2 & 0 & 2\lambda^2 + 2 & -g_3\lambda \\ g_2\lambda & 0 & g_3\lambda & \lambda^2 + 3 \end{pmatrix}^T$$

are equal to zero. In fact, otherwise the rank of $\boldsymbol{\Lambda}^\star$ is maximal, and (3) cannot take place.

The rank of matrix $\boldsymbol{\Lambda}^\star$ is less then 2 if and only if all minors of second order are equal to zero. This fact leads to restrictions

$$g_2 = g_3, \tag{5}$$

and

$$f(\lambda) = 2(\lambda^2 + 1)(\lambda^2 + 3) + g_3^2\lambda^2 = 0. \tag{6}$$

For any value of g_3 the polynomial $f(\lambda)$ has purely imaginary roots only, hence if $g_2 \neq g_3$, then $rank\ \boldsymbol{\Lambda} = 2$, and, according to theorem, system $MDGK$ is asymptotically stable. However, if (5) holds, then system (1), (4) is marginally stable. Its characteristic polynomial

$$det(\boldsymbol{M}\lambda^2 + (\boldsymbol{D}+\boldsymbol{G})\lambda + \boldsymbol{K}) = \begin{vmatrix} 3\lambda^2 + h\lambda + 2 & g_1\lambda \\ -g_1\lambda & \lambda^2 + h\lambda + 1 \end{vmatrix} \begin{vmatrix} 2\lambda^2 + 2 & g_3\lambda \\ -g_3\lambda & \lambda^2 + 3 \end{vmatrix}$$

has two pairs of purely imaginary roots and four roots with negative real part. The last correspond to variables $x_1, x_2, \dot{x}_1, \dot{x}_2$ – the "direct dissipative part" of state vector, and this does not depend from magnitude of g_1.

Over against, the borderline between marginal stability and asymptotic stability of the system is tied with restriction (5), which determines a set in subspace of system parameters, associated with $\boldsymbol{x}$ being a component of state vector, where asymptotic stability is lost.

3. Passive stabilization of Lagrange's gyroscope permanent rotations

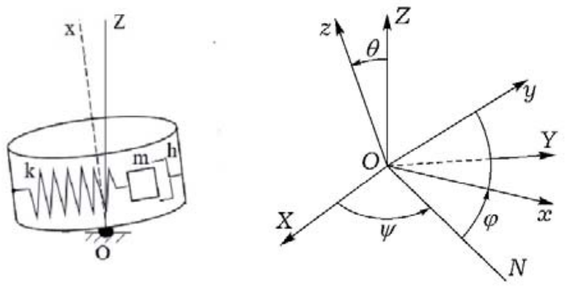

Figure 1. Rigid body with dashpot.

Consider a rigid body with a fixed point O, and introduce it into consideration two coordinate systems: the fixed one $OXYZ$ and system $Oxyz$ which is connected with the body. It is assumed that the body is dynamically symmetric, and the center of mass C of the body belongs to the axis Oz. As generalized coordinates that determine the position of the coordinate system $Oxyz$ with respect to the fixed one, we choose the Euler angles θ, φ, ψ which are introduced in a common way (Fig.1). Inside the body a dashpot is situated which is considered as mass m that can oscillate along the line which is orthogonal to axis of symmetry Ox and intersects it in point O_1. It is connected with carrier by viscoelastic spring with stiffness $\varkappa$ and coefficient of damping $\hbar$. Since the body is dynamically symmetric, we can assume that dashpot axis is collinear to the principal axis of inertia of the body (for example, the second). The corresponding radius vector in the coordinate system connected with the body can be written as

$$\boldsymbol{r}_1 = l_1 \ \boldsymbol{e}_X + \eta \ \boldsymbol{e}_Y = (l_1, \eta, 0),$$

where η is the distance from point O_1.

Taking into account the formula $\boldsymbol{v}_1 = \boldsymbol{\acute{r}}_1 + \boldsymbol{\omega} \times \boldsymbol{r}_1$, where $\boldsymbol{\acute{r}}_1$ is the relative derivative on time (relative velocity) and $\boldsymbol{\omega}$ is the angular velocity of the rigid body, the following expression for kinetic energy of dashpot holds

$$\mathcal{K}_1 = \frac{1}{2} m[\dot{\eta}^2 + 2 \ l_N \dot{\eta}\omega_3 + \eta^2(\omega_1^2 + \omega_3^2) - 2 \ l_N \ \eta\omega_1\omega_2 + l_N^2 \ (\omega_2^2 + \omega_3^2)].$$

Components of angular velocity vector are given by the kinematic Euler relations

$$\omega_1 = \dot{\theta}\cos\varphi + \dot{\psi}\sin\theta\sin\varphi, \ \omega_2 = -\dot{\theta}\sin\varphi + \dot{\psi}\sin\theta\cos\varphi, \ \omega_3 = \dot{\varphi} + \dot{\psi}\cos\theta. \tag{7}$$

The generalized inertia tensor of the system may be written as

$$\widetilde{\boldsymbol{I}} = \begin{pmatrix} I_1 + m\eta^2 & -ml_1 \ \eta & 0 \\ -ml_1 \ \eta & I_2 + ml_1^2 & 0 \\ 0 & 0 & I_3 + m(\eta^2 + l_1^2) \end{pmatrix}. \tag{8}$$

The potential forces are presented by gravitational force and elasticity of the spring. Hence, the potential energy of the system is given by the following formula

$$\Pi = g\sin\theta \ [(Ml + ml_1)\sin\varphi + m\eta \ cos\varphi] + \frac{1}{2}\varkappa\eta^2,$$

where M is the mass of the rigid body, $l = |OC|, \ l_1 = |OO_1|$.

The kinetic energy of the system is as follows

$$\mathcal{K} = \mathcal{K}_0 + \mathcal{K}_1 = \frac{1}{2}\langle\boldsymbol{\omega}, \tilde{\boldsymbol{I}}\boldsymbol{\omega}\rangle + m\langle\boldsymbol{\omega}, \boldsymbol{r}_1 \times \boldsymbol{\acute{r}}_1\rangle + m\boldsymbol{\acute{r}}_1^2 = \frac{1}{2}\sum_{j,s=1}^{4} a_{js} \ \dot{\xi}_j \ \dot{\xi}_s.$$

Here $\boldsymbol{\xi} = (\theta, \varphi, \eta, \psi)^T$, generalized coordinate ψ is cyclic, and

$$a_{11} = I_1 cos^2\varphi + \widetilde{I}_2 sin^2\varphi + 2ml_N\eta sin\varphi \ cos\varphi, \ a_{12} = 0, \ a_{13} = 0,$$

$$a_{14} = sin\varphi\ [(I_1 - \widetilde{I}_2)sin\varphi\ cos\varphi + ml_N\eta(sin^2\varphi - cos^2\varphi)],\ a_{22} = \widetilde{I}_2 + m\eta^2,\ a_{23} = ml_N,$$

$$a_{24} = cos\theta(\widetilde{I}_2 + m\eta^2),\ a_{33} = m,\ a_{34} = ml_N cos\theta,$$

$$a_{44} = I_1 sin^2\theta\ sin^2\varphi + \widetilde{I}_2(sin^2\theta\ cos^2\varphi + cos^2\theta) + m\eta^2 cos^2\theta - 2ml_N\eta sin^2\theta\ sin\varphi\ cos\varphi.$$

Excluding the cyclic velocity

$$\dot{\psi} = \frac{1}{a_{44}}(\beta_\psi - a_{14}\dot{\theta} - a_{24}\dot{\varphi} - a_{34}\dot{\xi}),$$

where β_ψ represents cyclic constant, we can write the following Routh kinetic potential

$$L_R = \frac{1}{2a_{44}}\sum_{j,s=1}^{3}(a_{js}a_{44} - a_{j4}a_{s4})\ \dot{\xi}_j\ \dot{\xi}_s + \frac{\beta_\psi}{a_{44}}\sum_{j=1}^{3}a_{j4}\ \dot{\xi}_j - W, \quad W = \frac{\beta_\psi^2}{2a_{44}^2} + \Pi.$$

Then equations of the motion of mechanical system under study are

$$\frac{d}{dt}\frac{\partial L_R}{\partial \dot{q}_j} - \frac{\partial L_R}{\partial q_j} = Q_j,\ Q = (0, 0, -\hbar\dot{\eta})^T,\ (j = \overline{1,3}). \tag{9}$$

Stationary motions of mechanical system are governed by equality $grad\, W = \mathbf{0}$ or

$$g\cos\theta\ [(Ml + ml_N)\sin\varphi + m\eta\ cos\varphi] - \frac{\beta_\psi^2}{a_{44}^2}\frac{\partial a_{44}}{\partial\theta} = 0,$$

$$g\sin\theta\ [(Ml + ml_N)\cos\varphi - m\eta\ sin\varphi] - \frac{\beta_\psi^2}{a_{44}^2}\frac{\partial a_{44}}{\partial\varphi} = 0,$$

$$gm\eta\sin\theta\ cos\varphi - m\frac{\beta_\psi^2}{a_{44}^2}(\eta\ cos^2\theta - l_N sin^2\theta\ sin\varphi\ cos\varphi) + \varkappa\eta = 0.$$

It is easy to see that the last system has a solution $(\pi/2, \pi/2, 0)$ and equations (9) has equilibrium

$$\theta^0 = \frac{\pi}{2},\ \varphi^0 = \frac{\pi}{2},\ \eta^0 = 0,\ \dot{\theta}^0 = 0,\ \dot{\varphi}^0 = 0,\ \dot{\eta}^0 = 0, \tag{10}$$

which describes permanent rotations of the body with angular velocity β_ψ/I_1 and with "frozen" mass m. To investigate the stability of solution (9) let us introduce the following perturbations

$$\theta = \widetilde{\xi}_1 + \frac{\pi}{2},\ \varphi = \widetilde{\xi}_2 + \frac{\pi}{2}.$$

For our purpose it is sufficient to get linear approximation of (9), i.e. terms of second order from L_R :

$$L_R^{(2)} = \frac{1}{2}[\widetilde{I}_2(\dot{\xi}_1^2 + \dot{\xi}_2^2) + m\dot{\eta}^2] + ml_1\xi_2\eta + \frac{\beta_\psi}{I_1}\{\dot{\xi}_1[(\widetilde{I}_2 - I_1)\widetilde{\xi}_2 + ml_1\eta] - \widetilde{I}_2\dot{\xi}_2\widetilde{\xi}_1 - ml_1\dot{\eta}\xi_1\} +$$

$$+\frac{\beta_\psi^2}{2I_1^2}[(\widetilde{I}_2 - I_1)(\widetilde{\xi}_1^2 + \widetilde{\xi}_2^2) - 2ml_1\widetilde{\xi}_2\eta] + \frac{g}{2}(Ml + ml_1)(\widetilde{\xi}_1^2 + \widetilde{\xi}_2^2) + mg\widetilde{\xi}_2\eta - \frac{1}{2}\varkappa\eta^2.$$

Introducing the dimensionless parameters

$$\xi_3 = \frac{\eta}{l_1},\ \tau = \frac{\beta_\psi}{I_1}t,\ a = \frac{I_2 + ml_1^2}{I_1},\quad p = \frac{ml_1^2}{I_1},\quad \mu = \frac{MglI_1}{\beta_\psi^2},$$

$$\mu_1 = \frac{mgl_1I_1}{\beta_\psi^2},\ h = \frac{\hbar}{\beta_\psi},\ \kappa = \frac{\varkappa I_1}{\beta_\psi^2}, \tag{11}$$

we finally arrive to system (1) with the matrices

$$\boldsymbol{M} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & p \\ 0 & p & p \end{pmatrix},\ \boldsymbol{D} = \mathrm{diag}(0, 0, h),\quad \boldsymbol{G} = \begin{pmatrix} 0 & 2a-1 & 2p \\ -2a+1 & 0 & 0 \\ -2p & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{K} = \begin{pmatrix} a-1-\mu-\mu_1 & 0 & 0 \\ 0 & a-1-\mu-\mu_1 & -1-\mu_1 \\ 0 & -1-\mu_1 & \kappa \end{pmatrix}.$$

To satisfy the requirements of paragraph 1 of theorem 4, matrix $\boldsymbol{K}$ must be positive, and hence the following restrictions are yielded:

$$a - 1 - \mu - \mu_1 > 0,\quad \kappa > \frac{p^2(1+\mu_1)^2}{a-1-\mu-\mu_1}. \tag{12}$$

If a gyro is upstanding (the case of top), parameters μ, μ_1 are positive, and the first inequality (12) requires $a > 1$ ($I_2 + ml_1^2 > I_1$), i.e. the generalized inertia ellipsoid is stretched (rotations around major axis). Also the angular velocity of rotation must be high enough. Then the second inequality (12) gives the lower limit value for stiffness of the spring $\varkappa$.

In order to make sure that stabilization is in effect – solution (10) is asymptotically stable – we have to consider the following matrix

$$\boldsymbol{\Lambda}^\star = \begin{pmatrix} a\lambda^2 + a - 1 - \mu - \mu_1 & -(2a-1)\lambda & -2p\lambda \\ (2a-1)\lambda & a\lambda^2 + a - 1 - \mu - \mu_1 & p(\lambda^2 - 1 - \mu_1) \end{pmatrix}^T.$$

If its rank is equal to 2, the motion is asymptotically stable, if not – it is marginally stable. The last case means that columns of matrix $\boldsymbol{\Lambda}^\star$ are proportional, and therefore

$$2(a\lambda^2 + a - 1 - \mu - \mu_1) = (2a-1)(\lambda^2 - 1 - \mu_1), \tag{13}$$

$$(a\lambda^2 + a - 1 - \mu - \mu_1)^2 + (2a-1)^2\lambda^2 = 0. \tag{14}$$

Expressing λ^2 from (13) and substituting into (14) we have

$$(2a-1)^2[(\mu_1^2+4\mu_1+4)a^2-2(\mu\mu_1+2\mu+2\mu_1+4)a+\mu^2+4\mu+\mu_1+4]=$$

$$=(2a-1)^2\{[(\mu_1+2)a-\mu-2]^2+\mu_1\}.$$

Triangle inequalities for moments of inertia $I_1 < 2I_2$ imply $a > 1/2$, and the system (13) - (14) is inconsistent. Consequently, $rank\ \mathbf{\Lambda}^\star = 2$, and the motion is asymptotically stable. It happens because energy transfer between "pure dissipative" variable ξ_3 and two others occurs, and this transfer with respect to ξ_1 is implemented by GF influence only for any values of gyroscopic terms (without exceptions).

The last feature surprisingly changes when gyroscope is pendent (hanging down). To analyze this case we can add a sign "-" before g in formulas (11), now μ, μ_1 are negative. With this change, if

$$a = \frac{2+\mu \pm \sqrt{-\mu_1}}{2+\mu_1}, \tag{15}$$

then $rank\ \mathbf{\Lambda}^\star = 1$, and the motion is marginally stable. This inference can be easily verified, because there are purely imaginary eigenvalues $\pm\ i\ (1+\sqrt{-\mu_1})$. Equality (15) determines two critical values for angular velocity value ω. Notice that mass, stiffness and viscosity of dashpot don't affect condition (15) – only position of point O_1 (parameter l_1) is essential.

Remark. We proved that inequalities (12) are the sufficient conditions of asymptotic stability of the motion studying. At the same time they give the necessary conditions of asymptotic stability. In fact, if at least one of these inequalities takes the opposite sign, then the matrix $\boldsymbol{K}$ has positive eigenvalue, and according to paragraph 2 of theorem 4 (as $rank\ \mathbf{\Lambda}^\star = 2$) the solution (10) is unstable.

4. Discussion and concluding remarks

In this paper we turned our attention to the role played by gyroscopic forces in stability issues for systems with incomplete energy dissipation. Influence of these forces can seriously differ from the case of full energy dissipation. In particular, some stabilizing effect are possible which are not available in common frames of the Kelvin-Chetaev theorems. As an example the stabilization of symmetrical rigid body rotations is considered. It is shown that energy dissipation in dashpot is conveying to the whole body due to presence of GF, and without it this stabilization is not possible.

References

[1] W. Thomson, P.G. Tait, Treatise on Natural Philosophy, vol. I, part I, New Edition, Cambridge University Press, Cambridge (1879).

[2] S.H. Crandall, The effect of damping on the stability of gyroscopic pendulums, ZAMP Z. angew. Math. Phys. 46, 761-780 (1995).

[3] R. Hryniv, P. Lancaster, Stabilization of gyroscopic systems, ZAMM Z. angew. Math. Mech. 81, 675-681 (2001).

[4] S.A. Agafonov, Stability and motion stabilization of nonconservative mechanical systems, J. Math. Sci. 112, 4419-4497 (2002).

[5] O. Kirillov, Nonconservative stability problems of modern physics, De Gruyter Studies in Mathematical Physics, Berlin (2013).

[6] N.I. Zhinzher, Effect of dissipative forces with incomplete dissipation on the stability of elastic systems, Izv. Ross. Akad. Nauk. MTT 1, 149-155 (1994).

[7] Skliba J., Problem of the gyroscopic stabilizer damping, Applied and Computational Mechanics, 3 (2009), 205212. Puzyryov V., Awrejcewicz J., Asymptotical stability of the motion of mechanical systems with partial energy dissipation, Nonlinear Dynamics, to appear. E.J. Routh, Treatise on the Stability of a Given State of Motion, Particularly, Steady Motion, MacMillan, London, (1877).

[8] Hao Dong, Jing Zeng, Liang Wu, and Huanyun Dai, Analysis of the Gyroscopic Stability of the Wheelset, Shock and Vibration, Volume 2014, Article ID 151625.

[9] N.G. Chetaev, Stability of Motion, Pergamon Press, 1961.

[10] D.R. Merkin, Introduction to the Theory of Stability, Springer, Berlin (1997).

[11] E.E. Zajac, Kelvin - Tait - Chetaev theorem and extensions, Journal of the Astronautical Sciences, 11, 46-49 (1964).

[12] R.E. Robertson, Notes on the Thomson-Tait-Chetaev stability theorem, Journal of the Astronautical Sciences, 15, 319-324 (1968).

[13] T.J. Moran, A simple alternative to the RouthHurwitz criterion for symmetric systems, ASME Journal of Applied Mechanics, 37 (1970), 11681170.

[14] Puzyrov V., Awrejcewicz J., Asymptotical stability of the motion of mechanical systems with partial energy dissipation, Nonlinear Dynamics (to appear).

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