Nonlinear dynamics and chaotic synchronization of contact interactions of multi-layer beams

J. Awrejcewicz, M.V. Zhigalov , V.A. Krysko-jr., U. Nackenhorst,
I.V. Papkova, A.V. Krysko

Abstract: In this work a theory of nonlinear chaotic dynamics of multi-layer beams consisting of equally distance located layers coupled only via the boundary conditions is proposed. Wavelet analysis is applied to study a chaotic phase synchronization of vibrating multi-layer beams. Two-layer beam package serves as an example of application of the given theoretical background, where three types of the nonlinearities (geometric, physical and design) are used.

1. Introduction

Investigation of chaotic dynamics of one-layer structures, as well as control of various regimes of such structures is reported in numerous papers and monographs including works of the authors of this paper [1-9]. However, chaotic dynamics of mechanical structures consisting of multi-layer beams coupled only via the boundary conditions and problems related to the phase synchronization and its control has not been reported in the existing literature.

In addition, we apply here to the so far stated problems Winkler's hypothesis regarding a transversal contact of thin walled structures. In the case of static problems we acknowledge important results reported by Kantor [10], which are further used in this paper.

2. Problem formulation

Consider a package composed of multi-layer beams shown in Figure 1. In the general case, beams may have arbitrary thickness as well as arbitrary material properties, but in order to simplify the considerations we consider beams of equal thickness, width and length (h, a, b), as well as with the same physical parameters: Young modulus $E(x, z, \varepsilon_0, e_i)$, Poisson's coefficient , shear modulus $G_0(x, z, \varepsilon_0, e_i)$, and the specific material density ρ.

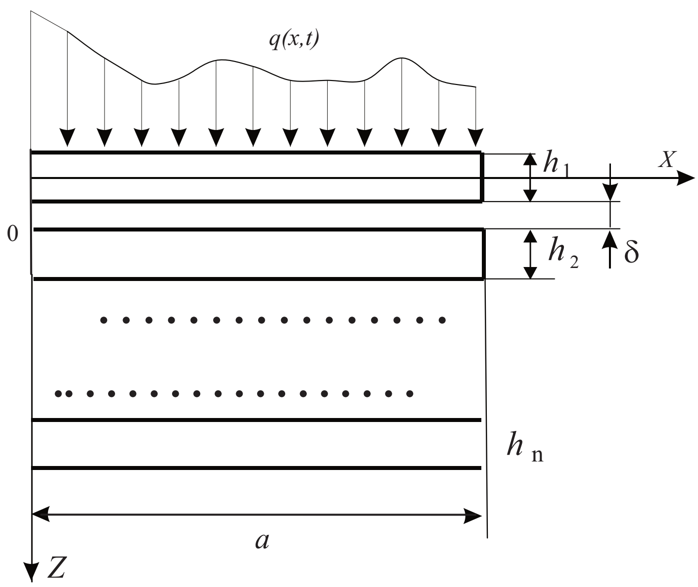


Figure 1. A multi-layer beam.

The following notation is introduced: $w_l(x,t)$ - beam deflection; $u_l(x,t)$ - middle beam surface displacement; t - time; ε_l - damping coefficient; e_{il}, σ_{il} and e_{sl}, σ_{sl} - intensity of deformations, stresses and plastic flow, respectively; K - Winkler's coefficient; δ - clearance between beam layers. Hencky's theory of small elastic-plastic deformations [11] is applied while estimating $E = E(x,z,\varepsilon_0,e_i)$. The following non-dimensional parameters are introduced:

$$
\begin{gathered}
x=\bar{x}a, \quad z=\bar{z}h, \quad h_l=\bar{h}_l h, \quad w=\bar{w}_l h, \quad E=\bar{E}G_0, \quad b=\bar{b}b_0, \quad \delta=\bar{\delta}h, \\
K=\bar{K}\frac{h^4 b}{a^4}, \quad t=\bar{t}\,\frac{a^2}{h}\sqrt{\frac{\rho}{G_0 b}}, \quad \varepsilon=\bar{\varepsilon}\,\frac{a^2}{h^2}\sqrt{\frac{\rho}{G_0 b}}, \quad l=1,2.
\end{gathered} \tag{2.1}
$$

Taking into account the hypotheses of Euler-Bernoulli, Kàrmàn as well as applying the method of variable parameters of stiffness [12], equations of motion of the beam in non-dimensional form follow (bars over non-dimensional parameters are already omitted)

$$
\begin{gathered}
bh\ddot{u}_l=\frac{\partial}{\partial x}\left[E\left(u_l'+\frac{1}{2}\left(w_l'\right)^2\right)+E_{1l}w_l''\right], \; bh\ddot{w}_l+\varepsilon\dot{w}_l= \\
q_l^*+\frac{\partial^2}{\partial x^2}\left[E\left(u_l'+\frac{1}{2}\left(w_l'\right)^2\right)-E_{1l}w_l''\right]+\frac{\partial}{\partial x}\left[w_l'\left\{E\left(u_l'+\frac{1}{2}\left(w_l'\right)^2\right)-E_{1l}w_l''\right\}\right],
\end{gathered} \tag{2.2}
$$

where $E_i = b\int_{-h/2}^{h/2} Ez^i dz$, $q_l^* = q_l + q_{kl}$, $i=0,1,2$, $l=1,2.$ Stress q_l^* acting on the beam is generated by the external periodic load $q = q_0 \sin(\omega_{lp} t)$ acting on the upper beam and contact stresses q_{kl}. Jump phenomena are included in the beam interaction phenomena. Contact problems are defined via formula $q_{kl} = (-1)^l K \frac{E}{h}(w_1 - \delta - w_2)\psi$, $(l = 1,2)$, where K is the proportional factor between the contact pressure and the clamp, $\psi = \frac{1}{2}[1 + sign(w_1 - \delta - w_2)]$. Observe that an occurrence of the multiplier ψ in equations of beam motion implies evidence of the new nonlinearity type, i.e. design nonlinearity, and hence in the deformation process the computational scheme is changed. Zero value initial conditions are introduced. Although the boundary conditions can be arbitrarily taken, but we limit the considerations to the following ones:

$$u_l(0,t) = u_l(1,t) = w_l(0,t) = w_l(1,t) = w_l''(0,t) = w_l''(1,t) = 0;\ l = 1,2. \tag{2.3}$$

Diagram of the beam material deformation $\sigma_{li}(e_{il})$ can be taken arbitrarily. In this work the following one is applied:

$$\sigma_{li} = \sigma_{ls}\left[1 - \exp\left(-\frac{e_{il}}{e_{sl}}\right)\right]. \tag{2.4}$$

3. Methods of analysis

Integration of equations of motion and taking into account geometric and physical nonlinearities is carried out with a help of the Finite Difference Method (FDM). In order to apply the method of variable stiffness parameters [12], the beam is divided into n_z layers regarding beam thickness. In addition, on each time step the method of variable stiffness has been applied for each node to define all necessary relations.

The so far described algorithm aimed on finding of solutions modeling the beam vibrations has been validated regarding convergence of the spatial and time meshes by considering a stationary problem and using a relaxation method [13], and the obtained optimal parameters follow: $n_x = 30$, $t = 2 \cdot 10^{-5}$, $n_z = 12$.

Solutions to problem (2.2) yield the values $w_l(x,t)$, $u_l(x,t)$, which are analyzed via qualitative theory of differential equations and dynamical systems. In particular, phase portraits, Poincaré maps, FTT (Fast Fourier Transform), the largest Lyapunov exponent, the contact pressure, time and space histories, energetic spectra of the wavelet transform, and the phase differences are reported in the case of phase synchronization regimes.

Wavelet analysis [14] allows investigating of frequency characteristics of a signal versus time, because the signal character can be essentially changed within time, and hence a direct application of the FFT can yield erroneous conclusions. In references [6]-[8] it has been shown that among various wavelets the Morlet wavelets are most suitable for investigation of the studied mechanical systems. Wavelet transformation allows for computation of the phase $\varphi_s(t) = \arg W(s,t),$ where $W(s,t) = |W(s,t)|\, e^{i\phi_s(t)}$ for each of the time interval s, i.e. each of the time intervals can be described via an associated phase.

It should be emphasized that during the phase synchronization a locking of chaotic signals occurs, whereas the amplitudes of the signals remain independent from each other. Phase locking yields frequency locking, and the frequency of a chaotic signal is defined as the averaged velocity of the phase changes $\langle \varphi(t) \rangle$. All time intervals associated with the energy transmission are associated with synchronization. Phase synchronization occurs on the time intervals *s:* $|\varphi_l - \varphi_{l+1}| < const,$ where φ_l are continuous beam phases, corresponding to the time intervals s. In the further reported numerical examples the so far mentioned zones are marked in black (drawing e).

4. Analysis of results and examples

We consider two beams coupled via boundary conditions of the following form:

$$u_l(1,t) = u_l(l,t) = w_l(1,t) = w_l(l,t) = \frac{\partial w_l^2(0,t)}{\partial x^2} = \frac{\partial w_l^2(0,t)}{\partial x^2} = 0;\ l = 1,2.$$

Boundary conditions:

$$w_l(x,0) = \frac{\partial w_l\,(x,0)}{\partial t} = u_l(x,0) = \frac{\partial u_l\,(x,0)}{\partial t} = 0;\ l = 1,2.$$

Transversal load is governed by formula

$$q_l = q_{0l} \sin\left(\omega_{lp} t\right). \tag{4.1}$$

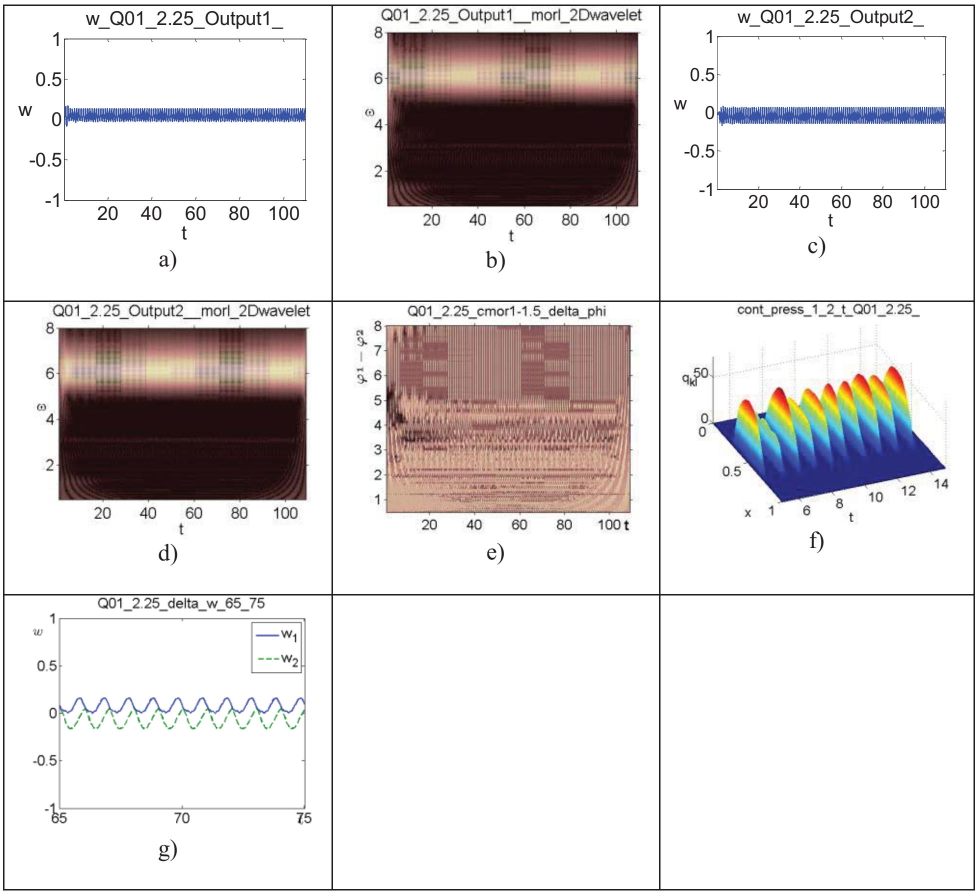
The load (4.1) is acting only on one upper side of the beam, whereas the bottom located beam moves only through interaction with the upper beam. Excitation frequency $\omega_{1p} = 6.28,$ the constraint between beams $\delta = 0.05,$ whereas the damping coefficient of the surrounding medium $\varepsilon = 1.4.$ Depending on the beam deflection magnitude and beam material type one may distinguish two groups of the problems regarding two beams: A – both beams are elastic (beam material is linear), B – both beams are made from a non-linear material.

Below are reported results obtained while investigating problems A and B. In Tables 1-4 vibrations of the beam middle surface point (x=0.5) of the first and second beam are shown and the following notation is applied: *a*,*c* – vibration time history of each beam; *b*,*d* – time-frequency wavelet spectrum; *e* – time-frequency phase difference beams localization to follow phase synchronization;

f – space-time contact pressure distribution; *g* – simultaneous vibrations of the middle surface point of the first and second beam in the given time intervals.

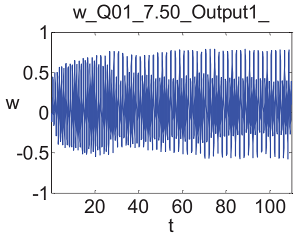
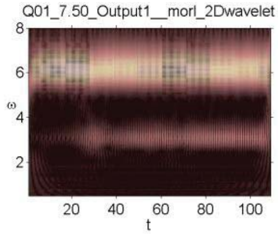
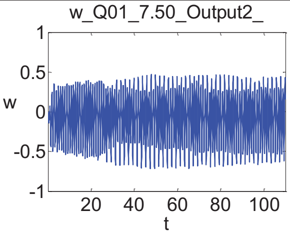
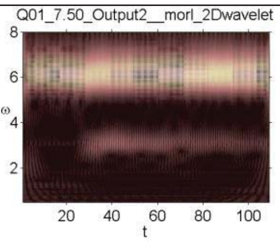
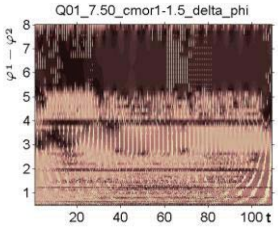
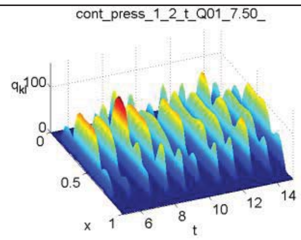
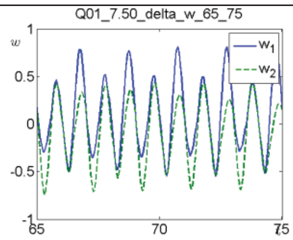
Problems of type A. Load amplitude action $q_{01} = 2.25$ on the upper beam results in a small deflection ($w < 0.25h$, and hence it means that the geometric non-linearity does not play here a crucial role). Owing to the reported wavelet-spectra b), d) vibrations are manifested only by one frequency (equal to the excitation load frequency), and phase synchronization e) is not observed. Comparison of deflection magnitudes imply a regular dynamics (g).

Table 1. Dynamical characteristics for $q_{01} = 2.25$ (see text for more details).



For $q_{01} = 7.5$ (Table 2) the wavelet spectra present a two-frequency quasi-periodic vibrations of both beams, but phase synchronization takes place in the vicinity of the external load frequency.

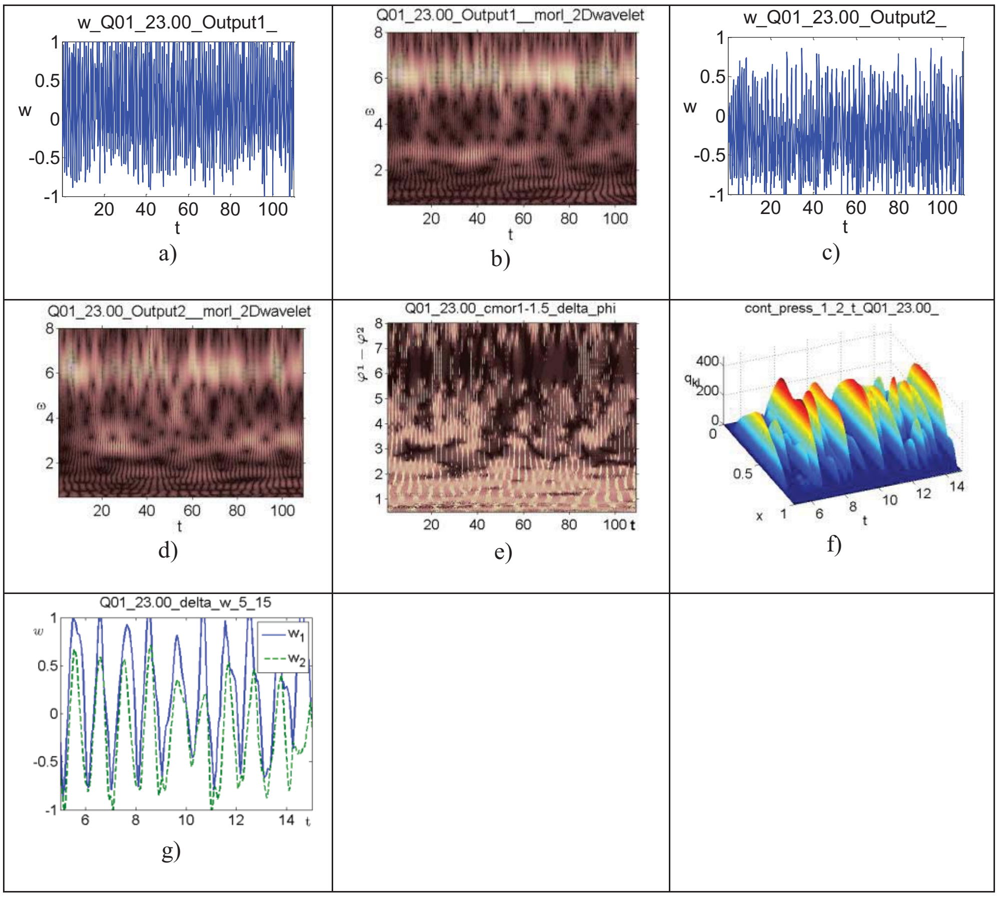
Table 2. Dynamical characteristics for $q_{01} = 7.5$ (see text for more details).

 a)	 b)	 c)
 d)	 e)	 f)
 g)		

Analysis of the contact pressure shows that it exhibits a slight chaotic behavior. The contact zones boundaries with sharp peaks are visible, which are characteristic for the kinematic Euler-Bernoulli model of the contact interactions between two beams. Time histories of vibrations of two beams (g) imply the synchronization phenomenon and signal amplitudes are coupled with each other, i.e. we can speak about a full synchronization (phase and amplitude locking).

Increase of the external load amplitude up to $q_{01} = 23.0$ (Table 3) forces the system to exhibit a chaotic dynamics of both beams and the phase synchronization appears not in the vicinity of the excitation frequency as we demonstrated earlier, but on the whole frequencies interval. The related drawing of the phase synchronization (e) exhibits different evaluation in time of the synchronization process. Frequencies matched with synchronization essentially change keeping constant values in the neighborhood of the excitation frequency. Contact pressure (f) possesses clearly exhibited chaotic behavior. Deflections of both beams (g) are close to each other on the whole time interval, i.e. there is a full synchronization either regarding the phases on signals.

Table 3. Dynamical characteristics for $q_{01} = 23.0$ (see text for more details).



Problems of type B. Results reported in Table 4 illustrate a transition of vibrations from asynchronous to synchronous, whereas the chart of the phases difference implies that the synchronization takes place not on the excitation frequency, but in the frequencies interval [6; 2.5]. In Table 4 (f) – (h) one may follow how the synchronization develops in time through the deflection changes. The phase synchronization begins at t >40, and it takes place on the frequency $\omega = 4.0$. For t >46 a full synchronization takes place in the wide interval of the analyzed frequencies.

In the problems with inclusion of the physical material non-linearity a dynamical stability loss of the first beam occurs, and the beam starts to vibrate in the neighborhood of a new equilibrium configuration without an interaction with the second beam. For the damping coefficient $\varepsilon = 1.4$ the second beam vibrations are eventually damped. In result of those investigation one may expect, that there is a possibility to control chaotic vibrations of the beams via either the constraint between beams or the beam material choice. In Table 5 it is shown how vibrations of one beam are damped.

Table 4. Dynamical characteristics for $q_{01} = 2.5$ (see text for more details).

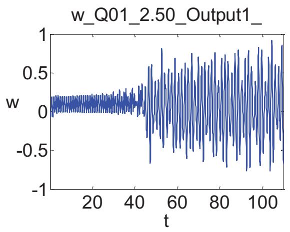
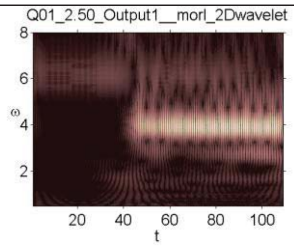
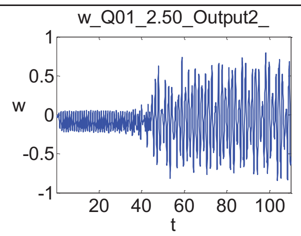
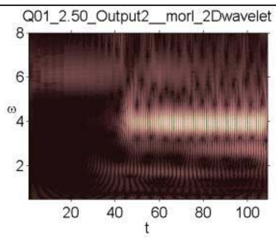
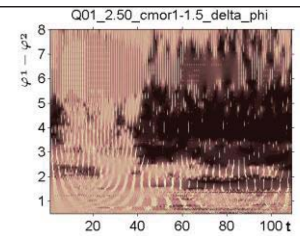
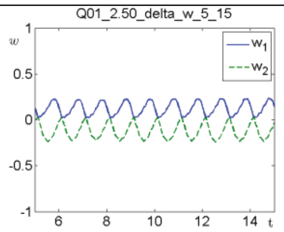
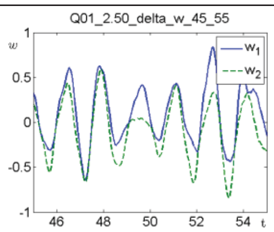
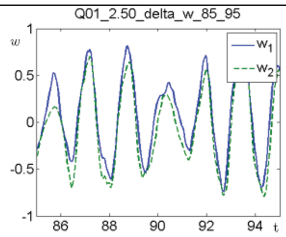
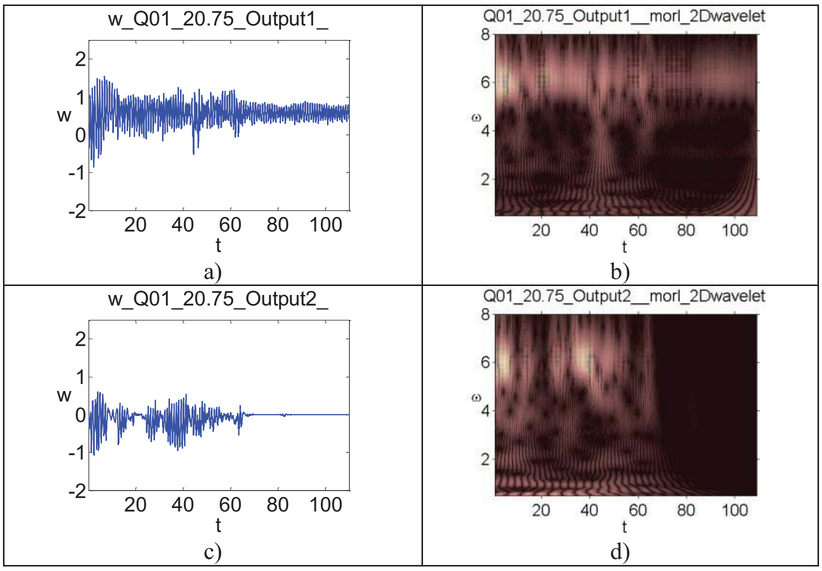
 a)	 b)	 c)
 d)	 e)	 f)
 g)	 h)	

Table 5. Time histories and wavelet evolutions of the beams for $q_{01} = 20.75$



5. Conclusions

In this work first a theory of non-linear vibrations and chaotic synchronization of the contact interaction of multi-layer beams has been developed. The buckling phenomenon (dynamic stability loss) has been detected, which can be understood as splitting of the multi-layer beams package. In technology and engineering field this phenomenon can be found in design of rails. The proposed algorithm can be applied for investigation of different type of problems including non-linear dynamics of plates, shells and structural members. Application of the wavelet analysis allows to understand and study the phase synchronization of chaotic vibrations, as well as to analyze other types of synchronization. Based on the obtained results it is recommended controlling chaotic vibrations with a help of the beams constraint variation and the proper choice of the beam material. The proposed theory and computational method can be extended to a study of coupled multi-layer beams.

Acknowledgement.

This work has been supported by the grant RFFFI No. 12-01-31204, 10-08-91332-ННИО_a, and the National Science Centre of Poland under the grant MAESTRO 2, No. 2012/04/A/ST8/00738, for years 2013-2016.

References

1. Awrejcewicz J., Krysko V.A.: *Introduction to Asymptotic Methods*. Taylor & Francis Group , Boca Raton. 2006.
2. Awrejcewicz J., Krysko V.A., Krysko A.V.: *Thermodynamics of Plates and Shells*. Springer, New York, 2007.
3. Krysko V.A., Awrejcewicz J.: *Chaos in Structural Mechanics*. Springer, New York, 2008.
4. Krysko V.A., Shchekutorova T.V.: Chaotic vibrations of conical shells. *Izvestia RAS*. *MTT* 4, 2004, 140-150 (in Russian).
5. Krysko V.A., Zhigalov M.V., Saltykova O.A., Desiatova A.S.: Dissipative dynamics of geometrically non-linear Euler-Bernoulli beams. *Izvestia RAS. MTT* 6, 2008, 128-136 (in Russian).
6. Krysko V.A., Papkova I.V., Soldatov V.V.: Analysis of non-linear chaotic vibrations of shallow rotational shells using wavelets. *Izvestia RAS. MTT* 1, 2010, 107-117 (in Russian).
7. Krysko V.A., Zhigalov M.V., Saltykova O.A., Soldatov V.V.: Investigation of chaotic vibrations of beams using the Sheremetev-Pelekh model and wavelets. *Problems of Machine Design and Durability*, 2010, 12-17 (in Russian).
8. Krysko V.A., Krysko A.V., Saltykova O.A., Zhigalov M.V.: On the influence of transversal shears on chaotic dynamics of elastic beams. *PMTF*, 52(5), 2011, 186-193 (in Russian).
9. Awrejcewicz J., Saltykova O., Krysko A., Mrozowski J., Zhigalov M.: Analysis of regular and chaotic dynamics of the Euler-Bernoulli using finite difference and finite element methods. *Acta Mechanica Sinica*, 27(1), 2011, 36-43.
10. Kantor B.Ya.: *Contact Problems of Non-Linear Theory of Rotational Shells*. Naukova Dumka, Kiev, 1990 (in Russian).
11. Iliushin A.A.: *Plasticity. Fundamentals of General Mathematical Theory of Plasticity*. AN SSSR, Moscow, 1963 (in Russian).
12. Birger I.A.: General Methods of Solutions in Theory of Plasticity. *PMM*, 15(6), 1951, 765-770 (in Russian).
13. Feodosev V.I.: On the solution of non-linear stability problems of deformable systems. PMM, 27(2), 1963, 265-274 (in Russian).
14. Debnath L.: *Wavelet Transforms and Their Applications*. Birkhäuser Verlag, Boston, 2002.
14. Pikovskii A., Rosenblum M., Kurtis Yu.: *Synchronization. Fundamental Non-Linear Phenomenon*. Tekhnosfera, Moscow, 2003 (in Russian).

J. Awrejcewicz, Department of Automation, Biomechanics and Mechatronics, Łódź University of Technology, POLAND, 90-924 Lodz, 1/15 Stefanowski St.; awrejcew@p.lodz.pl

M.V. Zhigalov , Saratov State Technical University, Department of Mathematics and Modeling, RUSSIAN FEDERATION, 410054 Saratov, Politehnicheskaya 77; zhigalovm@yandex.ru

U. Nackenhorst, Institute Mechanics and Computational Mechanics, Leibniz University Hannover, Appelstr. 9A, Hannover Germany 30167 nackenhorst@ibnm.uni-hannover.de

V. A. Krysko-jr., student of third year students and the Faculty of Architecture and Civil Engineering, Saratov State Technical University, RUSSIAN FEDERATION, 410054 Saratov, Politehnicheskaya 77; vadimakrysko@gmail.com

I.V. Papkova, Saratov State Technical University, Department of Mathematics and Modeling, RUSSIAN FEDERATION, 410054 Saratov, Politehnicheskaya 77; ikravzova@mail.ru

A.V. Krysko, Saratov State Technical University, Applied Mathematics and Systems Analysis, RUSSIAN FEDERATION, 410054 Saratov, Politehnicheskaya 77; anton.krysko@gmail.com