Mathematical model and numerical simulation of a control system with feedback actuated by a stepper motor

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Abstract: Coupled electromechanical interactions observed in a closed-loop control system actuated by a stepper motor are studied. Analysed system consists of a PID controller used to maintain constant vertical inclination of a single DOF pendulum. This pendulum serves as a tensioner for a string being pulled by a rotary motion of the electric stepper motor. The second end of the string is being fed to the motor with a variable speed. The mathematical model governing dynamics of the mentioned system consists of a non-linear system of four ordinary differential equations. In addition to the mathematical model, various ways of control of the electric stepper motor have been applied. In particular, the rotor control using the micro-stepping approach has been used to model the real dynamics of the motor controller. Comparison of the proposed theoretical considerations and modelling of the studied system fits well with the experimental investigations.

1. Introduction

Stepper motors found variety of uses in consumer device applications, such as ink-jet printers, scanners, etc. They offer good performance in open-loop systems, while maintaining low cost of both motor and the corresponding driver [1]. However, in more sophisticated set-ups, where there is a need for more reliability of drive positioning, an optical encoder may be used to enhance system's effectiveness [6, 2, 4]. That is because open-loop stepper motor operations are prone to positioning errors as a consequence of missing a step or stalling, especially under variable load conditions.

The simplest way to ensure positioning reliability is to use the encoder to track drive's position and correct it accordingly after each sequence, as is the case in the realistic physical system described in this paper. While there are many different solutions used in positioning systems, stepper motors offer certain unique features that make them desirable in a variety of set-ups (e.g., they maintain a holding torque when in a standstill).

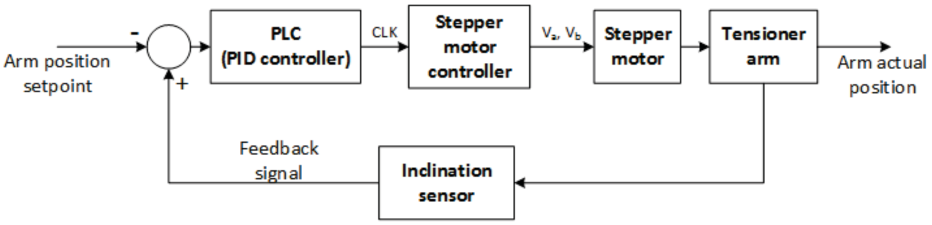


Figure 1. Control system with feedback.

The goal of the following research is to create a mathematical model that could be used to evaluate motor's performance in a variety of scenarios. In particular it has to allow investigations of the system behaviour under varying load conditions and using different control methods.

2. Physical system

The physical system being considered here is a part of an industrial device dedicated to measuring and cutting PVC bead-strings (used for instance in window and door blinds). As the beads are being fed to the device form an injection moulding machine at a variable speed, cutting device speed needs to be appropriately adjusted. Moreover, the strings must be sufficiently rigid for cutting to be possible. This is achieved by fitting the device with a rotational tensioning mechanism (essentially a single DOF pendulum), which angular position is measured and transmitted to a PLC.

The drive system consists of a low power hybrid stepper motor powered by an appropriate driver, which in turn is controlled by the PLC. There is an incremental optical encoder on motor's shaft that ensures its correct position after each positioning sequence (in this particular case before cutting the string). Motor's speed is set by a digital PID controller (inside the PLC). Previously mentioned measurement of tensioner's inclination functions as a feedback to the PID controller making it a closed loop system (Fig. 2). Frequency of the clock signal (labelled "CLK"), coming from the PLC's pulse output, regulates motor's demanded speed. Up to that point all signals are solely digital, including the information from the digital inclination sensor, which operates through RS-232 interface. Stepper motor controller converts the clock signal to an appropriate combination of voltages (V_a and V_b) applied to stepper motor's windings.

3. Mathematical model

3.1. Stepper motor

The dynamics of a stepper motor, as can be found in references [1, 5, 7, 8, 9], is governed by the following non-linear system of ordinary differential equations:

$$\begin{aligned}
\frac{di_A}{dt} &= \frac{1}{L}\left[V_A - Ri_A + K_m\omega\sin(N_r\Theta)\right], \\
\frac{di_B}{dt} &= \frac{1}{L}\left[V_B - Ri_B + K_m\omega\cos(N_r\Theta)\right], \\
\frac{d\omega}{dt} &= \frac{1}{J}\left[-K_m i_A\sin(N_r\Theta) + K_m i_B\cos(N_r\Theta) - B\omega - \tau_l\right], \\
\frac{d\Theta}{dt} &= \omega,
\end{aligned} \tag{1}$$

where: i_a, i_b, V_a, V_b are currents and voltages in phases A and B; R, L are winding's resistance and inductance, respectively; K_m is the motor torque constant, B is viscous friction coefficient, N_r is the number of rotor teeth, J is rotor's moment of inertia, τ_l is the load torque, ω is the rotor speed and Θ is its angular position.

First two of the above equations determine the current in motor's windings (phase A and B), while the two latter correspond to Newton's 2nd Law of motion.

3.2. Controller

Stepper motor controller is compelled to provide appropriate voltage on motor's connectors and change them as required by the demanded move sequence. The dual H-bridge is the most basic circuit that can be used as a dual-phase stepper motor controller. It consists of four transistors that are switching power sequentially, allowing the rotor to move in the desired direction. While this approach has a big advantage of simplicity and low cost, it doesn't allow for a very good drive performance. Because of that, for more demanding tasks, dedicated integrated circuits are commonly used.

Those more advanced circuits allow micro-stepping, which by applying appropriate ratio between currents in both phases, enables the rotor to take positions between the full steps. This reduces the vibrations excited during motion and makes drive's operation more smooth. More importantly however, they use supply voltage that is much higher than motor's nominal voltage while cutting it off when the current in phase goes above the nominal value. This kind of circuit is called a chopper and allows the windings to be energised much more rapidly (as for each step current rises much faster), which results in much better performance of the motor, namely higher maximum speed and more torque.

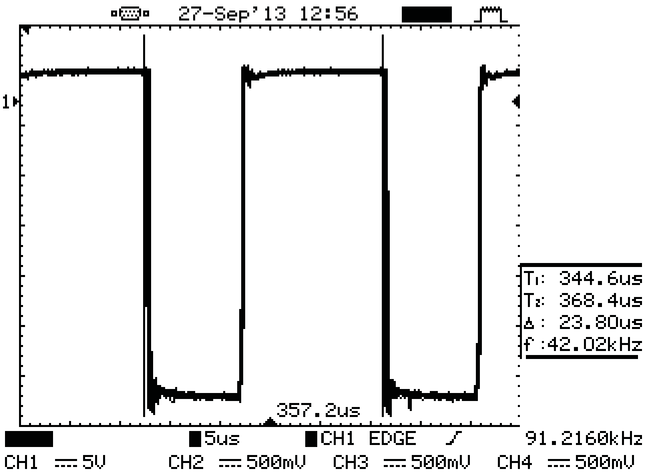


Figure 2. Voltage measurement on phase A using a digital oscilloscope.

To recreate above conditions, voltages substituted in (1) are given by the following rule:

$$V = \begin{cases} V_{cc} & \text{if } i < I_m \\ 0 & \text{if } i \geq I_m. \end{cases} \tag{2}$$

In reality, controller's output voltage is a subject to pulse-width modulation at a constant frequency of 42 kHz. This was measured on the physical object, as seen in a waveform in Fig. 3.2. If the system is controlled with micro-stepping approach, maximum allowed current I_m is determined by motor's nominal current and the appropriate current ratio for the particular micro-step. The latter is determined by following equations:

$$I_{ma} = I_n \sin(\frac{n}{2d}\pi) \quad \text{for } n \in \{1, 2, \ldots, 4d\}, \tag{3}$$

$$I_{mb} = I_n \cos(\frac{n}{2d}\pi) \quad \text{for } n \in \{1, 2, \ldots, 4d\}, \tag{4}$$

where I_n is the nominal current, n is the current micro-step and d denotes the division factor. Please note that there are several other methods of determining desired current while micro-stepping, that may be more suitable to a particular motor model.

3.3. Tensioning mechanism

The tensioning mechanism is a subject of control in the system. The position of tensioning arm is at all times restricted by the length of the string currently present in the device (see Fig. 3.3). The length in turn, depends on the one hand on the feed speed and on the other

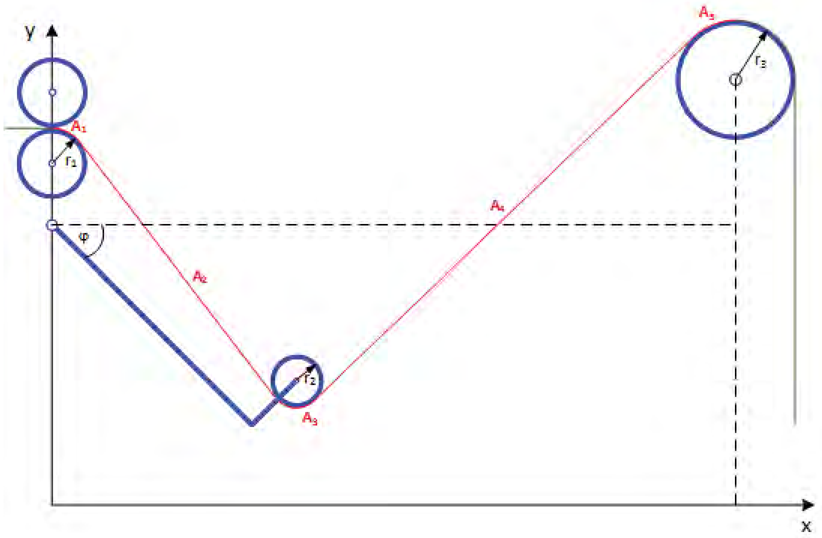


Figure 3. Tensioning mechanism.

on motor's speed. Tensioner's angular position and string length are tied by a following third-order approximation:

$$\varphi(a) = 1072{,}9a^3 - 3864{,}7a^2 + 4812{,}7a - 2002, \tag{5}$$

where: a is string's length and φ is arm's position. This was obtained by calculating $a = f(\varphi)$ using geometric relations and approximating the inverse function numerically. The above equation is very close to actual relation in angle range the device operates in.

4. Computer simulation

The above mathematical constraints have been applied in a computer model created using *scicos* which is a part of numerical computation software *scilab*. All of the mentioned subsystems were recreated and connected together, as seen in the diagram (Fig. 4). Simulation parameters, namely motor's specification, according to the data sheet are given in Tab. 1.

Computer model allows applying different kinds of loads to the motor. In particular, load from tensioning arm is approximated. Assuming that at angle $\varphi = \frac{\pi}{2}$, the tensioner is in vertical position and at $\varphi = 0$ in horizontal position respectively, the simulated load on the motor is assumed to be:

$$\tau_l = \tau_{max} \cos(\varphi). \tag{6}$$

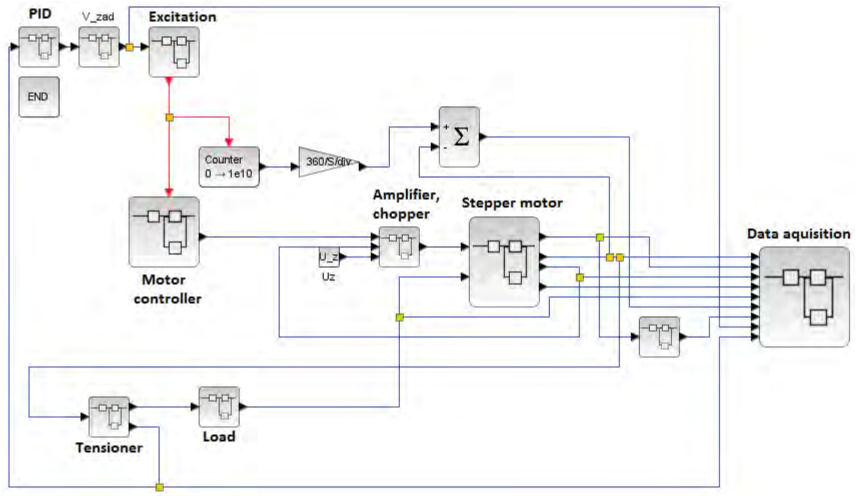


Figure 4. Computer model in *scicos*.

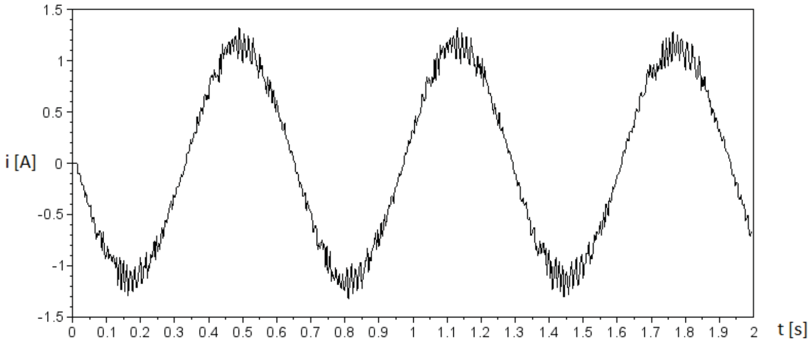
Apart from the above, the external load can be set to zero or given as step function with a constant load starting from time t_0.

As for the stepper motor's excitation frequency (CLK signal), it can be easily set as a constant value, a polynomial function or a value determined by a PID controller. In the latter case, all of the PID term's constants can be set, which allows tuning the device using the computer simulation instead of testing it on site.

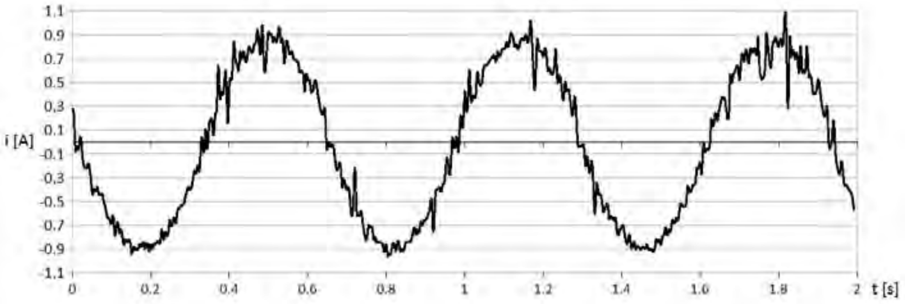
5. Results

The following plots were made using the computer model described above. Among the objects of interest are phase currents of the stepper motor, as well as rotor's mechanical position. To evaluate the behaviour of the model, different load and excitation conditions were tested.

Additionally, to evaluate the simulation results, some simple measurements on the physical object were attempted (compare [3]). In particular, phase current was measured. This was achieved by adding a serial resistor into the phase circuitry. The resistance of it is several orders of magnitude lower than the winding's resistance, so that motor performance is not affected by its presence. By connecting an oscilloscope to both ends of the resistor, one can easily measure voltage drop on the resistor, which is an indirect way of measuring phase current in the circuit, as those two physical quantities are proportional. The downside of this simple method, however, is that the recorded waveforms contain a lot of noise and are not very accurate. This is because of the fact that the measured voltage drop is very low (less than 100 mV).



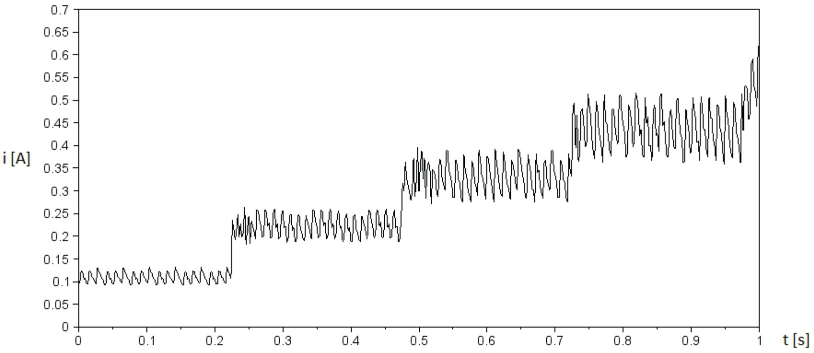
(a) Simulation results



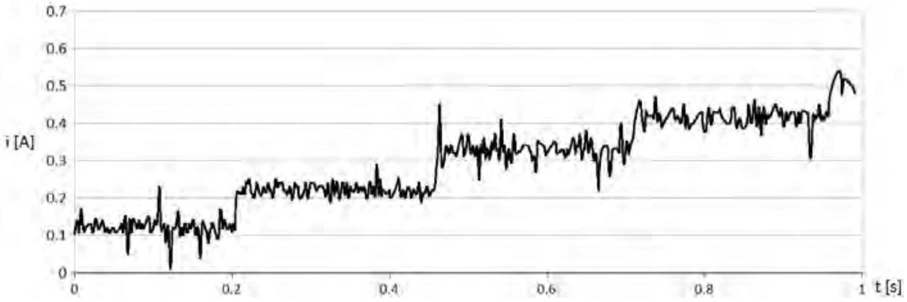
(b) Physical object measurements

Figure 5. Current in phase A during a test run with excitation frequency $f = 100\mathrm{Hz}$ and microstepping 1/16.

Investigations shown in Fig. 5 confirm that mathematical model's simulation results are compliant with the physical object. However subtle differences can be observed, such as the amplitude of the phase current. That is likely to be associated with the fact that the amplifier and chopper are not yet accurately modelled. Figure 6 shows current oscillations when the motor is advancing to the next step. As micro-stepping is used in the considered scenario, those oscillation's amplitude is proportional to maximum allowed current in a given micro step, as can be seen in (3) and (4). Although simulation results allow to reconstruct motor's behaviour, because of the noise measurement's accuracy is limited. Operation of the system as a whole is presented in Fig. 7. In this particular example, excitation frequency is regulated by a PID controller with non-zero "P" and "I" terms. Observe that it accurately resembles behaviour of the actual physical device.



(a) Simulation results



(b) Physical object measurements

Figure 6. Current in phase A during a test run with excitation frequency $f = 4\text{Hz}$ and microstepping 1/16.

6. Concluding remarks

The qualitative results obtained during simulations fit well with the expected behaviour of an electric stepper motor. In particular, the proposed solutions to simulating real stepper motor controllers, allow for realistic simulation conditions, resembling commercially available devices. This can lead to a better understanding of pros and cons of certain control methods as well as allowing to a broader usage of electric stepper motors in environments, where their features are desirable, offer more performance or lower cost than their DC or asynchronous counterparts. The mathematical model of a stepper motor system allows for investigating and avoiding some of the phenomena that can hinder system's performance. Among those are electro-mechanical resonances that result in increased vibrations, stalling, losing steps,

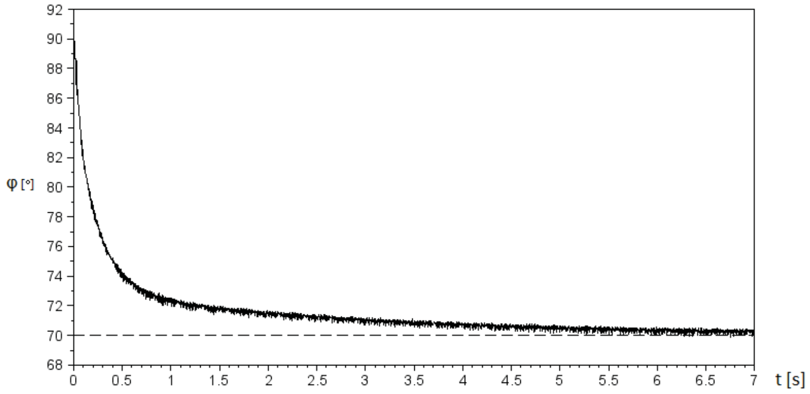


Figure 7. Tensioner angular position during operation with a PID controller with a setpoint of $\varphi = 70^o$ and microstepping 1/16.

Table 1. Stepper motor properties assumed in the computer simulation.

R	$5 \quad \Omega$
L	8.6×10^{-3} H
B	$8 \times 10^{-4} \quad \mathrm{N \cdot m \cdot s/rad}$
K_m	$0.55 \quad \mathrm{N \cdot m/A}$
I_n	$1 \quad \mathrm{A}$
J	$11 \times 10^{-6} \quad \mathrm{kg \cdot m^2}$
N_r	50

noisy operation and decreased performance and reliability. In further research, more physical object measurements will be made, which will open possibilities for fine-tuning the computer model. In particular, precise phase current measurements need to be done to verify the simulation results. This will require building a circuit that allows amplifying the measured signal as well as filtering out the background noise and interferences. Moreover, physical properties of the motor and the whole system should be verified as they can deviate slightly from the values provided in the data sheets.

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