Dynamic Properties of Two-Axle Freight Wagon with UIC Double-Link Suspension as a Non-smooth System with Dry Friction

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Abstract The influence of chosen parameters on the lateral dynamic behavior of the two-axle freight wagon with UIC double-link suspension is presented. This type of suspension uses the advantages of the studied non-smooth mechanical system, since dry friction is used to damp the system vibrations. Mathematical models of this suspension with and without lateral bump-stop are derived owing to non-smooth mechanics assumptions being based on the Coulomb law regarding friction and implemented into the MBS program. Numerical simulations of dynamics of the analyzed system are performed on a straight track followed by the methods appropriate for predicting the dynamic stability of railway vehicles. A dynamic reply of the vehicle to the railway track excitation, in the form of the initial condition, is monitored and studied. The carried out analysis mainly concerns the investigation of limit cycle dynamics exhibited by the system elements in terms of safety of the freight wagon. In addition, an influence of the friction coefficient coupling the interacting elements of suspension and being responsible for damping properties of our non-smooth system is analyzed. Furthermore, an occurrence of a low critical speed is illustrated and its physical meaning is explained, and the usefulness of applying the lateral bump-stop in the structure of UIC double-link suspension is justified.

1 Introduction

Many of freight wagons in Europe are equipped with the UIC link suspension in which damping is provided only by dry friction in pivoted joints of linkages. The structure of this suspension is quite simple, but its elements have strongly nonlinear

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characteristics. Such a type of suspension was examined by many researchers [1, 2]. Nonlinear simulation model for freight wagons with UIC double-link suspension in two-axle freight wagon was developed, for instance, by Hoffmann in [1], but without the lateral bump-stop limiting lateral displacements of the lower link. The purpose of the presented article is to compare the dynamic properties of the two-axle freight wagon equipped with standard UIC double-link suspension with and without lateral bump-stop. The mathematical model of the UIC double-link suspension, prepared according to non-smooth mechanics assumptions, was implemented into the MBS program [6]. The numerical examinations were conducted on the straight track. They included different values of coefficient of friction in joints of the UIC double-link suspension without and with lateral bump-stop, as well as the state of loading the car body (empty/fully loaded). In the case of the leaf spring, a piecewise linear and progressive characteristic in vertical direction was taken into account. The utilized MBS program enabled to use pre-calculated and tabulated wheel–rail contact parameters generated for different lateral positions for S1002 and UIC60 real, nonlinear profiles of the wheel and rail. We recognize the wagon movement as stable, if dynamic reply of the vehicle to the railway track excitation has the form of decaying lateral oscillations of wheel sets, returning to the center line of the track. At sufficiently high speed of the vehicle, the oscillations following an external disturbance grow and lead to a limit cycle. In general, freight wagons have low critical speeds. Fully developed limit cycle oscillation is usually termed “hunting.” The lowest vehicle speed at which sustained oscillations appear is named the critical speed.

2 Elements of the Freight Wagon with the UIC Double-Link Suspension

The considered two-axle freight wagon with UIC double-link suspension consists of the body and two wheel sets are shown in Fig. 1. In the structure of the wagon the leaf spring ***2*** is located between the axle box ***4*** and the car body ***1*** (Fig. 2). Wheel sets are guided in lateral and longitudinal direction, utilizing the links ***3***. Longitudinal displacements of wheel set are blocked by guiding fork ***6***. Brackets ***5*** enable the transfer of vertical load from the car body to the axle boxes and wheels. Each of the wheel sets can move relatively to the car body in the range of lateral and longitudinal clearances, equal to 20 and 22,5 mm accordingly.

Upper and lower links ***3***, tumbling blocks ***7***, pins ***8***, C-washers ***9*,** and links ***10***, shown in Fig. 3 and in Fig. 4, are the main elements of the UIC double-link suspension. Each tumbling block rests on the pin and is secured in the lateral direction by the C-washer. When the wheel set moves relative to the car body, the upper and lower links roll or slide over the tumbling blocks. Rolling in the joint is possible, because the radii of the links’ cross sections are smaller than the radii of the half hole in the tumbling blocks.

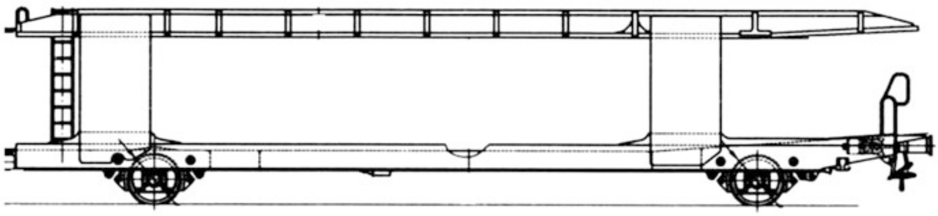

Fig. 1 Side view of the two-axle freight wagon adapted to transport of cars [4]

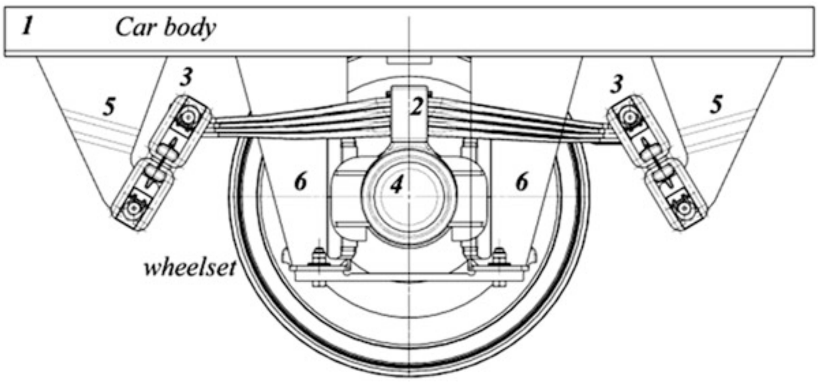

Fig. 2 Sketch of wheel set guiding with the UIC double-link suspension

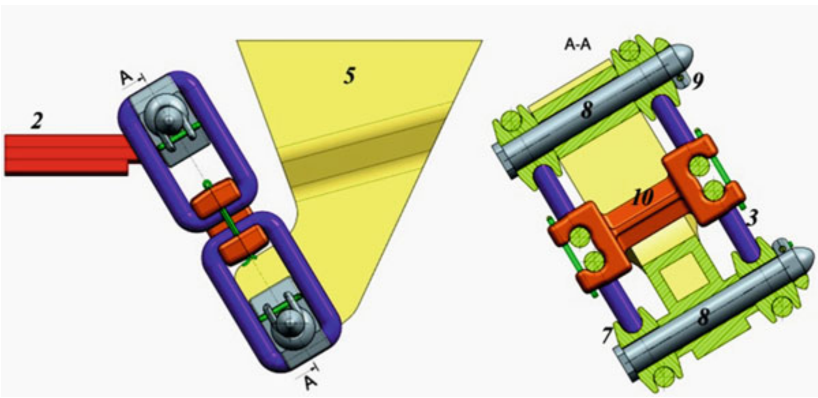

Fig. 3 Elements of UIC double-link suspension without the lateral bump-stop

3 Mathematical Model of the UIC Double-Link Suspension

Authors of this article used the mathematical models of standard UIC double-link suspension proposed by Piotrowski [5]. The main assumptions, according to these models, were the following: contacting elements of joints are cylindrical; the Coulomb law of dry friction is applied for the description of friction in the

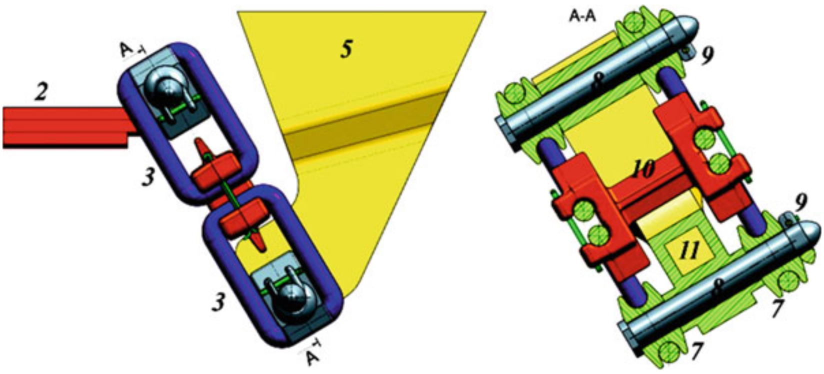

Fig. 4 Elements of UIC double-link suspension with the lateral bump-stop

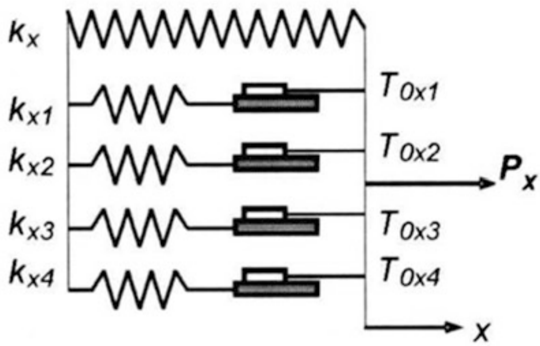

Fig. 5 Rheological model of the UIC double-link suspension for longitudinal direction in the case without and with the lateral bump-stop [5]

joints; the elements of the joints are assumed to be rigid; the lateral and longitudinal displacements of the suspension are not coupled. Rheological models, used to describe the properties of the UIC double-link suspension, have the form of elastic elements with dry friction, composed of springs and dry friction sliders. For longitudinal direction the model is presented in Fig. 5. For lateral direction the model of suspension with the lateral bump-stop has the form shown in Fig. 6 [4]. When there is a lack of the bump-stop, the spring k_2 and slider u_0 are removed.

In case of the UIC suspension with the lateral bump-stop, the two links are active, if the lateral displacement of the leaf spring pivots $Y < u_0$. When $Y > u_0$, only the upper link is active and the stiffness of the suspension is equal to $k + k_2$.

The break force T_{0y} and spring with stiffness of k_1, describing the elastic element with dry friction, may be replaced by the differential equation for the force T (Fig. 7). Then the continuity condition for the slider and spring has a form

$$\dot{T}/k_1 + v_S = \dot{\varsigma} - \dot{Y} \tag{1}$$

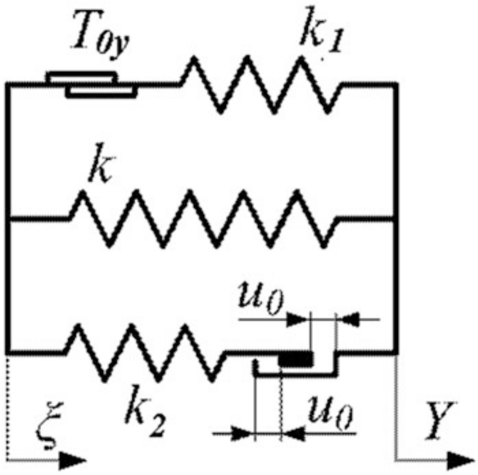

Fig. 6 Rheological models of the UIC double-link suspension with lateral bump-stop for lateral direction [4]

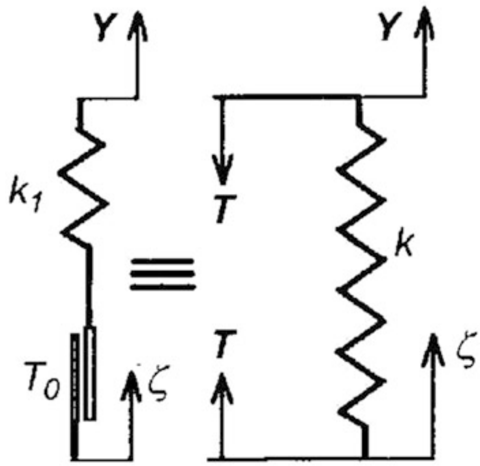

Fig. 7 The slider and the spring replaced by friction force T [5]

where v_s is the velocity of sliding and T_0 is the break force (friction force). The non-smooth relations, basing on the Coulomb law of dry friction and describing the characteristics of friction slider, are as follows [5]:

$$\Omega : \quad T \in [-T_0, +T_0] \,, \tag{2}$$

$$v_S \in -K\left(\dot{T}, D\Omega\right). \tag{3}$$

The cone **K** is described by the velocity of sliding:

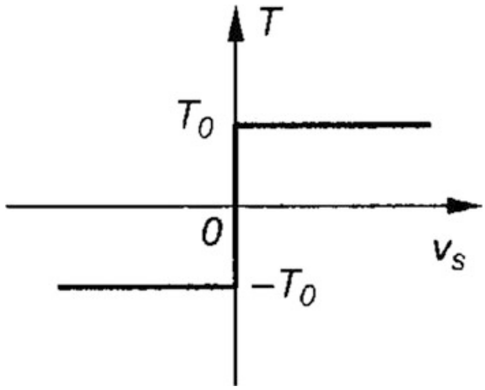

Fig. 8 Relation between the velocity of sliding and the force of friction

$$v_s \in \begin{cases} \{0\} & if \quad |T| < T_0 \\ R^+ & if \quad T = +T_0 \\ R^- & if \quad T = -T_0 \end{cases},$$

where the function $[.]^+$ is defined as

$$[u]^+ = \begin{cases} u & if \quad u \geq 0 \\ 0 & if \quad u < 0 \end{cases}. \tag{4}$$

When $|T_0| = T_0$, it is necessary to consider the differential succession of non-smooth relation (4). The characteristics shown in Fig. 8 are non-smooth, multi-valued, and non-differentiable. It was applied to the vehicle simulation model. In that way the models of the suspension have been described by the differential equations of the first order, implemented in the MBS model of the 2-axle freight wagon in the following manner, for the lateral and longitudinal direction:

$$\dot{T}_y = \begin{cases} k_{y1}\left(-\dot{Y}\right) & if \quad \left|T_y\right| < T_{0y}, \\ -\left[-k_{y1}\left(-\dot{Y}\right)\right]^+ & if \quad T_y = +T_{0y}, \\ \left[k_{y1}\left(-\dot{Y}\right)\right]^+ & if \quad T_y = -T_{0y}, \end{cases} \tag{5}$$

$$\dot{T}_{xi} = \begin{cases} k_{xi}\left(-\dot{X}\right) & if \quad |T_{xi}| < T_{0xi} \\ -\left[-k_{xi}\left(-\dot{X}\right)\right]^+ & if \quad T_{xi} = +T_{0xi}, \\ \left[\, k_{xi}\left(-\dot{X}\right)\right]^+ & if \quad T_{xi} = -T_{0xi} \end{cases} \quad (i = 1, 4). \tag{6}$$

In formulas (5) and (6), the lateral deflection is described by Y, while X indicates the longitudinal deflection of the suspension. The restoring forces P_y, P_x of the suspension in lateral and longitudinal directions are defined as

Table 1 Parameters of the UIC double-link suspension for the lateral direction without the bump-stop

f	T_{0y}/Q	k_y/Q [1/m]	k_{y1}/Q [1/m]
0,20	0,36518E-01	0,34534E+01	0,10519E+02
0,50	0,83620E-01	0,34718E+01	0,10155E+02

Table 2 Parameters of the UIC double-link suspension for the lateral direction with the bump-stop

f	T_{0y}/Q	k_y/Q [1/m]	k_{y1}/Q [1/m]	k_{y2}/Q [1/m]
0,20	0,36518E-01	0,34534E+01	0,10519 IE+02	0,34534E+01
0,50	0,83620E-01	0,34718E+01	0,10155E+02	0,34658E+01

Table 3 Parameters of the UIC double-link suspension model for the longitudinal direction

f	T_{0x1}/Q	T_{0x2}/Q	T_{0x3}/Q	T_{0x4}/Q	k_x/Q	k_{x1}/Q	k_{x2}/Q	k_{x3}/Q	k_{x4}/Q
0,2	0,750E−2	0,536E−2	0,69E−3	0,32E−2	0,712E+1	0,428E+1	0,245E+1	0,308E+0	0,107E+1
0,5	0,183E −1	0,117E −1	0,21E −2	0,89E −2	0,661E +1	0,448E +1	0,229E +1	0,415E +0	0,126E +1

$$P_y = k_y Y + T_y, \qquad P_x = k_x X + \sum_{i=1}^{i=4} T_{xi}. \tag{7}$$

The parameters of the model of the UIC double-link suspension without and with lateral bump-stop have been prepared for dimensions located in the middle of the tolerance field, given by the technical specification of DIN 5545 Standards [4]. Four values of the coefficient of friction between rolling/sliding elements of joints have been assumed: $f = 0{,}2$; 0,3; 0;4, 0,5. Values of stiffness and dry friction force for lateral/longitudinal directions were scaled with the vertical load Q of the suspension in the following manner: k/Q, k_1/Q, k_2/Q, k_{xi}/Q, T_{0y}/Q, T_{0xi}/Q, $(i = 1, 4)$. Chosen unit parameters are presented for $f = 0{,}2$ and $f = 0{,}5$ in Tables 1–3.

4 The MBS Model of the Two-Axle Freight Wagon with the UIC Double-Link Suspension

A multi-body simulation model of the two-axle freight wagon (18° of freedom) was prepared in the MBS program [6]. The UIC double-link suspension was modeled as nonlinear, massless element. Mathematical models of this suspension were involved into multi-body model in the form of the differential equations (5) and (6), for each of wheel set. These differential equations have been integrated with the wheel set template in the MBS program. For the nominal UIC60 profile for rails and nominal S1002 profile for wheels with nominal radii equal to 0,42 m, the one-point contact model (based on Kalker's rolling contact theory [3]) was assumed. The nonlinear wheel–rail geometry and pre-tabulated contact functions according to the

Table 4 Mass and inertial parameters for two-axle freight wagon with UIC double-link suspension

Part	Mass (kg)	Moments of inertia (kg m^2)		
		I_x	I_y	I_z
Empty car body	13,450	282,000	318	321,000
Fully loaded car body	27,450	595,140	618,140	596,970
Wheel set	990	535	74	535

FASTSIM algorithm of Kalker [6] were used. The coefficient of friction between wheel and rail, accepted equal to 0,4. The track model was treated as a uniform structure with the mechanical properties described by the parameters given in [6]. The inertial parameters used in simulations for the empty and loaded freight wagon are presented in Table 4. The other parameters are as follows: the base of the wagon equal to 10 m, track gauge equal to 1,435 m, rail inclination equal to 1:40, and the border displacement of the leaf spring pivoting in lateral direction $u_0 = 10$ mm. The distance of the wagon center of gravity from the rail level was accepted as 1,527 and 2,162 m for empty and fully loaded car body appropriately.

5 Results of Numerical Simulation

The simulations have been performed for two cases: without and with the lateral bump-stop in UIC double-link suspension. Lateral vibrations were caused by the initial condition introduced in the form of lateral displacement of leading wheel set, equal to 0,005 m, relative to the track center line. As a response we can observe the hunting motion of wheel sets. We stated that critical speed of empty and fully loaded wagon is very small and close to 10 m/s. Freight wagons are operated usually with the maximum velocity smaller or equal to 120 km/h. In order to judge the properties of the suspension, further investigations were done for velocity bigger than critical speed of the wagon. As a criterion of evaluation, the lateral displacements of leading wheel set were accepted. In the range of velocity from 10 to 20 m/s, an advantage of UIC double-link suspension without lateral bump-stop, with the coefficient of friction between rolling/sliding elements equal to 0,2 was stated. But for velocity close to 35 m/s, lateral displacements of leading wheel set of empty wagon violently grow and undesirable wheel flange contact between wheels and rails is observed (Fig. 9). Simultaneously, the lateral amplitudes of leading wheel set of empty wagon with the UIC double-link suspension equipped with the lateral bump-stop achieve the values close to 5 mm for velocity equal to 15 m/s (Fig. 10). The limit cycle of this wheel set is presented in Fig. 11. For velocity from 20 to 40 m/s, lateral amplitudes of leading wheel set for this case are smaller than for the compared type of suspension without the bump-stop. In addition they are smaller than 1 mm. It means that there is no flange contact between wheels and rails.

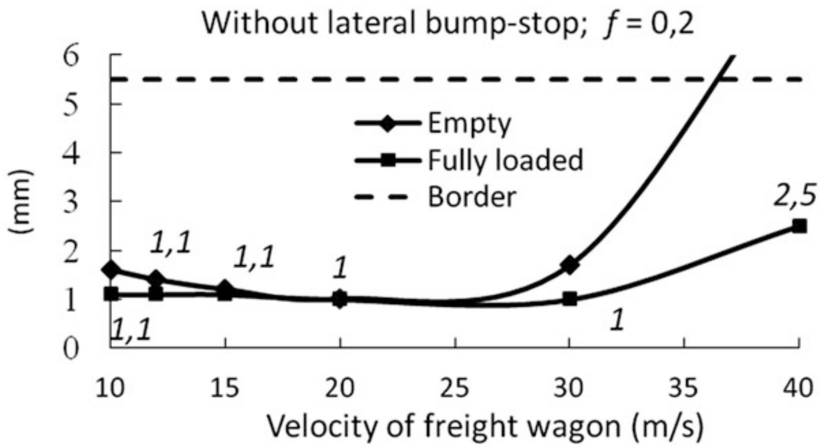

Fig. 9 Amplitudes of lateral displacements of leading wheel set for wagon with the suspension without the lateral bump-stop and coefficient of friction between rolling/sliding elements of joints equal to 0,2, in the function of wagon velocity

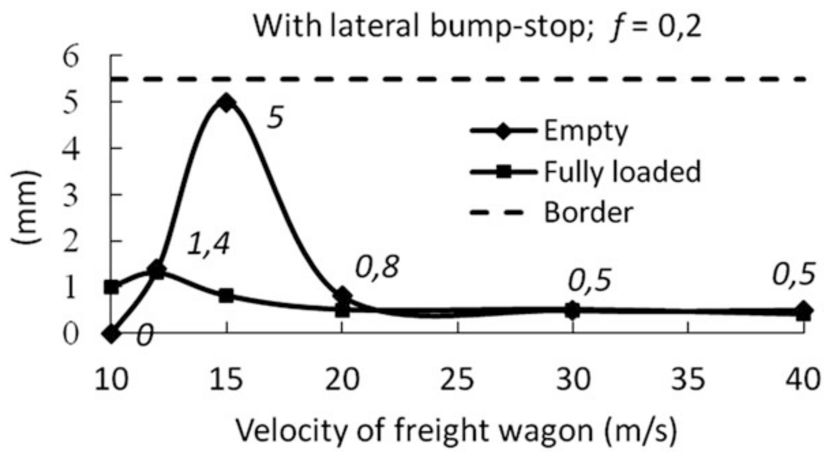

Fig. 10 Amplitudes of lateral displacements of leading wheel set for wagon with the suspension with the lateral bump-stop and coefficient of friction between rolling/sliding elements of joints equal to 0,2, in the function of wagon velocity

In the context of previous conclusion according to the UIC double-link suspension with lateral bump-stop, we need to explain the cause of the large lateral amplitude of leading wheel set, close to 0,005 m, examining the case presented in Fig. 10 for empty wagon. Therefore an analysis of a resonance between the lateral excitation frequency of the wheel sets f_w and the yaw eigenfrequency of car body f_{yaw}, as well as the roll eigenfrequency f_{roll} of the car body motion, was conducted. These frequencies can be approximated in the following manner [1]:

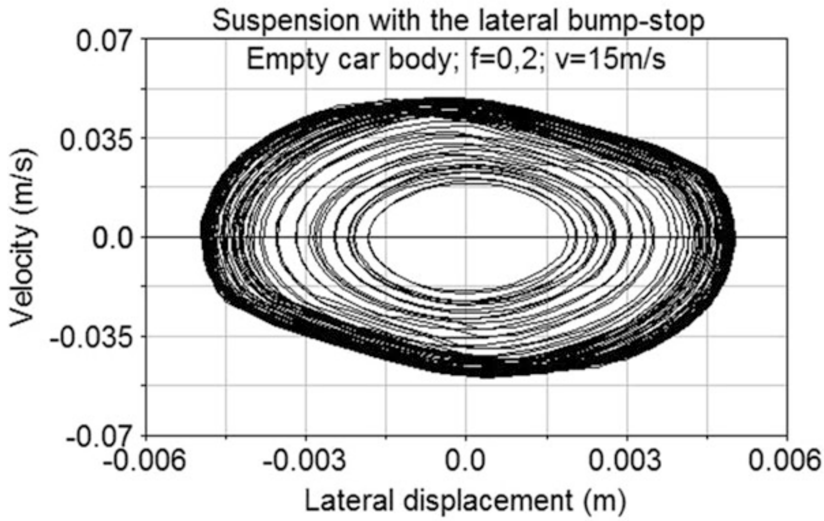

Fig. 11 Limit cycle of leading wheel set of empty wagon with the UIC double-link suspension equipped with the lateral bump-stop, for $f=0{,}2$ and $v=15$ m/s

$$\begin{aligned} f_w &= v/2\pi\sqrt{\lambda/b_0 r_0},\\ f_{yaw} &= 1/2\pi\sqrt{4a^2 k_y/I_z},\\ f_{roll} &= 1/2\pi\sqrt{4\left(L_1^2 k_z + L_2^2 k_y\right)/I_x}, \end{aligned} \tag{8}$$

where a is the half wheelbase, k_y and k_z denote appropriately the lateral stiffness of suspension and the vertical stiffness of the leaf spring, I_z and I_x are the yaw and the roll moments of inertia of the car body, v is the speed of vehicle, λ denotes the wheel conicity, $2b_0$ and r_0 are the distance between the nominal rolling circles and the nominal rolling radius of wheel, and L_1 and L_2 indicate the lateral and vertical distances between the center of car body mass and the UIC suspension, respectively.

Values of parameters in formulas (8) were assumed to be known. The value of wheel conicity parameter was accepted as $\lambda=0{,}13$, which corresponds to lateral displacements of wheel set in the scope up to 6 mm. Using Klingel's formula, we got from the equations (8) the value of lateral excitation frequency of the wheel set f_{w} equal to 1,53 Hz. Applying the spectrum analysis (fast Fourier transform) of lateral oscillations of wheel sets we obtained more probable value of f_{w} equal to **1**,**5** Hz. In the next step the hysteresis loop presented in Fig. 12 and according to lateral direction of the UIC double-link suspension with the bump-stop was analyzed. The following values of k_y stiffness were estimated: $k_{y1}=113470$ N/m, $k_{y2}=225860$ N/m, and $k_{y3}=560760$ N/m. Next, the yaw and roll eigenfrequencies of the empty car body were calculated: $f_{\mathrm{yaw1}}=1{,}05$ Hz, $f_{\mathrm{yaw2}}=1{,}49$ Hz, $f_{\mathrm{yaw3}}=2{,}35$ Hz, $f_{roll1}=0{,}93$ Hz, $f_{roll\,2}=1{,}16$ Hz, and $f_{roll\,3}=1{,}68$ Hz. We noticed that the values of the lateral excitation frequency of the wheel set $f_{\mathrm{w}}=1{,}5$ and yaw

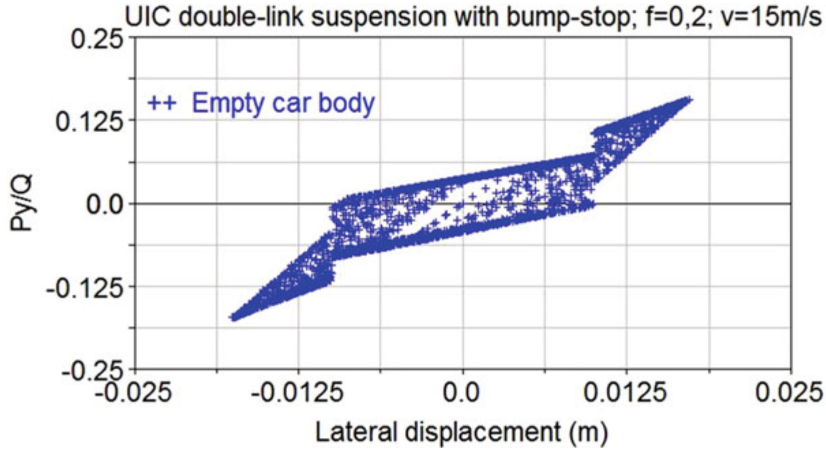

Fig. 12 Hysteresis loops for empty wagon equipped with the UIC double-link suspension with the bump-stop, for velocity equal to 15 m/s and coefficient of friction between rolling/sliding elements equal to 0,2

eigenfrequency $f_{\mathrm{yaw2}} = 1{,}49$ Hz are almost identical. It means that lateral oscillations of wheel set and car body yaw motion are in the resonance. That is the direct cause of the high leading wheel set amplitude of examined freight wagon, equipped with UIC double-link suspension with the lateral bump-stop. Similar analysis for empty wagon running with velocity greater than 20 m/s and for fully loaded wagon in the range of velocities from 10 m/s to 40 m/s showed that instability is caused by small damping of the lateral displacements of wheel set. Hysteresis loop for fully loaded wagon equipped with the UIC double-link suspension with the bump-stop, for velocity equal to 15 m/s and coefficient of friction between rolling/sliding elements equal to 0,2, is shown in Fig. 13. Similarly, but more worse situation according to hunting motion of leading wheel sets we can observe for coefficients of friction between rolling/sliding elements of joints greater than 0,2, especially for $f = 0{,}5$ (Figs. 14 and 15). Also in these cases the high lateral displacements of leading wheel sets are caused first through a resonance between the lateral excitation frequency of the wheel sets f_{w} and the yaw eigenfrequency f_{yaw} of the empty and fully loaded car body.

When the speed of wagon increases, we still can observe the limit cycles of wheel sets caused by small damping properties of the suspension. According to earlier assumptions, the lateral and longitudinal directions of the UIC double-link suspension should not be coupled. In practice, wheel sets have the possibilities to yaw and in that way the longitudinal displacement of the leaf spring occurs. In the end, suspension works in two directions; however the longitudinal restoring forces P_x are almost four times smaller than lateral restoring forces P_y (Fig. 16).

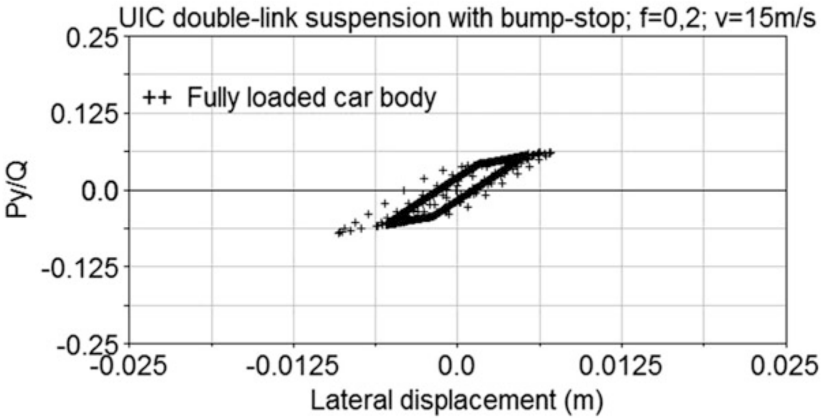

Fig. 13 Hysteresis loop for fully loaded wagon equipped with the UIC double-link suspension with the bump-stop, for velocity equal to 15 m/s and coefficient of friction between rolling/sliding elements equal to 0,2

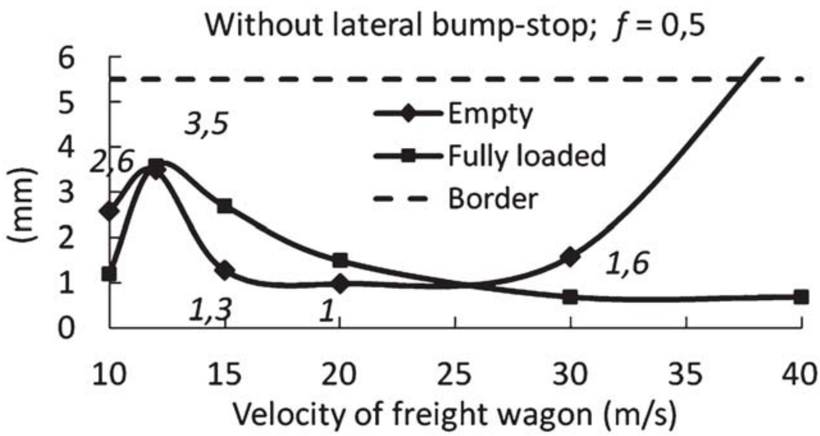

Fig. 14 Amplitudes of lateral displacements of leading wheel set for wagon with the suspension without the lateral bump-stop and coefficient of friction between rolling/sliding elements of joints equal to 0,5, in the function of wagon velocity

6 Conclusions

In the presented article the influence of chosen parameters on the lateral dynamic behavior of the two-axle freight wagon with the UIC double-link suspension was showed. Mathematical models of this suspension with and without lateral bump-stop were derived owing to non-smooth mechanics assumptions being based on the Coulomb law regarding friction.

Very small and close to 10 m/s, critical speed of empty and fully loaded freight wagon with the UIC double-link suspension was stated.

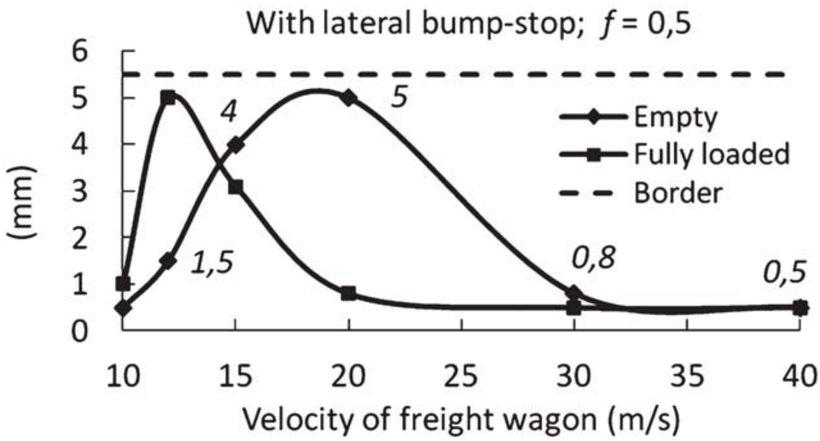

Fig. 15 Amplitudes of lateral displacements of leading wheel set for wagon with the suspension with the lateral bump-stop and coefficient of friction between rolling/sliding elements of joints equal to 0,2, in the function of wagon velocity

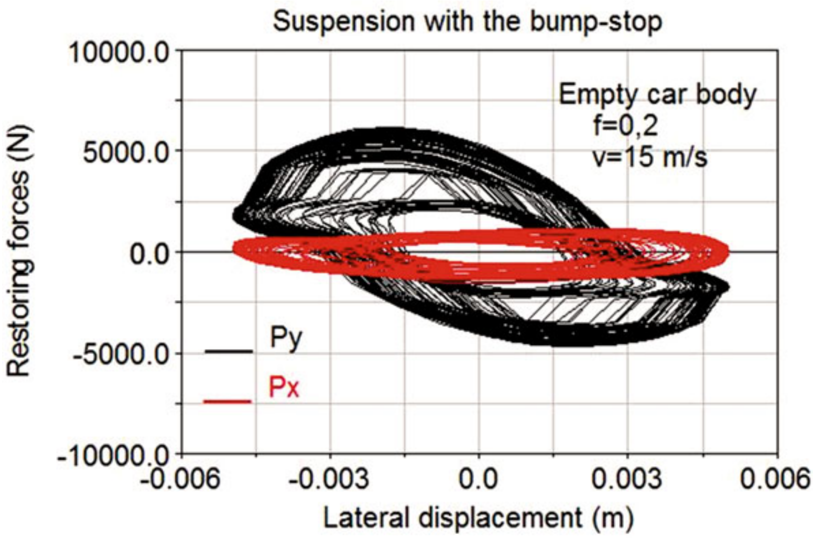

Fig. 16 Restoring forces in the function of lateral displacement of leading wheel set for empty wagon equipped with the UIC double-link suspension with the bump-stop, for velocity equal to 15 m/s and coefficient of friction between rolling/sliding elements equal to 0,2

An advantage of the UIC double-link suspension without lateral bump-stop, for the coefficient of friction between rolling/sliding elements equal to 0,2, was showed for velocity smaller than 30 m/s.

The appearance of high lateral amplitudes of wheel sets, of the empty or fully loaded freight wagon with the UIC double-link suspension with or without lateral bump-stop, was interpreted as a resonance between the lateral excitation frequency of the wheel sets and the yaw eigenfrequency of the car body.

It was also demonstrated, that if the speed of two-axle freight wagon increases, we still can observe the limit cycles of wheel sets, caused by small damping properties of the suspension, for two analyzed cases of suspension.

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