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Shell Structures: Theory and Applications

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Editors

Wojciech Pietraszkiewicz
The Szewalski Institute of Fluid-Flow Machinery, Polish Academy of Sciences, Gdańsk, Poland

Ireneusz Kreja
Faculty of Civil and Environmental Engineering, Gdańsk University of Technology, Gdańsk, Poland

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Investigation of nonlinear dissipative chaotic dynamics of plates and shells

V.A. Krysko, M. Zhigalov, V. Soldatov, S. Mitskevitch, E.S. Kuznetsova & K.F. Shagivaleev
Department of Mathematics and Modeling, Saratov State Technical University, Russia

J. Awrejcewicz & J. Mrozowski
Department Automatics and Biomechanics, Technical University of Łódź, Poland

ABSTRACT: A general theory of chaotic nonlinear dissipative dynamics of shallow shells and plates is developed and validated via a computational example.

1 INTRODUCTION

Some important aspects of nonlinear dynamics of continual mechanical objects as well as their modeling, governing equations and computational approaches are addressed in references Awrejcewicz & Krysko (2003), Awrejcewicz *et al.* (2007), Awrejcewicz & Krysko (2008), Awrejcewicz *et al.* (2008), Sun & Zhang (2001). In our work the governing partial differential equations, either in hybrid form or regarding displacements, modeling of the studied shells and plates dynamics are derived through application of the Hamilton principle and applying the hypotheses of Kirchhoff, Timoshenko and Sheremetev-Pelekh. In other words the following assumptions are made: (i) a normal to a middle surface remains normal after deformation process; (ii) a normal rotates during deformation process; (iii) a normal being perpendicular to a middle shell surface is rotated and curved during deformation process.

Geometrical nonlinearities are included into our theory using assumptions introduced by T. von Kármán. Material of a shell-type construction can be isotropic, transversally isotropic and orthotropic, and an object analyzed is considered elastic. The developed theories include axially (non-axially) symmetric circled shells, rectangular and sector-type shells as well as closed cylindrical shells. In order to reduce the evolutionary PDEs (governing dynamics of flexible shells) to a system of ODEs, the Ritz and Bubnov-Galerkin methods in higher approximations are applied, as well as the methods of finite differences of orders h^2, h^4, and finite element method (FEM) in the Bubnov-Galerkin form. The obtained ODEs are solved via various variants of the Runge-Kutta methods. The convergence of the mentioned numerical algorithms is rigorously discussed and a priori estimations as well as computation validations are given. Nonlinear vibrations of the mentioned continuous objects are studied via qualitative theory of differential equations using the following classical analysis of: time histories, FFT (Fast Fourier Transform), wavelet transforms, Poincaré sections, autocorrelation functions, as well as Lyapunov largest exponent computation. The infinite dimensional original problems are treated as those with many degrees-of-freedom. The same problem is solved via a few different approaches in order to get reliable results.

2 THEORY AND COMPUTATIONAL EXAMPLE

As an example we study chaotic vibrations of a cylindrical panel with a rectangular projection plane and subjected to transversal sign-changeable load action and satisfying the assumption (i). Non-dimensional equations of motion and deformation compatibility equations are as follows:

$$\frac{1}{12(1-\nu^2)}\left[\nabla_\lambda^4 w\right] - L(w,F) - \nabla_k^2 F + \frac{\partial^2 w}{\partial t^2} + \varepsilon\frac{\partial w}{\partial t} + \nabla_p^2 w - q(t) = 0, \tag{1}$$
$$\nabla_\lambda^4 F + \frac{1}{2}L(w,w) + \nabla_k^2 w = 0,$$

where:

$$\nabla_\lambda^2 = \lambda^{-1}\frac{\partial^2}{\partial x^2} + \lambda\frac{\partial^2}{\partial y^2},\quad \nabla_k^2 = k_x\frac{\partial^2}{\partial y^2} + k_y\frac{\partial^2}{\partial x^2},$$
$$\nabla_p^2 = p_x\frac{\partial^2}{\partial y^2} + p_y\frac{\partial^2}{\partial x^2},$$

and $L(w,w)$, $L(w,F)$ – are the well-known nonlinear operators. Non-dimensional quantities are introduced in the following way:

$$x = a\bar{x},\ \ y = a\bar{y},\ \ k_x = \bar{k}_x\frac{2h}{b^2},\ \ k_y = \bar{k}_y\frac{2h}{a^2},$$
$$q = \bar{q}\frac{E(2h)^4}{a^2b^2},\ \ p_x = \bar{p}_x\frac{E(2h)^3}{b^2}, \tag{2}$$
$$p_y = \bar{p}_y\frac{E(2h)^3}{a^2},\ \ \tau = \frac{ab}{2h}\sqrt{\frac{\rho}{Eg}},\ \ \lambda = \frac{a}{b},$$

where a, b denote shell dimensions regarding x and y, respectively; t denotes time; ε – linear damping coefficient; F – stress (Airy's) function; w – deflection function; h – shell thickness; $\nu = 0.3$ – Poisson's coefficient; g – gravity acceleration; E – Young's modulus; $p_x(y,t) = p_x^0 \sin(\omega_p t)$, $p_y(x,t) = p_y^0 \sin(\omega_p t)$ – longitudinal loads; k_x and k_y – shell curvature regarding x and y, respectively; $q(x,y,t) = q_0 \sin(\omega_p t)$ – transversal load sign-changeable (bars over non-dimensional quantities are omitted in equations (1)). The governing equations are supplemented by initial and boundary conditions. In the latter case we apply simple supports on flexible non-stretched ribs in a tangential plane.

In order to solve the obtained governing equations the Bubnov-Galerkin method in higher approximations is applied, i.e. we assume that

$$w = \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} A_{ij}(t)\varphi_{ij}(x,y),$$
$$F = \sum_{i=0}^{N_1}\sum_{j=0}^{N_2} B_{ij}(t)\psi_{ij}(x,y). \quad (3)$$

In order to define approximated values of w and F, the following co-ordinate system of functions $\{\varphi_{ij}(x,y), \psi_{ij}(x,y)\}$ $(i,j = 0,1,2\ldots)$ is substituted into (1). They should be linearly independent, continuous together with their derivatives up to the fourth order in domain Ω $\{0 \le x \le a;\ 0 \le y \le b\}$, and they must satisfy the boundary conditions.

After application of the Bubnov-Galerkin procedure in higher approximations, the following second order systems of ODEs and algebraic equations are obtained:

$$\mathbf{G}(\ddot{\mathbf{A}} + \varepsilon\dot{\mathbf{A}}) + \mathbf{SA} + \mathbf{C}_1\mathbf{B} + \mathbf{D}_1\mathbf{AB} = \mathbf{Q}q + \mathbf{H}_1,$$
$$\mathbf{C}_2\mathbf{A} + \mathbf{PB} + \mathbf{D}_2\mathbf{AA} = \mathbf{H}_2, \quad (4)$$

where: $\mathbf{G} = \|G_{ijkl}\|$, $\mathbf{S} = \|S_{ijrskl}\|$, $\mathbf{C}_1 = \|C_{1ijkl}\|$, $\mathbf{C}_2 = \|C_{2ijkl}\|$, $\mathbf{D}_1 = \|D_{1ijrskl}\|$, $\mathbf{D}_2 = \|D_{2ijrskl}\|$, $\mathbf{P} = \|P_{ijkl}\|$, – square matrices of dimension $2 \cdot N_1 \cdot N_2 \times 2 \cdot N_1 \cdot N_2$, $\mathbf{A} = \|A_{ij}\|$, $\mathbf{B} = \|B_{ij}\|$, $\mathbf{Q} = \|Q_{ij}\|$ – matrices of dimension $2 \cdot N_1 \cdot N_2. \times 1$.

The second equation of system (4) is solved regarding matrix $\mathbf{B}$ via an inversed matrix method on each time step:

$$\mathbf{B} = \left[-\mathbf{P}^{-1}\mathbf{D}_2\mathbf{A} - \mathbf{P}^{-1}\mathbf{C}_2\right]\mathbf{A} + \mathbf{P}^{-1}\mathbf{H}_2. \quad (5)$$

Multiplying the first equation of (4) by $\mathbf{G}^{-1}$, the following Cauchy problem for first order ODEs is obtained:

$$\dot{\mathbf{R}} = -\varepsilon\mathbf{R} + \mathbf{G}^{-1}\mathbf{D}_1 AB - \mathbf{G}^{-1}\mathbf{S}A -$$
$$-\mathbf{G}^{-1}C_1 B + q\mathbf{G}^{-1}\mathbf{Q} + \mathbf{G}^{-1}\mathbf{H}_1, \quad (6)$$
$$\dot{\mathbf{A}} = \mathbf{R}.$$

The obtained differential equations are solved using the fourth order Runge-Kutta method.

As a computational example we study the square panel with the fixed parameters ($\lambda = 1, K_Y = 36$, $K_X = 0$) and with the following boundary conditions:

$$w = M_x = N_x = \varepsilon_y = 0, \quad (x \leftrightarrow y)$$
$$\text{for} \quad x = 0;1, \ y = 0;1, \quad (7)$$

and the following initial conditions

$$w|_{t=0} = 0, \dot{w}|_{t=0} = 0. \quad (8)$$

The following loading parameters acting on the panel are applied: compressing forces $p_y = 3p_x$, $p_x = 0.5$, and the transversal load $q = q_0 \sin\omega_p t$ ($\omega_p = 17$). For the studied case we have $\varphi_{ij}(x,y) = \psi_{ij}(x,y) = \sin(i\pi x)\sin(j\pi y)$.

In Table 1 the following indicators are reported: time history (signal), the largest Lyapunov exponent, FFT and 2D Morlet's wavelet spectra (larger values of the wavelet coefficients correspond to more bright colors) for the middle panel point (0,5;0,5). First row corresponds to one term approximation (in this case we deal with a Duffing-type approximation), whereas second row corresponds to 25 approximating series terms for the same amplitude of transversal load. One may see from the Table 1 that the obtained results depend essentially on the used approximation terms. For instance, in the case of one term approximation a transition into chaotic state is realized via classical Feigenbaum scenario through a period doubling sequence, contrary to the results associated with 25 terms approximation.

3 RESULTS

Let us briefly emphasize some interesting nonlinear behavior of the studied object. For the load amplitude $q_0 = 3530$ and taking 25 terms the largest Lyapunov exponent is negative. Hence, we deal with regular dynamics. However, the FFT spectrum as well as the wavelet-spectrum are similar to that of chaotic dynamics. Observe that energy associated with a driven frequency is essentially higher than energies sum of the remaining frequencies (it is validated by wavelet-spectrum), therefore we have rather regular but complex vibrations instead of chaotic ones.

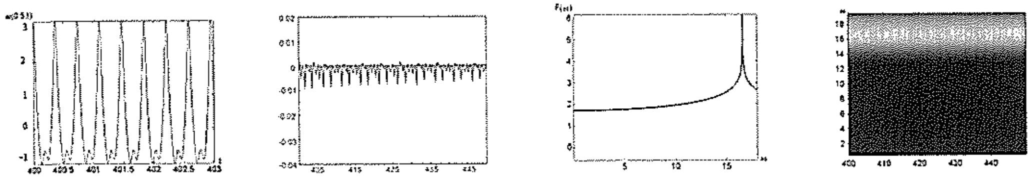
For $q_0 = 3976{,}5$ and for one term approximation, one observes two period doubling bifurcation points and the largest Lyapunov exponent is negative. However, taking into account 25 approximating terms vibrations are chaotic, which is proved by the Fourier spectrum, the wavelet spectrum and the Lyapunov exponent value. For $q_0 = 5000$ for both approximations we have chaotic vibrations (in the case of one term approximation chaos is associated with two frequencies $\omega_1 = 5{,}5$ and ω_p, whereas in the case of 25 terms approximation chaos is associated with external frequency ω_p).

4 CONCLUSIONS

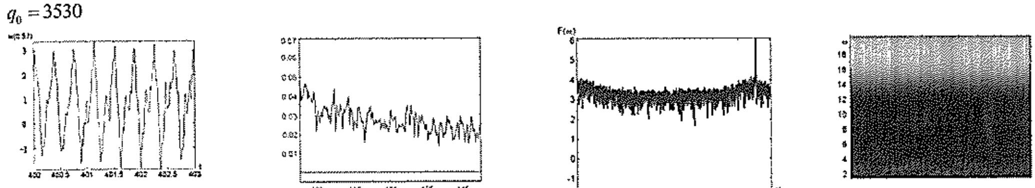
The following main conclusion results from our analysis: reduction of a continuous system into one

Table 1. Time evolution of signals (time histories), Lyapunov exponents, Fourier spectra and wavelet spectra for different values of excitation amplitude q_0.

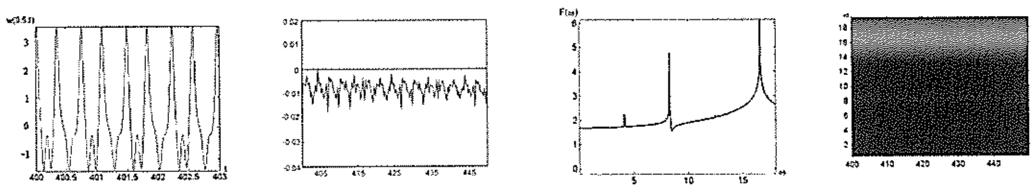
Signal $w(0{,}5;t)$ | Lyapunov exponent | Fourier spectrum | Wavelet-spectrum



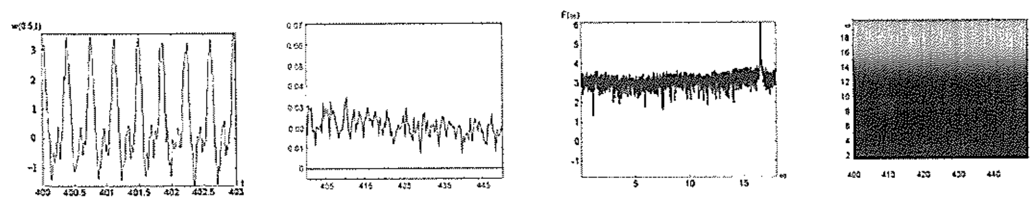
$q_0 = 3530$



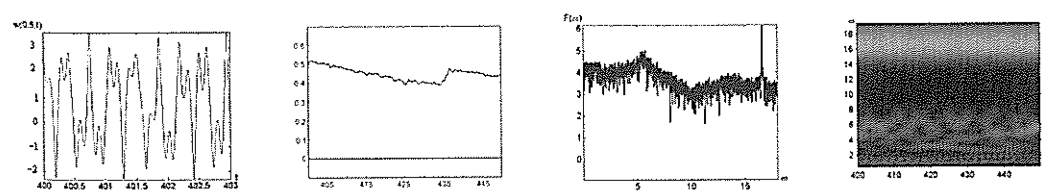
$q_0 = 3530$



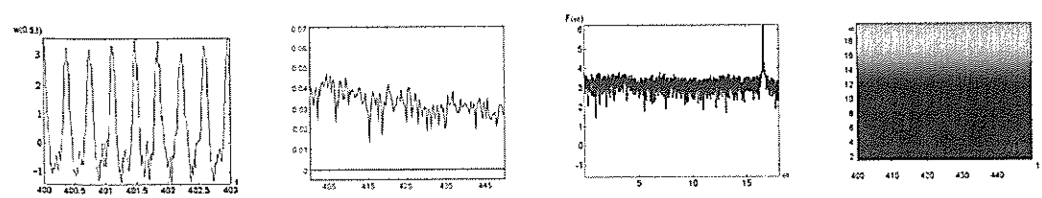
$q_0 = 3976{,}5$



$q_0 = 3976{,}5$



$q_0 = 5000$



$q_0 = 5000$

Duffing-type equation yields improper results regarding its chaotic dynamics. In addition, our computational results showed that the system transition into chaos is realized via various scenarios (one term approximation corresponds to the Feigenbaum scenario, whereas higher order approximation allows to detect the modified Ruelle-Takens-Newhouse scenario).

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